The Scaling Theory XII: Universal Timer - The Zeroth Law of Motion

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20.1. Clocks, Classical and Relativistic

Time in special theory of relativity is identified by the readings of ideal clocks, and global time in an inertial frame S is achieved through clocks' synchronization with respect to a master clock at a given point using light signals[17]. The latter procedure requires the velocity of light be constant within S, which is fulfilled in special relativity through a postulate asserting that light travels rectilinearly in vacuum at a constant velocity c which is independent of the relative velocity between the source and observer.

On the other hand, the classical view of time is expressed by Newton in his celebrated Principia[18]: *Absolute, true mathematical time, of itself and from its own nature, flows equably without relation to anything external.* Therefore, while time in special relativity is identified by clocks readings, each at its site in its own frame, clocks in classical mechanics (used here as opposed to relativistic) are merely useful tools to determine time, and *ideal clocks* in the latter, should run uniformly everywhere and independently of its state of motion, measuring accordingly the same durations. If s is an inertial frame that is translating uniformly at a constant velocity $\vec{u} = u\vec{\imath}$ relative to S, then the contiguity of an S and an s observers, say $B \in S$ and $b \in s$, defines in both frames the same event which is characterized by the same instant of time, but with coordinates that can assume any chosen values in each frame. The instant (or instants) of observation of this event by an observer in S (or in s) residing elsewhere was overlooked. It was implicitly assumed instead that information is transmitted immediately, or as to say in no time. In any case, clocks in the classical view is a handy way to measure time which flows uniformly everywhere and with no connection to the state of motion of the clock (or observer) that measure it. The ambiguity in the phrase "ideal clocks" was eliminated through demanding implicitly that all observers' clocks conform to a "universal timer", with the properties of being usable everywhere, and not biased to any observer. The positions of distant stars in the firmament served at night as a world-wide timer by which a sidereal day could be defined as a unit of time. The position of the sun, with its development through out the year is taken into account, served also as a timer during the daylight. Mechanical instruments (and later electronic) called clocks, were designed to read the time defined by the stars motion relative to earth, and it were considered reliable as much as it did conform to the universal timer defined by the earth's configuration with respect to the firmament.

The classical view of contiguity is contrasted here with the relativistic view through an example. Consider a train s passing by a train station S at a constant speed $\vec{u} = u\vec{\imath}$. An observer $B \in S$ sees a passenger b on board just passing by. He looks at his own clock, it indicates $T = 0$; it is midnight. In fact B's clock is synchronized with all clocks in the train station; they all indicate midnight. The observer B asserts that the event of being contiguous to the passenger b took place at $T = 0$, and is represented thus by $(X_0, 0,0,0)$, where$(X_0, 0,0)$ is its own position in S. By Lorentz transformations, the event of b being contiguous to B should be describable in s by

$$(x, 0,0, t) = \gamma(X_0, 0,0, -uX_0/c^2),$$

with $\gamma = 1/\sqrt{1-u^2/c^2}$. Thus, while all clocks in S (the train station) read $T = 0$, the corresponding clock at b reads $t = -\gamma u X_0/c^2$. The observer B may look through the train window and see b's clock reading t, while his own clock is reading $T = 0$. Suppose now that the observer B decides to refer his position to a new origin of coordinates in S, say his own position, and hence $X_0 = 0$. In this new case, B reads on the clock b the instant $t = 0$. Thus and *just by changing his mind about the point which he wishes to choose as the origin of his coordinates, the observer B alters the reading of the s-clock that is contiguous to him! Moreover, an s- clock originally reading* $-\gamma u X/c^2$ *is altered to read* $-\gamma u(X - X_0)/c^2$.

The arguments in (part XI) according to which the classical concepts of time and length can be implemented, when observations through light signals are discarded, are extended here to a synchronous rotating frame; and Newton's laws are supplemented by "*a rotational law of inertia*", which serves in itself to define and measure time. The case of observations through light's signal is dealt with in XIII.

In this piece of work

- Global time readings that are not influenced by temperature, gravity, magnetic or electric fields at the point in which the timer is placed, are sought.
- The issue of defining a global time in one chosen frame S through clock's synchronization using light signals, is circumvented by means of one universal timer. Our method of establishing a global uniform time in S, which is inherited in any frame, inertial or not, is an idealization to that adopted in astronomy[40].
- The use of clocks is an essential need in physical measurements, although we could in principle dispense with clocks altogether in favor of a universal timer. Indeed, our treatment is compatible with the use of clocks provided they run at the same rate as does the global time established in S; and in essence, it replaces all clocks by a single timer by which the concepts of past, present, and future are well defined *for all observers, regardless of the state of their motions*. If it happened that the readings of one, or a set of clocks, deviated from the time specified by the universal timer, showing retardation or advance, then this has to be attributed to the environment's effect on the concerned clocks as physical devices, and not to time itself.

20.2 A Universal Timer

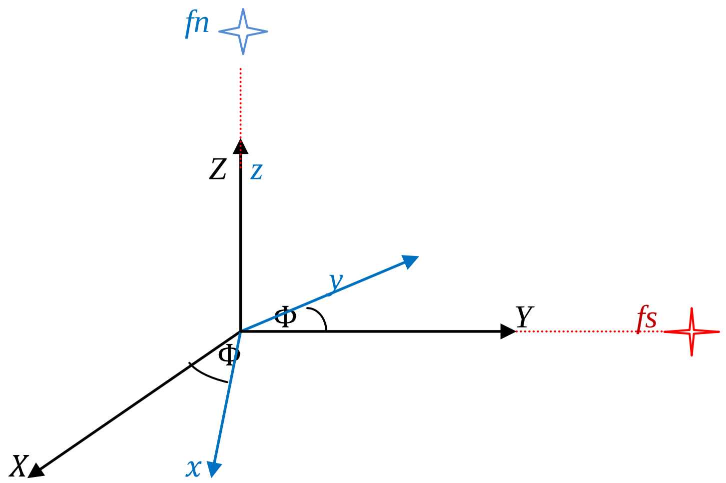


Fig.1. The spaceship frame $s \equiv oxyz$ rotates uniformly relative to the frame of fixed stars $S \equiv OXYZ$.

To simplify discussions, we may temporarily imagine a huge spaceship *s* with spherical shape, and sufficiently far from the influence of the neighboring matter. The inhabitants of this spaceship, with engines shut off, observe that their ship rotates relative to the distant universe about its axis $n'on$, with *o* is the center of *s*, and the vector $\overrightarrow{on}$ pointing to a very distant star *fn*, which they call the pole star. Let $s \equiv oxyz$ be a rectangular Cartesian frame attached to the spaceship *s* with origin *o* at the ship's center and *oz* along the axis of rotation, i.e. *oz* is along $\overrightarrow{on}$ where *n* is the north pole of the ship. Let $S \equiv OXYZ$ be a rectangular Cartesian frame with origin *O* permanently at *o*, *OZ* along *oz*, and *OY* pointing to a fixed star *fs*. The frame *S* is thus a frame of fixed stars, which can be considered stationary and identifiable with the absolute space (see part XI), while *s* rotates relative to *S* about *OZ*.

20.3. Global Time in *S*

A short-cut argument to set up a global time in the surface *ss* of *s*, as well as in *S*, is the following:

-the frame *s* rotates uniformly relative to *S*. If Φ is the rotation angle of *s* relative to *S* (Fig.1), then Φ is a linear function of time *t*.

- choose one particular instant of time at which the configuration of *s* relative to *S* is determined by an angle Φ_0 as the zero of timing on the surface *ss* as well as everywhere in *S*. i.e. $t = 0$ everywhere in *S* corresponds to the configuration $\Phi = \Phi_0$.
- define the unit of time as the duration of a complete revolution of the spaceship *s* about its axis, and call it a *sidereal day* (or just a *day*). Thus, elapsing of one day corresponds to a new configuration of *s*, determined by $\Phi = \Phi_0 + 2\pi$.
- a global time *t* in *S* at any latter instant is given in sidereal days by

$$(20.1) \qquad t = (\Phi - \Phi_0)/2\pi \quad \Phi_0 \leq \Phi < \infty.$$

Because *t* is a homogeneous function in $\Phi - \Phi_0$ and the zero of time is arbitrary, we can extend the latter relation to incorporate negative instants of time:

$$(20.2) \qquad t = (\Phi - \Phi_0)/2\pi \quad -\infty \leq \Phi < \infty.$$

The relation (20.2) defines a global time in *S* as well as on *ss*, and inside, through the angle of rotation.

In the Newtonian view therefore, simultaneous time readings at all points in S (and in any other frame) are rationally identified by the rotation angle $(\Phi - \Phi_0)$, as much as the existence of the system of coordinates is rationally erectable in no time and extending indefinitely. Note that in the same way the fixed stars serve as a timer for the spaceship, the frame *s* of the spaceship serves also as a timer in the inertial frame of fixed stars *S*.

The following section puts the above considerations in a measurable fashion, and can be skipped.

20.4. Global Timing in *ss* Through Local Measurements

We neglect temporarily the aberration effect, and also suppose that matter is distributed in a spherically symmetric sense in the spaceship s. Relative to the frame *s*, every point on the surface *ss* of the spaceship surface is determined by its longitude and latitude angles $\emptyset$ and θ respectively. Consider the fixed star *fs* and assume that at some instant of time $t = 0$, an observer at $(\emptyset = 0, \theta_0)$ in *ss* sees the star *fs* rising, or as to say just appearing from his "eastern horizon" which coincides at $t = 0$ with the direction of OY. Since the star is very distant, all observers with the same longitude $\emptyset = 0$, but any latitude θ -to be called *Greenwich longitude*- see the star rising at the same instant $t = 0$. Due to the spaceship rotation, and after a lapse of time, all observers with another longitude

$\emptyset = \emptyset_0$ $(0 < \emptyset_0 < 2\pi)$ see together the star rising at the same moment t_0. It is evident that the event "*fs* is rising at us" is simultaneous with respect to all observers that have the same longitude, and that the instant of time associated with this event is related to their longitude $\emptyset_0$. Owing to the spaceship rotation every observer will see the star *fs* rising again after *one sidereal day*. The inhabitants of the ship have no reason denying them to *trust that their spaceship rotates uniformly* about its axis $n'on$ with respect to the fixed stars. Or equivalently, the distant universe is rotating uniformly in an opposite sense about $n'on$ relative to *s*, while *s* is stationary. Thus, the ship inhabitants *trust* to employ the rotation of their ship to define and measure time.

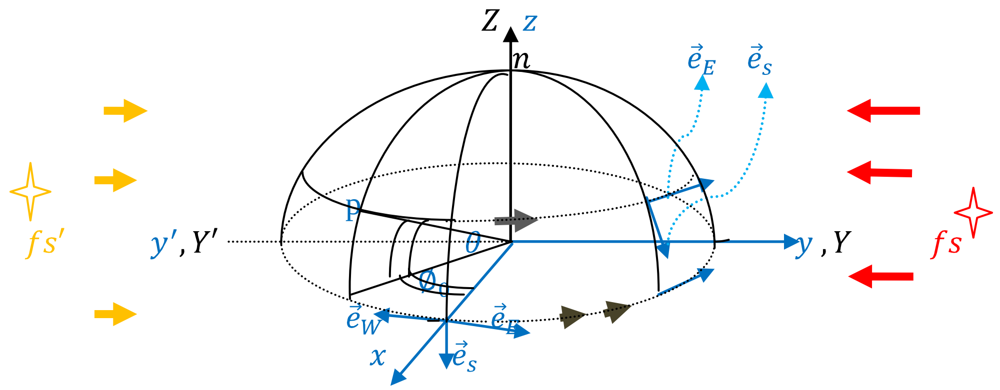


Fig.2. The configuration of *ss* relative to *S* at the instant of *fsrise* at the locations $(\emptyset = 0, \theta)$. At this instant *s* and *S* are coincident. For convenience, the longitudes $\emptyset$ of point in *ss* are measured clock-wise. $\vec{e}_E, \vec{e}_W, \vec{e}_S$ are the unit vectors at an arbitrary point *p* of *ss* pointing east, west, and south respectively. fs' is a fixed star that lies diametrically opposite to *fs* relative to *s*.

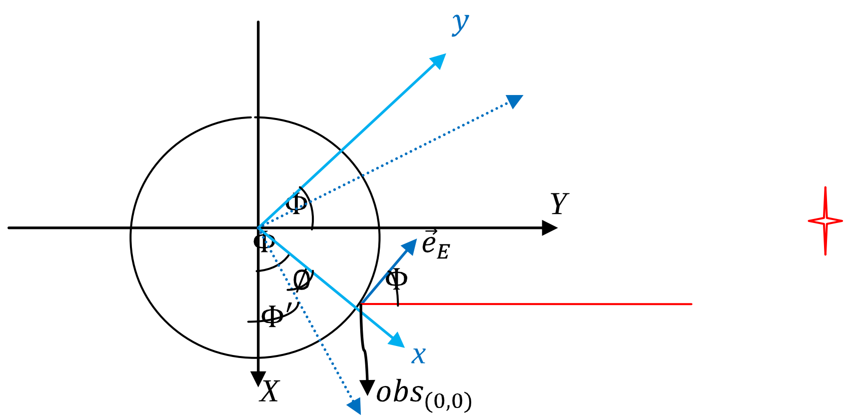
Giving each observer in *ss* his longitude, which is obtained through geometric measurements, a global uniform time can be set up on *ss* in three steps: (i) to choose a longitude at which a particular *fsrise* (i.e. *fs'* rise) corresponds to zero instant of time (ii) to synchronize time allover *ss* (iii) to establish a uniform time allover *ss*.

(i) All observers in *ss* with longitude $\emptyset = 0$ agree to identify the instant $t = 0$ with one particular event of the form "*fs* is rising at us", which we call the *first fsrise at* $\emptyset = 0$ (or at Greenwich) and denote by G_0.

(ii) Trusting that their spaceship rotates uniformly with respect to the remote universe, all *ss* observers with longitude $\emptyset$ associate with the first fsrise at their location, which succeeds G_0, the instant $t_\emptyset = \emptyset/2\pi$ sidereal day.

(iii) To set up time at each point of *ss* that is at synchrony with all other points, each observer add to the instant corresponding to his own *fsrise* a duration $\Phi'/2\pi$ sidereal day, where Φ' is the angle of rotation of *s*, since his own *fsrise*. The subsequent discussion, which is also well known to most readers, explains how this is achieved at each point through local measurements.

For simplifications, we consider first an equatorial observer $obs_{(0,0)}$ at $(\emptyset = 0, \theta = 0)$. At the instant $t = 0$, $obs_{(0,0)}$ sees the first fsrise. The star *fs* describes during the following half day a semi-circle in his sky lying in his vertical plane which comprises his east direction. The angle between his east horizon and the line of sight to *fs* is equal to the rotation angle Φ of *s* relative to *S* since *fs* has risen at $obs_{(0,0)}$. Consider a rotation by an angle Φ that transports an equatorial observer $obs_{(\emptyset,0)}$ with longitude $\emptyset$ from darkness to a position in daylight specified by an angle $\Phi' = \Phi - \emptyset$ with the *X*-axis of *S*. The angle Φ' is the rotation angle of the latter observer after his own fsrise which takes place at $\emptyset/2\pi$. The time in sidereal days registered by $obs_{(\emptyset,0)}$ when he is at an angle Φ' with the *X*-axis of *S* is



(20.3) $$t = \frac{\Phi'}{2\pi} + \frac{\emptyset}{2\pi} = \frac{\Phi}{2\pi}.$$

It is easy to see that $\Phi' = \Phi - \emptyset$ is the angle between the east horizon of the observer $obs_{(\emptyset,0)}$ and the line of sight to *fs*. The latter relation shows that the time reading by any observer $obs_{(\emptyset,0)}$ is independent of his longitude (and latitude) and depends solely on the rotation angle of *s* relative to *S* since the first *fsrise* G_0. The latter rotation angle Φ can be measured by each observer simply by measuring the latitude (= height) angle Φ' between his east horizon and *fs*, and then adding to it his own longitude $\emptyset$. For $obs_{(\emptyset,0)}$, the first *fsset* takes place half a day after his *fsrise*, but he can continue reading time by just adding the angle between his east horizon (which corresponds now to $\Phi = \pi$) and the line of sight to another fixed star fs' which happened to be diametrically just opposite to *fs*, and thus rising when *fs* is setting. Since the east directions are parallel for all observers that have the same longitude $\emptyset$, all observers with the same longitude measure the same latitude angle of *fs* with their east horizon, and thus time read by any $obs_{(\emptyset,\theta)}$ is given by (20.3). The time reading of any instant of time during the first day of every observer $obs_{(\emptyset,\theta)}$ is thus established. For subsequent days it is also true that

(20.4) $$t = \frac{\Phi'}{2\pi} + \frac{\emptyset}{2\pi} = \frac{\Phi}{2\pi} \qquad 0 \le \Phi < \infty.$$

The last relation applies also to all points inside the spaceship *s*, and can be extended to comprise the past.

20.5. The Zeroth Law of Motion

This law, which could have preceded Newton's laws of motion, can be phrased as follows:

In the absence of all external influences other than the influence of the distant stars, a spherical body with an internal spherically symmetric distribution of matter rotates relative to the fixed stars about a fixed axis by equal angles in equal times. Or in a more compact language: *When subject only to the influence of* *the remote universe, a spherically symmetric body rotates uniformly about a fixed axis relative to the fixed stars.* The state of being not rotating is of course included as a special case.

It is clear that Newton's three laws of motion lack a criterion for judging *uniformity*. The *zeroth law of mechanics, or the law of time* as it may rightfully be called, serves to set up this criterion through relating uniform time to equal spatial rotational measurements. This law in essence asserts that time flows in proportion to the angle of rotation of a body subject only to the influence of fixed stars. The axial semi-plane comprising the semi-circle $\emptyset = 0$ in *ss* may serve as the hand of a universal clock in *S*, with time is read by the angle Φ between this hand and its initial configuration in *S*, where *S* is the frame of fixed stars.

Adopting the rotation angle as a measure of time in *S*, we may dispense with time in any equation $f(\vec{r}, d\vec{r}/dt\,, d^2\vec{r}/dt^2, t, m) = 0$ and replace it with Φ to obtain

$$f(\vec{r}, 2\pi\, d\vec{r}/d\Phi\,, (2\pi)^2\, d^2\vec{r}/d\Phi^2, \Phi/2\pi\,, m) = 0\,,$$

In particular, the law of inertia and Newton's second law are expressed as

$$\overrightarrow{\Delta r} = \vec{v}\Delta t = \vec{v}\,\frac{\Delta\Phi}{2\pi},$$

$$\vec{f} = m\frac{d^2\vec{r}}{dt^2} = m(2\pi)^2\frac{d^2\vec{r}}{d\Phi^2}.$$

The latter replacements do not make things any easier, but it links the development in time of any physical system directly to the standard system of "uniformity".

The reader may wonder why not the law of inertia itself can serve to define time. In fact it can, and that what we have done in (part XI) when set up a global time in a stationary inertial frame *S* through employing a special form of the law of inertia to light's photons, and thus making a timed frame of *S*.

20.6. The Timed Inertial Frame of Fixed Stars *S*

In terms of a given unit of length *ls* (a given rod, say a meter) the geometric distance between any two points inside, and on, the surface *ss* of the spaceship *s* can be specified. Being a subspace of the 3-physical space, the geometry in the spaceship is Euclidean, and the surface *ss* of *s* is a sphere of radius a. Thus the metric form of *ss* is $dss^2 = a^2(d\theta^2 + sin^2 d\emptyset^2)$. In addition to the distance in *ss*, and inside, a global time in the spaceship has been set up in a way described in a previous section. The reference frame *ss*, though not inertial, is endowed with a global time; it is what we have called (in Part XI) a *synchronous frame*.

Our task now is to induce in *S* geometric length and time from those in *ss*. Let's revert to the spaceship *s*, and imagine a shell of observers just above *ss* and not rotating relative to the fixed stars; they are thus a part of the stationary frame *S*. With respect to the shell of observers, *shob*, the spaceship rotates at an angular velocity ω, or equivalently, *shob* rotates relative to spaceship at an angular velocity $(-\omega)$. To set up a global time that is read locally by each member of $shob \subset S$, it is sufficient to imagine that each member adopts the time reading of his contiguous *ss*' observer. Indeed, if an instant $t = \Phi/2\pi$ is read in *ss* then this determines the same instant in *shob*, and the synchrony between *ss* and *shob* persists for all subsequent instants since *shob* have also faith that *s* rotates uniformly relative to them. In other words, the *shob* measure time also by the rotation angle Φ, and as a result, all clocks in *s* and in *shob* are synchronized; they all read time as determined by the rotation angle Φ and thus synchrony persists for good.

Distance in *shob* is induced from *ss* through an isometric contiguity of *ss* an *shob* as follows: At an instant $t_0 = 0$ the contiguity of *ss* and *shob* forms a bijection

$$contig: ss \to shob, \qquad p(\theta, \emptyset) \in ss \to P(\theta, \emptyset) \in shob$$

which endows *shob* with a system of coordinates $(\theta, \emptyset)$. Let $P(\theta, \emptyset) = contig(p(\theta, \emptyset))$ and $P'(\theta', \emptyset') = contig(p'(\theta', \emptyset'))$. The distance $d(P, P')$ between $P, P' \in shob$ is taken to be equal to the distance $d(p, p')$ between $p, p' \in ss$. It is clear that the mapping $shob \times shob \to \mathbb{R}$ defined by

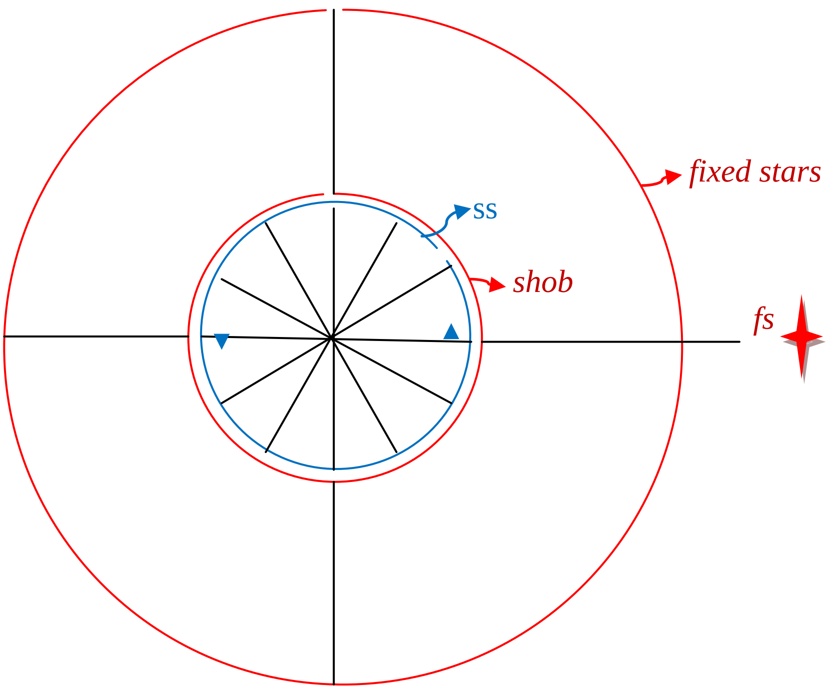
$$d(P, P') = d(p, p')$$

is independent of the instant of the contiguity of (p, p') and (P, P'). This means that $d(P, P')$, as it should be, is equal to the distance $d(p, p')$ for all instants of time if contiguity holds at one instant of time. In other words

$$d\big(P(\theta, \emptyset), P'(\theta', \emptyset')\big) = d\big(p(\theta, \emptyset + \omega t), p'(\theta', \emptyset' + \omega t)\big) \qquad \forall t,$$

where we look at *ss* from the coordinated frame $shob \subset S$ and see it rotating at angular velocity (ω). Accordingly the metric form in *shob* is given by

$$dshob^2 = a^2(d\theta^2 + sin^2 d\emptyset^2) = dss^2,$$



with $(\theta, \emptyset)$ *and* $(\theta + d\theta, \emptyset + d\emptyset)$ are the coordinates of two points of *shob* (or in *s* at the same instant $t_0 = 0$). We have thus set up time and distance in *ss* and *shob* purely through geometric measurements, and with the unit of length ($ls = 1m$) and time (*sidereal day*) are the same in *ss* and *shob*; light signals do not enter in this set up at all. Our task now is to set up time and distance in *S*. Being Euclidean, the metric form of the space in *S* is

$$dS^2 = dR^2 + R^2(d\theta^2 + sin^2 d\emptyset^2)$$

The frame *S* is stationary; it is the frame of fixed stars, and a unit of length *ls*, which is inherited from *ss* is already available. The frame *S*, being a timed inertial frame, can now be endowed with geometric distance either by means of rulers or through light signals; both ways are equivalent (see timed frames (part XI)). Practically, we cannot use solid rods to measure the distance between two points in the space S, or in *shob* in particular, except when these points are close enough (in a cabin of *shob* or in S), and we appeal therefore to light signals to relate time and distance measurements in the frame of fixed stars S. Employing the postulate: light propagates within S at a constant velocity c, we may measure geometric distance through light signals (see timed frames (part XI)). We must keep in mind however that a unit of length should be already available in S, which is the case here, since the unit of length ls in ss is inherited in *shob*, and accordingly, base lines on *shob* and inside (or on ss and inside) are already available for triangulation. Now if $Q \in S$ is a source of light (or a point) outside *shob* then the coordinates $(R, \theta, \emptyset)$ of Q are easily determined. One way of doing so, is to assign to Q the same angular coordinates $(\theta, \emptyset)$ of the observer $q \in shob$ that sees Q overhead (we shall replace soon the observers in *shob* by observers in ss).The radial coordinate R is easily calculated by means of triangulation. Indeed, if $q' \in shob$ sees Q in his horizon, i.e., the line of sight from q' to Q is tangential to *shob*, then the angle $\angle(oQ, oq') = \varphi$ is known and $R = a/cos\varphi$. It is clear however that observation from *shob* is illusive because it is utterly impractical; it can be performed only as a "thought observation". These observations however can be done certainly by the conjugate observers in ss, with the provision that they are done at the same instant of time. Indeed, if the observers $qss \in ss\ and\ q'ss \in ss$ are conjugate to $q, q' \in shob$ at the same instant $t_0 = 0$ then the same reasoning above applies and the coordinates of Q are $(R = a/cos\varphi\,, \theta, \emptyset)$, where a is the radius of ss and $(\theta, \emptyset)$ are the coordinates of $qss \in ss$. The aberration effect should also be taken into account.

We very well know however that the synchronization process need not to actually be done, and that it can't be done practically for any reference frame. In fact synchronization is not a scheme for implementation, but rather a "thought" method to justify the concept of global time. What happens in reality is that we observe light emanating from a source that is usually not stationary in our frame of reference, and by knowing just its velocity and geometric distance from us at the instant of emission, we can determine the travel time of the signal that we have received. That what the scaling theory does.

We finally note that the described procedure provides only an approximate universal time in a geocentric system. This is because the orbital motion of the earth round the sun is only almost uniform during short periods of time, and in real terms, the system earth-moon falls permanently to the sun due to sun's gravitational field. Moreover, the earth is a bit bulged at its equator, and not a perfect sphere. As a result the rotation axis of the earth precesses about the direction perpendicular to the ecliptics making a full round in about 2540 years[30].