Faults in Michelson-Morley

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The Michelson-Morley experiment is one of the best-known experiments in the fields of physics and astrophysics, and arguably one of the most important. Without it, and without the Lorentz factor which it made popular, such concepts as *length contraction*, *time dilation*, *mass increase* and *frames of reference* would possibly never have been developed.

The present paper has the following objectives:

1) After giving a brief background and run-up to the experiment, to do all of the (easy!) maths. This is essential to understanding the experiment.
2) To then identify various types of error associated with Michelson's argumentation and to address these.
3) To show why *length contraction* is not a "solution" to the Michelson-Morley experiment.
4) And finally to identify a new and extremely important error in Michelson's mathematics.

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In 1887 the physicist Albert Michelson (1852-1931) and the chemist Edward Morley (1838-1923) conducted the now-famous Michelson-Morley experiment in Cleveland, Ohio, America.

This experiment, however, was actually the sequel to an earlier experiment conducted by Michelson. And since it is hardly possible to fully understand the latter without the former, we will begin here with the first of these two experiments and papers.[1]

In 1881 Michelson was in Potsdam, Germany, where the first experiment took place. This attempted to verify the existence of the "ether" - a medium in the vacuum of space through which light was thought to move in waves, similar to the motion of sound in the media of air, water, etc. - on the basis that, given the obvious fact that the earth was not stationary within the ether (as was apparent from its orbital motion around the sun), differences in the time required for the light to travel two distances of equal length, but in different directions, should be detectable.

The light used was emitted from a single source, then split into two beams, the times taken by these beams to travel a given distance (and back again) in these two

[1] *The Relative Motion of the Earth and the Luminiferous Ether*, Albert Abraham Michelson, American Journal of Science, 1881, 22: 120-129; and *On the Relative Motion of the Earth and the Luminiferous Ether*, Albert Abraham Michelson and Edward Morley, American Journal of Science, 1887, 34 (203): 333-345.

directions, one in the direction of the motion of the earth and one perpendicular to this, both on the horizontal plane, being compared.

The experiment failed, however, to detect any significant difference in the times taken by these beams to return.

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This result took those scientists acquainted with the experiment by surprise, the implication being that, no matter how fast or how slow one travelled, light always seemed to reach one from all directions at the same speed.

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The 1881 Experiment and Paper

For a quick run-through, Michelson's first paper can be subdivided into the following 12 segments (a to l), some of which will then be looked at more closely when examining the maths:

a) The 1881 paper begins with a very short introduction to the topic (5 lines), explaining the ether and discounting the significance of "indices of refraction" in this context.
b) A short list of assumptions follows.
c) Then a brief calculation is made to show how to determine "the velocity of the earth's motion through the ether."
d) Michelson next refers to James Clerk Maxwell (1831-1879), the Scottish physicist widely regarded as the scientist of the 19th century. Maxwell had previously considered the experiment and had decided that it would not be possible to conduct it conclusively, given the (low) levels of instrumental accuracy in those days. Michelson disagrees and presents his alternative.
e) A mathematical argument follows, intended to show that the sought result can indeed be calculated.
f) Michelson next takes account of the motion of the earth in the context of the experiment.
g) Then he rotates his experiment through 90° for a comparative calculation.
h) Next he outlines the experimental approach at some length, also giving two diagrams of the setup.
i) This is followed by the actual history of the experiment as attempted by him, explaining the results obtained in several series of observations, further diagrams being given and his results also being shown in tabular form.
j) Then he critically assesses the results obtained.
k) He next summarizes these results stating his conclusions.
l) Finally he closes, expressing his thanks for support received.

(Anyone interested in the full article should "Google" it out of the Web)

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Since this paper is primarily concerned with the concepts applied and the mathematics employed, it restricts itself, in what follows, to segments b, c, e, f and g.

b) Assumptions

b1) Space - or the ether - is at rest.
b2) The time required for light to pass from one point to another on the earth's surface depends on the direction in which it travels.
b3) V = velocity of light (nowadays we use c).
b4) v = the speed of the earth with respect to space.
b5) D = the distance between two given points (on the earth).
b6) d = the distance through which the earth moves while light travels from one of the given points to the other (i.e. on the outbound journey).
b7) d_1 = the distance through which the earth moves while light travels from one of the given points to the other, this time in the opposite direction (i.e. on the return journey).

c) Initial Calculations

Given that the trajectory of the line joining the two points coincides with (= is parallel to) the trajectory of the earth's motion, that T = the time required for light to pass from the one point to the other (in the direction of the earth's motion), that T_1 = the time required for light to pass from the one point to the other, though now in the opposite direction (i.e. against the direction of the earth's motion), and that T_0 = the time required for light to travel from one point to the other if the earth were at rest, then:

c1) $T = (D+d)/V = d/v$ and $T_1 = (D-d)/V = d_1/v$

c2) From these we find $d = Dv/(V-v)$ ... and $d_1 = Dv/(V+v)$ ...

c3) whence $T = D/(V-v)$ and $T_1 = D/(V+v)$

Implying that

c4) $T - T_1 \approx 2T_0 (v/V)$

so that

c5) $v = V(T - T_1) / 2T_0$

If it were now possible to measure $T - T_1$ [says Michelson], since V and T_0 are known, we could find v the velocity of the earth's motion through the ether.

e) Further Calculations

The total time taken in going and returning is therefore

e1) $T + T_1 = D/(V-v) + D/(V+v) = 2DV/(V^2-v^2)$

As for the other direction, light travelling at right angles to the earth's motion would be entirely unaffected and the time of going and returning would be

e2) $2D/V = 2T_0$.

The difference between the times $T + T_1$ and $2T_0$ is thus

e3) $2DV (1/(V^2 - v^2) - 1/V^2) = 2DV (v^2/V^2(V^2-v^2)) = \tau$

e4) or *nearly* $2T_0 (v^2/V^2)$.

In the time τ the light would travel a distance of

e5) $V\tau = 2VT_0(v^2/V^2) = 2D(v^2/V^2)$.

That is, the actual distance travelled by the light in the first case is greater than in the second by the quantity

e6) $2D(v^2/V^2)$.

f) Practical Calculations

(See "Proofs" section below)

g) Rotation Through 90°

(See "Proofs" section below)

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Proofs

It is important to address these initially in the context of the 1881 experiment, since it was an error made here by Michelson that led to the 1887 experiment in the first place.

Michelson himself doesn't actually give these proofs, but the maths is so easy and the proofs are so obvious that we can reconstruct them as follows (the proofs being shown in green):

(c1) $T = (D+d)/V = d/v$ and $T_1 = (D-d)/V = d_1/v$

This is clear since the time taken for light to cover the distance between the two points in the direction of the earth's motion is $D/V + d/V = (D+d)/V$ and this time is equivalent (= identical) to the time taken for distance d at velocity v, i.e. d/v. As for the time taken for the light to travel in the opposite direction, this is $D/V - d/V = (D-d)/V$ which is equivalent (= identical) to the time taken for distance d_1 at velocity v, i.e. d_1/v.

(c2) From these we find $d = Dv/(V-v)$ ... and $d_1 = Dv/(V+v)$ ...

The proof will be given later.

(c3) whence $T = D/(V-v)$ and $T_1 = D/(V+v)$

Proof 1	$(D+d)/V$	= T, taken from (c1)
=	$(D+Dv/(V-v))/V$	substituting Dv/(V-v) for d, from (c2)
=	$D/V + Dv/V(V-v)$	separating out the numerator
=	$(D(V-v) + Dv) / V(V-v)$	taking a common denominator
=	$(DV - Dv + Dv) / V(V-v)$	multiplying out the numerator
=	$DV / V(V-v)$	adding the numerator
✓	$D/(V-v)$	dividing by V, above and below
Proof 2	$(D-d)/V$	= T_1, taken from (c1)

=	$(D-Dv/(V+v))/V$	substituting $Dv/(V+v)$ for d_1, from (c2)
=	$D/V - Dv/V(V+v)$	separating out the numerator
=	$(D(V+v) - Dv) / V(V+v)$	taking a common denominator
=	$(DV + Dv - Dv) / V(V+v)$	multiplying out the numerator
=	$DV / V(V+v)$	adding the numerator
✓	$D/(V+v)$	dividing by V, above and below

Implying that

(c4) $T - T_1 \approx 2T_0 (v/V)$

Proof 3	$D/(V-v) - D/(V+v)$	$= T - T_1$, taken from (c3)
=	$D(V+v) - D(V-v) / (V-v)(V+v)$	taking a common denominator
=	$DV + Dv - DV + Dv / (V-v)(V+v)$	multiplying out the numerator
=	$2Dv / V^2 - Vv + Vv - v^2$	adding the numerator
=	$2Dv / V^2 - Vv + Vv - v^2$	multiplying out the denominator
=	$2Dv / (V^2-v^2)$	adding the denominator
≈	$2Dv / V^2$	eliminating v^2 from (V^2-v^2) in the denominator
≈	$(2D/V)(v/V)$	redistributing
✓	$2T_0(v/V)$	since $2T_0 = D/V$, by definition from c

so that

(c5) $v = V(T - T_1) / 2T_0$

Proof 4	$T - T_1 = 2T_0 (v/V)$	taken from (c4)
⇒	$2T_0 (v/V) = T - T_1$	swapping sides
⇒	$v/V = (T - T_1) / 2T_0$	dividing both sides by $2T_0$
✓	$v = V(T - T_1) / 2T_0$	multiplying both sides by V

If it were now possible to measure $T - T_1$ [says Michelson], since V and T_0 are known, we could find v, the velocity of the earth's motion through the ether.

e) Further Calculations

The total time taken in going and returning is therefore

(e1) $T + T_1 = D/(V-v) + D/(V+v) = 2DV/(V^2-v^2)$

Proof 5	$D/(V-v) + D/(V+v)$	adding T and T_1 from the equations of (c3)
Proof 6	$D/(V-v) + D/(V+v)$	Proof 5
=	$(D(V+v) + D(V-v)) / (V+v)(V-v)$	taking the same denominator and then adding
=	$(DV+Dv+DV-Dv) / (V^2 +Vv - Vv -v^2)$	multiplying out, both above and below
✓	$2DV/(V^2-v^2)$	adding, both above and below

As for the other direction, light travelling at right angles to the earth's motion would be entirely unaffected and the time of going and returning would be

(e2) $2D/V = 2T_0$.

The difference between the times $T + T_1$ and $2T_0$ is thus

(e3) $2DV (1/(V^2 -v^2) - 1/V^2) = 2DV (v^2/V^2(V^2-v^2)) = \tau$

Proof 7	$2DV/(V^2-v^2) - 2D/V$	from Proof 6 (result) minus (e2)
✓	$2DV (1/(V^2-v^2) - 1/V^2)$	extracting 2DV
& Proof 8	$2DV/(V^2-v^2) - 2D/V$	from Proof 6 (result) minus (e2)
=	$(2DV(V)-2D(V^2-v^2))/V(V^2-v^2)$	taking the same denominator and then adding
=	$(2DV^2-2DV^2+2Dv^2)/V(V^2-v^2)$	multiplying out, above
=	$2Dv^2/V(V^2-v^2)$	subtracting, above
✓	$2DV (v^2/V^2(V^2-v^2))$	multiplying by V, above and below

So with this Michelson had identified the difference (τ) between the time taken for the journey there and back in the direction of the earth's motion ($T+T_1$) and the time taken for the journey there and back perpendicular to the direction of the earth's motion ($2T_0$). He next argues that the values obtained for τ are

(e4) *nearly* $2T_0 (v^2/V^2)$, i.e. $\approx 2T_0 (v^2/V^2)$

Proof 9	$2DV (v^2/V^2(V^2-v^2))$	the result of Proof 8
=	$2DVv^2/V^2(V^2-v^2)$	removing the outer brackets
=	$2(D/V)v^2/(V^2-v^2)$	dividing by V^2, above and below
=	$2 T_0 v^2/(V^2-v^2)$	substituting $2T_0$ for 2D/V, as in (e2)
≈	$2 T_0 v^2/(V^2)$	eliminating v^2 from (V^2-v^2) in the denominator
✓	$2 T_0 (v^2/V^2)$	repositioning the brackets
=	τ	from (e3)

He next argues that in time τ the light would travel a distance of

(e5) $V\tau = 2VT_0(v^2/V^2) = 2D(v^2/V^2)$.

Proof 10	$2T_0 (v^2/V^2)$	= τ, as taken from (e4)
✓	$2VT_0 (v^2/V^2)$	multiplying τ by V, as here in (e5)
=	$2VD/V (v^2/V^2)$	substituting D/V for T_0, as in (e2)
✓	$2D(v^2/V^2)$	cancelling out the Vs

That is, the actual distance travelled by the light in the first case is greater than in the second by the quantity

(e6) $2D(v^2/V^2)$.

f) Practical Calculations

Considering the velocity of the earth in its orbit, the ratio $v/V \approx 1/10,000$ and $v^2/V^2 \approx 1/100,000,000$ millimetres or in wave lengths of yellow light, 2,000,000, then in terms of the same (yellow-light) unit, $2D(v^2/V^2) = 4/100$.

If, therefore, an apparatus is so constructed as to permit two pencils of light, which have travelled over paths at right angles to each other, to interfere, the pencil which has travelled in the direction of the earth's motion will in reality travel 4/100 of a wavelength further than it would have done, were the earth at rest. The other pencil being at right angles to the motion (of the earth) would not be affected.

g) Rotation Through 90°

If now the apparatus be revolved through 90° so that the second pencil is brought into the direction of the earth's motion, its path will have lengthened (by) 4/100 wavelengths. The total change in the position of the interference bands would be 8/100 of the distance between the bands, a quantity easily measurable.

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Now we can begin to take a critical look at the 1881 experiment and paper.

Scathing over f and g first, for someone like myself who has never seen an interferometer (the device used in Michelson's experiment), the arguments here relating to interference bands and the doubling of the magnitude with a 90° change in the directions of the arms are a bit difficult to follow. Fortunately this is not something we need to understand, so we will simply ignore it, too, and turn to the maths.

The essential problem found with this experiment and paper by a contemporary of Michelson's, the Dutch mathematician and physicist Hendrik Antoon Lorentz (1853-1928), was that the beam travelling in the perpendicular direction was not, as Michelson had claimed - in connection with (e2) and under f, for example - "entirely unaffected" by the motion of the earth.

Fig. 1 below shows Michelson's concept. The light from source s is split at a, one beam travelling the distances ac and ca in alignment with the motion of the earth, while the other travels the same (earth) distance, ab and ba, though in the perpendicular direction.

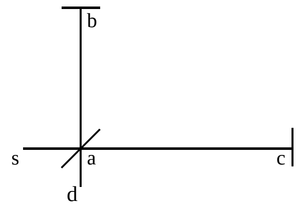

Fig 1
Michelson's initial (1881) view of the set-up

The length of ab and ac was Michelson's distance of D.

Lorentz believed instead that the "perpendicular" path travelled was that of the hypotenuse (AC), shown in Fig 2 below, and not AB (or ab in Fig. 1), as believed by Michelson.

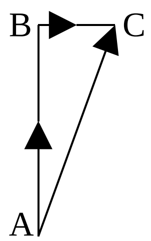

Fig. 2
Outbound journey in the perpendicular direction

Assuming the Lorentz criticism to be correct, this would mean that the "perpendicular" velocity would no longer be V in the direction AB, but would now be V in the direction AC (as shown in Fig. 2a, below). [Note that it must be V and not $\sqrt{(V^2+v^2)}$, since the latter value would imply a speed *greater* than the speed of light!]

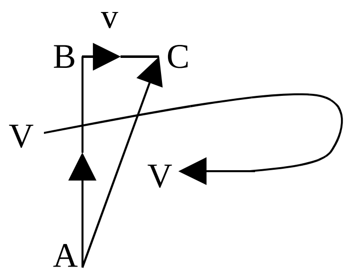

Fig. 2a
Outbound journey in the perpendicular direction

This in turn means that the strictly perpendicular component of velocity - i.e. between A and B - is (by Pythagoras) $\sqrt{(V^2-v^2)}$, where v is here the velocity of the earth. So we now have all three sides of the triangle "valued":

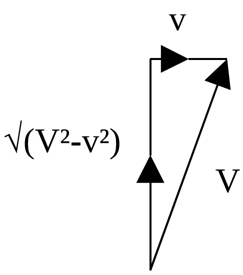

Fig. 2b
Outbound journey in the perpendicular direction

It is important to remember here, however, that the *real* velocity of light evinces itself along the hypothenus alone, the value for light in the perpendicular direction being a *fictitious* value, but one which nevertheless permits us to retain D and T_0 - for the moment, at any rate.

So whereas we previously had

$T_0 = D/V$

we now have

$T_0 = D/\sqrt{(V^2-v^2)}$.

In other words, in adapting the speed of light to the perpendicular we have increased the time required for this journey, $\sqrt{(V^2-v^2)}$ being smaller than V.

Lorentz claimed that this difference would half the value sought by Michelson in his 1881 experiment and paper, thereby casting doubts on Michelson's results.

Which caused Michelson to repeat his experiment with a more accurate set-up in 1887.

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The 1887 Experiment and Paper

In his 1881 paper Michelson begins and ends with references to aberration that otherwise appear to have nothing whatsoever to do with his experiment. At the outset of the 1887 paper, by contrast, he states that the entire experiment is intended to test one such theory of aberration.

This said, however, he then returns to an approach similar to that of his first paper, reintroducing his previous mathematics for calculating the speed of the earth, though with certain corrections made to his calculations of the perpendicular direction. This allows us to continue our critical look at the 1881 experiment and paper in the context of its 1887 sequel.

The following figure shows Michelson's own comparative depictions of the two experiments.

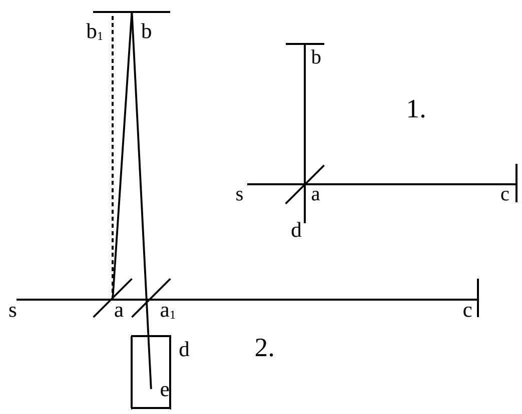

Fig. 3
Illustrating the difference between the 1881 experiment (Diagram 1.) and the 1887 experiment (Diagram 2.)

Michelson's Assumptions

The time required for light to pass from one point to another on the earth's surface depends on the direction in which it travels.

V = velocity of light.

v = the velocity of the earth in its orbit.

D = the distance between two given points (ab or ac, Diagram 1, Fig. 3).

T = the time light takes to pass from a to c.

T_1 = the time light takes to return from c to a.

(Notice that this time Michelson makes no mention whatsoever of the terms d and d_1, or of the time T_0, which were used in the 1881 experiment and paper.)

Michelson's Calculations (1887)

(c3) $T = D/(V-v)$ and $T_1 = D/(V+v)$

(see Proof 2)

The total time taken in going and returning is therefore

(e1) $T + T_1 = 2DV/(V^2-v^2)$

(see Proof 6)

The distance travelled in this time is

(j) $2DV^2/(V^2-v^2) = 2D\,(1+(v^2/V^2))$, neglecting terms of the fourth order.

The length of the other path is evidently

(k) $2D\,\sqrt{(1+v^2/V^2)} = 2D\,(1+v^2/2V^2)$, to the same degree of accuracy.

The difference is therefore

(l) $D(v^2/V^2)$

Turning the apparatus through 90°, the difference will now be in the opposite direction, hence the displacement of the interference fringes should be

(m) $2D(v^2/V^2)$

Considering only the velocity of the earth in its orbit, this would be $2D \times 10^{-8}$. If, as was the case in the first experiment, $D = 2 \times 10^6$ waves of yellow light, the displacement to be expected would be 0.04 of the distance between the fringes.

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Equations (c3) and (e1) have already been proven in connection with the 1881 paper. So let's first do the proofs for the remainder:

(j) $2DV^2/(V^2-v^2) = 2D\,(1+(v^2/V^2))$, neglecting terms of the fourth order.

Proof 11	$2DV/(V^2-v^2)$	as taken from Proof 6
✓	$2DV^2/(V^2-v^2)$	multiplying total time $(T+T_1)$ by velocity V
=	$2DV^2/V^2(1-(v^2/V^2))$	extracting V^2 from the terms in brackets
=	$2D/(1-(v^2/V^2))$	eliminating V^2, above and below
=	$(2D)(1/(1-(v^2/V^2)))$	separating 2D as a multiplicand of the rest
=	$(2D)((1+(v^2/V^2)/(1-(v^2/V^2)(1+(v^2/V^2)))$	multiplying by $(1+(v^2/V^2)$, above and below
=	$(2D)((1+(v^2/V^2)/(1+(v^2/V^2)-(v^2/V^2)-(v^2/V^2)^2))$	multiplying out the denominator
=	$(2D)((1+(v^2/V^2)/(1-(v^2/V^2)^2))$	adding the denominator
=	$(2D)((1+(v^2/V^2)/(1-(v^4/V^4)))$	multiplying out the "terms of the fourth order"
≈	$(2D)((1+(v^2/V^2)/(1))$	"neglecting terms of the fourth order"
✓	$2D(1+(v^2/V^2))$	dividing by 1

(k) $2D\,\sqrt{(1+v^2/V^2)} \approx 2D\,(1+v^2/2V^2)$, to the same degree of accuracy

Proof 12	$2D\,\sqrt{(1+(v^2/V^2))}$	"evident", according to Michelson
✓≈	$2D\,(1+v^2/2V^2)$	employing $\sqrt{(a^2+b)} \approx (a + (b/2a))$

Michelson's next point is that the difference between (j) and (k) is

(l) $D(v^2/V^2)$

Proof 13	$2D(1+(v^2/V^2)) - 2D(1+(v^2/2V^2))$	from Proofs 11 & 12
⇒	$2D+(2Dv^2/V^2) -2D-(2Dv^2/2V^2)$	expanding
⇒	$2(2Dv^2) - 2Dv^2)/2V^2)$	taking the same denominator
⇒	$(4Dv^2 - 2Dv^2)/2V^2)$	multiplying out the numerator
⇒	$2Dv^2/2V^2$	adding the terms of the numerator
✓	$D(v^2/V^2)$	dividing out and bracketing

Turning the apparatus through 90° is simply supposed to double the value obtained for (l), i.e.

(m) $2D(v^2/V^2)$

Proof 14	$D(v^2/V^2)$	from equation (l)
✓	$2D(v^2/V^2)$	multiplying by 2

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So now we come to one of the main objectives of this paper, to identify the faults contained in Michelson's 1881 and 1887 experiments and papers.

In this connection we will list 4 error categories:

- trivial errors (TEs)
- conceptual errors (CEs)
- statistical errors (SEs)
- mathematical errors (MEs)

evaluating their importance more or less in this order.

<u>Trivial Errors (TEs)</u>

Both the 1881 and 1887 papers contain a number of trivial errors. Taken in their order of appearance, the following can be mentioned:

TE(1) In c2, where we have indicated two omissions by "...", Michelson makes the absurd claim that "$d = Dv/(V-v) = d_1/v$ and $d_1 = Dv/(V+v) = d_1/v$". This is mistaken on a number of counts (not least in its equating distance with time!), so that we shall simply ignore it, perhaps as a bad printer's error.

TE(2) Under f, Michelson gives the approximation "$v^2/V^2 \approx 1/100,000,000$ millimetres". The velocity of the earth divided by that of light (again) does not give a distance, however, but is a simple ratio. Since this plays no major role in the calculations or argument, however, we shall again ignore it.

TE(3) At the start of his calculations in the 1887 paper (on page 336 of the "American Journal of Science") Michelson quotes T as being the "time light occupies to pass from a to c" and, on the following line, as the "time light occupies to pass from c to a". In the second case T_1 is clearly intended.

(Careful too in Wikipedia, which mistakenly gave the subsequent equation as T_1 = D/(V-v) instead of T_1 = D/(V+v), until it was corrected.)

Other errors that might have been mentioned were also noticed, but they play no major role in the experiment.

Conceptual Errors (CEs)

Now things become interesting. We will again mention only 3 conceptual errors, however, since the others are better dealt with under the next heading.

CE(1) We can speak, I believe, of conceptual error when we consider Michelson's self-assessment of his mathematical approach. First the mathematics was done, then its conclusions were adopted as secure predictions, then came the experiment, and then the experimental results. And if the predictions and the results failed to match? Then our view of the world was clearly mistaken, certainly not the mathematics!

CE(2) Although it should really have been obvious to him, since it is implicit in the assumption of light moving at a constant speed through space, or the ether, etc. - an accepted assumption in the context of the experiment - Michelson failed to realize that the light emitted from his source did not take on the velocity of this source.[2] . Without this conceptual error he would never have postulated his T_0 - really a conceptual error in its own right - as 2D/V (see e2) in the first place. And had he realized the complexity to which the correction of this error would give rise, he may never even have begun his experiment at all (one thinks here, again, of Clerk Maxwell's reservations). The error was noticed and corrected, as mentioned above, by Lorentz.

CE(3) Our third reference to conceptual errors made by Michelson, though one that is difficult to pinpoint, has to do with the concept of aberration. It is a point which relates to CE(2) and one which will also be touched on in the next section, but it nevertheless deserves a separate mention here, since it fundamentally relates to a common misconception associated with the Michelson-Morley experiment: that of a perpendicular beam, the path of which is truly at 90° to the beam in the direction of the earth's motion. The following statement, therefore, cannot be emphasized enough:

> Unless the earth is stationary in space, the so-called "perpendicular" angle cannot be 90°, but must be less.

In his 1881 experiment Michelson went to great pains to ensure a 90° angle, as he believed. After the criticism from Lorentz, however, he accepted that the angle must be less than 90° - though his preparatory procedure remained essentially unchanged. How does this tally? There are two essential arguments, here, that clarify the point:

[2] Later confirmed in an experiment conducted by Alväger et al., 1964 (CERN) [My thanks to Professor Schiller of the Institut für Experimentalphysik, Heinrich-Heine-Universität, Düsseldorf, for this reference]

1) The first one contradicts the very hypothesis of 90° for a moving body. Given an absolutely accurate perpendicular trajectory of 90°, the beam of light would theoretically return to *exactly* that point in space from which it was emitted. The moving earth, however, would meanwhile have moved on. (In fact, this too is pure theory, since the same applies to the mirror at 90°, so that the beam would miss this too and would not return at all!)

2) The second is a practical point associated with the experimental setup. In referring to the complicated procedure required to align the beams for the 1881 experiment, Michelson explains how the final minute adjustments were made using "plate b", to ensure that the interference bands were visible. What Michelson was actually doing here was "picking up" the angle required to obtain a returned beam. Or put another way, he was calibrating to eliminate aberration!

In terms of the conceptual adaptation required to accommodate the Lorentz correction, we now have a situation in which the distance to be travelled in the perpendicular direction is greater than 2D, meaning that the time required for this is greater than T_0, i.e. the distance between the interference bands must be smaller.

Statistical Errors (SEs)

This is where Michelson *really* went wrong.

Continuing with the total perpendicular direction first, in the 1881 experiment this was seen by Michelson as being simply the distance to the perpendicular reflector and back, i.e. 2D. The time taken for this round trip is therefore 2D/V. In the version corrected by Lorentz and accepted by Michelson, however, this is given in the 1887 paper as being "evidently" $2D \sqrt{(1+v^2/V^2)}$.

It is rather surprising that Michelson uses the word "evidently", here, since to him this was by no means evident - in the sense of "obvious" - in the context of the 1881 experiment and paper. The word can also be used in the sense of "apparently", however, which must be taken to have been meant by him here.

The derivation of this term is given in our Proof 11, although this proof is actually employed to derive the distance in the direction of the earth's motion. The maths can be applied to both directions now, however, since both have a "v" component. In the process Michelson neglects what he refers to as "terms of the fourth order", here v^4/V^4.

The calculation, based on the differences of Fig. 2, as regards the applicable velocities, is shown as follows:

$$D\,(V / \sqrt{(V^2-v^2)})$$
$$= D\,(V / \sqrt{V^2(1-v^2/V^2)})$$
$$= D\,(V / V\sqrt{(1-v^2/V^2)})$$

$= \quad D / \sqrt{(1-v^2/V^2)}$
$= \quad D \sqrt{(1+v^2/V^2)} / \sqrt{(1-v^2/V^2)}\sqrt{(1+v^2/V^2)}$
$= \quad D \sqrt{(1+v^2/V^2)} / \sqrt{((1-v^2/V^2)(1+v^2/V^2))}$
$= \quad D \sqrt{(1+v^2/V^2)} / \sqrt{(1^2-v^4/V^4)}$
$= \quad D \sqrt{(1+v^2/V^2)} / \sqrt{(1^2)}$
$= \quad D \sqrt{(1+v^2/V^2)}$

Using Michelson's own approximations for the velocities of the earth and of light, the term eliminated here is thus approx. $(10^{-4})^4 = 10^{-16}$. Comparing this with the "solution" later given by Lorentz to account for the zero result obtained by the Michelson-Morley experiment, namely *length contraction*, we have a ratio of $10^{-16} / 2x10^{-8} = 5x10^{-7}$ (i.e. a fifty millionth [of a two-hundred millionth]), which is indeed negligible.

Even though Michelson repeats this approximation (as shown in Proofs 11 and 12), the consequent adjustments remain negligible, in terms of the results obtained by him.

Continuing to think along these lines, however, let's now take a look at Proof 3 and Proof 9, in both of which he disregards the term v^2 in the component $1/(V^2-v^2)$. Before examining the magnitude of this adjustment, its significance can be anticipated from Fig. 2b, in which the relationships of the sides of the triangle to each other are:

$V^2 = (V^2-v^2) + v^2$
$\Rightarrow \quad v = \sqrt{(V^2 - (V^2-v^2))}$
$\Rightarrow \quad v = \sqrt{(V^2 - (V^2))}$ disregarding v^2
$\Rightarrow \quad v = \sqrt{(V^2 - V^2)} = 0$

In other words, by negating the element v^2 Michelson was effectively negating the motion of the earth, which is what he was also trying to measure!

In terms of magnitude, we can see in the following conversion

$1/(V^2-v^2)$
$\Rightarrow \quad 1/(V^2(1-v^2/V^2))$
$\Rightarrow \quad 1/(V^2(1-0/V^2))$ disregarding v^2, i.e. making $v^2=0$
$\Rightarrow \quad 1/(V^2(1))$
$\Rightarrow \quad 1/V^2$

Negating the factor v^2/V^2 is equivalent to negating

$(3x10)^2/(3x10^5)^2$
$= \quad (3x10^2)/(3x10^{10})$
$= \quad 10^2/10^{10}$
$= \quad 10^{-8}$

And this amounts to half the magnitude compensated by *length contraction!*

Not only does Michelson eliminate this value twice, once in Proof 3 and again in Proof 9, but the results of these proofs infiltrate almost his entire mathematics, from Proof 3 to Proof 10, with the exception of Proof 5.

To underline this point let's compare Proof 6 of (e1) with (e2), the former giving the total time for the light to travel there and back in the direction of the earth's motion, the latter the time for the perpendicular direction:

(e1) $T + T_1 = D/(V-v) + D/(V+v) = 2DV/(V^2-v^2)$

Proof 5	$D/(V-v) + D/(V+v)$	adding T and T_1 from the equations of (c3)
Proof 6	$D/(V-v) + D/(V+v)$	Proof 5
=	$(D(V+v) + D(V-v)) / (V+v)(V-v)$	taking the same denominator and then adding
=	$(DV+Dv+DV-Dv) / (V^2 +Vv - Vv -v^2)$	multiplying out, both above and below
✓	$2DV/(V^2-v^2)$	adding, both above and below

(e2) $2D/V = 2T_0$.

Notice that if we do here in Proof 6 what Michelson does elsewhere, namely to eliminate the v^2 component from the denominator, we end up with $T+T_1 = 2DV/V^2 = 2D/V = 2T_0$ - which implies no less than that the motion of the earth is zero!

At any rate, if *length contraction* serves as an explanation for the Michelson-Morley result, then it is entirely superfluous, since Michelson's own errors of precision in his calculations arguably already account for the same.

The fact that Lorentz failed to see this is astonishing.

This is, however, by no means the only precision error that can be mentioned in terms of the Michelson-Morley experiment.

Take the speed of light, for example: Michelson equates this to 300,000 km/s, which is normally an acceptable approximation. A closer approximation is given, however, by 299,792 km/s. Approximating this to 2998 x 10^2 km/s, let's now compare the two as

$$
\begin{aligned}
& (3000 \times 10^2) - (2998 \times 10^2) / 3000 \times 10^2 \\
= \quad & (2 \times 10^2) / (3000 \times 10^2) \\
= \quad & 2 / 3000 \\
= \quad & 2/3 \times 10^{-3}
\end{aligned}
$$

Now compare this magnitude, of 2 parts per 3,000, to that of the Lorentz solution:

$$
\begin{aligned}
& 2/3 \times 10^{-3} / 2 \times 10^{-8} \\
= \quad & 1/3 \times 10^{5} \\
\approx \quad & 7 \times 10^{4}
\end{aligned}
$$

In other words, the approximation used for the speed of light is in itself roughly 70,000 times greater than any compensation brought about by the *length contraction* explanation.[3]

[3] Or to take a different example, if a stopwatch triggered by a laser at the finishing post of a race course has an accuracy of up to a tenth of a second, you can record a runner's time to an accuracy of a tenth of a second, but not to a hundredth of a second.

And Michelson himself mentions, at the end of his 1887 experiment and paper, that the calculations were based on the assumption of an orbital velocity of roughly 30 km/s. Some estimates put this at closer to 26 km/s, however, which makes the *length contraction* "solution", with its compensation factor of a mere two-hundred millionth, all the more absurd.

Then there is the velocity of the solar system in the galaxy, which is often regarded as being something like 8 times that of the earth's orbital velocity, but does not form any part of the experimental calculations made by Michelson.

Need one say more.

In summarizing this section, then, it can be said that, in terms of its statistical accuracy, the Michelson-Morley experiment comes nowhere near establishing predictable criteria against which to compare the experimental results.

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Point four, on mathematical errors, has yet to be treated, but this can now be taken as more of an adjunct to the preceding account.

The mathematical errors mentioned so far are essentially errors of conception (90° angle) or of magnitude and the discarding of significant values (v^2). Now we come to a true mathematical error which, to the best of my knowledge, has not been previously detected.

Those who have followed the argumentation from the start and have a very good memory may have noticed that one proof has not as yet been given, that associated with (c2).

In fact, there are actually two "similar" proofs involved here, one for $d = Dv/(V-v)$ and one for $d_1 = Dv/(V+v)$, the first of which goes as follows:

Proof 15	$(D+d)/V = d/v$	as given from c1
⇒	$d = v(D+d)/V$	reversing the order and multiplying both sides by v
⇒	$d = (Dv+dv)/V$	multiplying out
⇒	$d = Dv/V + dv/V$	separating the terms in brackets
⇒	$d - dv/V = Dv/V$	subtracting dv/V from both sides
⇒	$V(d - dv/V) = Dv$	multiplying both sides by V
⇒	$dV - dv = Dv$	multiplying out
⇒	$d(V-v) = Dv$	extracting d from the terms in brackets
✓	$d = Dv/(V-v)$	dividing both sides by (V-v)

Nothing wrong with this, in the context of the 1881 experiment. So now let's take a look at the second equation, $d_1 = Dv/(V+v)$. Here Michelson clearly took essentially the same approach as for our Proof 15:

Proof 16	$(D-d)/V = d_1/v$	as given from c1
⇒	$d_1 = v(D-d)/V$	reversing the order and multiplying both sides by v
⇒	$d_1 = (Dv-dv)/V$	multiplying out
⇒	$d_1 = Dv/V - dv/V$	separating the terms in brackets

⇒	$d_1 + dv/V = Dv/V$	adding dv/V to both sides
⇒	$V(d_1 + dv/V) = Dv$	multiplying both sides by V
⇒	$d_1V + dv = Dv$	multiplying out
⇒	$d_1V + d_1v = Dv$	replacing d by d_1
⇒	$d_1(V+v) = Dv$	extracting d_1 from the terms in brackets
✓	$d_1 = Dv/(V+v)$	dividing both sides by (V+v)

Noticed the error? Since $d \neq d_1$ one cannot simply exchange these. Instead the correct equation - from the fourth-last term on - reads:

Proof 16	...	
⇒	$d_1V + dv = Dv$	multiplying out
⇒	$d_1V + dvd_1/d_1 = Dv$	adding d_1, above and below
⇒	$d_1V + d_1vd/d_1 = Dv$	rearranging the second term
⇒	$d_1(V + vd/d_1) = Dv$	extracting d_1 from the terms in brackets
!	$d_1 = Dv/(V + vd/d_1)$	dividing both sides by $(V + vd/d_1)$

And suddenly the whole Michelson-Morley experimental calculation falls apart, since this additional d/d_1-component is so omnipresent that there is now hardly a proof of the original Michelson equations that is correct!

As to the precise effects of this mistake on Michelson's calculations, I'm happy to leave this work to others busy in the field.

***

The last topic I would like to address in connection with the Michelson-Morley experiment is the Lorentz factor, which it helped to make so popular.

Looking again at Fig. 2b,

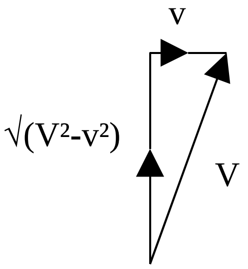

Fig. 2b
Outbound journey in the perpendicular direction

we can see that this factor, $1/\sqrt{(1-v^2/V^2)}$, can be readily derived as the ratio of the hypothenus proposed by Lorentz to Michelson's perpendicular (i.e. AC/AB in Fig. 2.):

1) $V / \sqrt{(V^2-v^2)} = V / \sqrt{(V^2(1-v^2/V^2))} = V / V\sqrt{(1-v^2/V^2)} = 1/\sqrt{(1-v^2/V^2)}$

In other words, the Lorentz factor can always be derived by taking the ratio of one of the shorter sides of a right-angled triangle to its hypothenus, e.g. L = H/S.

This in turn allows one to derive the shorter side (S = H/L) from the hypothenus:

2) $V / (1/\sqrt{(1-v^2/V^2)}) = V \sqrt{(1-v^2/V^2)} = \sqrt{\ } (V^2 (1-v^2/V^2) = \sqrt{\ } (V^2-v^2)$

It also allows us to derive the hypothenus from the shorter side (H = S L):

3) $\sqrt{(V^2-v^2)}\ (1/\sqrt{(1-v^2/V^2)}) = \sqrt{(V^2-v^2)} / \sqrt{(1-v^2/V^2)} = V\sqrt{(1-v^2/V^2)}/\sqrt{(1-v^2/V^2)} = V$

So as one can see, there is nothing magical about this "factor". It is intrinsic in every system that can be depicted in the form of triangular coordinates.

If therefore, in connection with the Michelson-Morley experiment, applying the Lorentz factor were to be taken as providing a solution to the result obtained, then certainly because the prediction for the experiment was incorrect to the value of this factor, and not because the physical world contracts to an extent required to validate Michelson's predictions. Here, with all of the errors we have pointed to, CE(1) above decidedly applies.

***

Summary

This paper has shown that Michelson's approach relied on the correctness of his mathematical calculations. The paper has given step-by-step proofs for all of Michelson's equations and has shown that these calculations were severely flawed.

It has identified the approximations made by Michelson and has shown that, when dealing with such small magnitudes, even such tiny approximations can be fatal.

It has explained how aberration can be accounted for, in the context of Michelson's experiments, without his having realized this (a point that also applies to equivalent modern-day experiments).

And it has shown that *length contraction* - at least of the magnitude proposed by Lorentz - cannot resolve the result obtained in the Michelson-Morley experiment.

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