The Scaling Transformations IIIb
Doppler's Effect and Ives-Stilwell Data

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In a previous work[1,2] we have presented lucid explanation of some well-known phenomena, mostly optical, using scaling transformations. In this article we discuss Doppler's effect as prescribed by the scaling theory in comparison with the relativistic prediction.

Consider a source of light *b* moving in the frame $S \equiv OXYZ$ with velocity $\vec{u} = u\vec{i}$, with $\vec{i}$ is the unit vector of the *X*-axis. Let $s \equiv oxyz$ be a frame of reference in which the source b is stationary. Suppose that the source *b* is radiating a monochromatic light of a characteristic wave-length λ_0. The light emitted from *b* is received by any *s* observer and in particular by the observer *o*, as a monochromatic light of the same wave-length λ_0. At any instant of actual observation the radius vectors $\vec{R}$ and $\vec{r}$ of the source *b* in *S* and *s* respectively are related by the scaling transformations

(1) $$\vec{R} = \Gamma(\theta,u)\vec{r}, \quad T = \Gamma(\theta,u)t,$$

with

$$\Gamma(\beta,\theta) = (1-\beta^2)^{-1/2}(\beta\cos\theta + \sqrt{1-\beta^2\sin^2\theta}),$$

$\theta = \angle(\vec{R},\vec{u})$, and where the passive view is being adopted. If the distance *r* corresponds to one wave length λ_0 in *s*, then the distance *R* corresponds to one wave length λ in *S* (which is the distance between two nodes, for example). Setting $R = \lambda$ and $r = \lambda_0$ in (1) yields the generalized Doppler's formula

(2) $$\lambda = \Gamma(\beta,\theta)\lambda_0,$$

which determines the wave length as measured by the observer *O* in the frame *S*. The last relation shows that $\lambda > \lambda_0$ for $0 < \theta < \pi/2$, and $\lambda < \lambda_0$ for $\pi/2 < \theta < \pi$. The source *b* is receding from the observer in the first case, and approaching it in the second.

In comparison, the generalized relativistic Doppler's formula corresponding to the case we have discussed is

(3) $$\lambda = \frac{1+\beta\cos\theta}{\sqrt{1-\beta^2}}\lambda_0$$

The generalized formula (2) and the relativistic one (3) both reduce, for $\theta = 0$, to the red shift relativistic Doppler's formula

(4) $$\lambda = \Gamma(\beta,0)\lambda_0,$$

corresponding to the source and the observer receding from each other, and for $\theta = \pi$, both reduce to the blue shift relativistic Doppler's formula

(5) $$\lambda = \Gamma(\beta,\pi)\lambda_0,$$

corresponding to the source and the observer approaching each other.

The wave length predicted by Einstein's formula is in general longer than the corresponding one of the current theory. The difference however is of second order in

$\beta \sin\theta$ times λ_0. Denoting the wave length predicted by Einstein's formula by λ_E and retaining λ to represent the wave length predicted by the current theory, we have

(6)
$$\lambda_E - \lambda = \left(\frac{1+\beta\cos\theta}{\sqrt{1-\beta^2}} - \frac{\beta\cos\theta + \sqrt{1-\beta^2\sin^2\theta}}{\sqrt{1-\beta^2}} \right) \lambda_0 = \frac{1-\sqrt{1-\beta^2\sin^2\theta}}{\sqrt{1-\beta^2}} \lambda_0 .$$

On neglecting powers of fourth order and higher in $\beta\sin\theta$ the formula (6) is approximated by the expression

(7)
$$\lambda_E - \lambda \approx \frac{1+\beta\cos\theta - \frac{1}{2}\beta^2\sin^2\theta}{\sqrt{1-\beta^2}} \lambda_0 = \lambda_E - \frac{1}{2}\beta^2\sin^2\theta\lambda_0 ,$$

which shows that the wavelength predicted by the current theory is less than the relativistic wave length by an angle depending term in the second power of $\beta\sin\theta$. This difference vanishes for $\theta = 0$ or $\theta = \pi$, resulting in identical predictions by both theories as it was already expressed by equations (4) and (5). The predictions of the two formulae become qualitatively distinct for $\theta = \pi/2$, in which case the relativistic formula (3) predicts a red shift given by

(8)
$$\lambda_E = \frac{\lambda_0}{\sqrt{1-\beta^2}}$$

whereas the relation (10.2) reduces to

(9)
$$\lambda = \Gamma(\beta, \tfrac{\pi}{2})\lambda_0 = \lambda_0 ,$$

which, contrary to the relativistic prediction, shows that there is no traverse Doppler's effect.

At his this stage it seems necessary to compare the predictions of the scaling theory with the results of the Ives-Stilwell experiment[3,4] which is claimed to confirm the predictions of the special theory of relativity. To specify the goal of the experiment, we denote the wavelengths emitted from an approaching and receding sources by λ_a and λ_r respectively. The Ives-Stilwell experiment was designed to measure the shift

(10)
$$\Delta\lambda = \tfrac{1}{2}(\lambda_a + \lambda_r) - \lambda_0 .$$

In the relativistic theory

(11)
$$\lambda_{Er} = \gamma(1+\beta\cos\theta)\lambda_0, \quad \lambda_{Ea} = \gamma(1-\beta\cos\theta)\lambda_0 ,$$

and the shift of wave length is

(12)
$$\Delta\lambda_E = (\gamma - 1)\lambda_0 \approx \tfrac{1}{2}\beta^2\lambda_0 .$$

In the scaling theory

(13a)
$$\lambda_r = \gamma(\beta\cos\theta + \sqrt{1-\beta^2\sin^2\theta}) ,$$

(13b)
$$\lambda_a = \gamma(-\beta\cos\theta + \sqrt{1-\beta^2\sin^2\theta}) ,$$

and the wavelength shift is

(14)
$$\Delta\lambda = (\gamma\sqrt{1-\beta^2\sin^2\theta} - 1)\lambda_0 \approx \tfrac{1}{2}\beta^2\cos^2\theta\lambda_0 = (\Delta\lambda_E)\cos^2\theta .$$

The last relation shows that the scaling theory predicts in general a smaller shift than the relativistic one, and the two prediction coincide for $\theta = 0$ or $\theta = \pi$.

In Ives-Stilwell's experiment[6-15] a small concave mirror is set at an angle $\theta = 7^0$ with the ions velocity to reflect the emitted radiation backwards. As (14) shows, the relativistic prediction should be scaled by a factor $\cos^2 7^0 \approx 0.985$ producing accordingly a smaller shift.

The following table in which the first and second columns are quoted from reference[4] displays some of the predictions of the special theory of relativity and the scaling theory together with the observed shift in Ives and Stilwell experiment.

the relativistic prediction $\Delta\lambda_E = \frac{1}{2}\lambda_0\beta^2$	observed shift (Ives-Stilwell)	the scaling prediction $\Delta\lambda = \frac{1}{2}\lambda_0\beta^2\cos^2\theta$
0.0202	0.0185	0.0198
0.0243	0.0225	0.0239
0.0280	0.0270	0.0275
0.0360	0.0345	0.0354
0.0478	0.0470	0.0470
0.0670	0.0670	0.0660
0.0686	0.0675	0.0675
0.0869	0.0900	0.0856

The table shows that the prediction of the scaling theory are competitive with the relativistic ones. It must be mentioned however that, on many aspects, the Ives-Stilwell results were judged as inconclusive[4].

References

[1] Viazminsky C P, "Tangible grounds for Lorentz and generalized Lorentz transformations", Apeiron, (2008) **15(1)**.

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[3] French A P. *Special relativity*. Butler & Tanner Ltd, Frome and London, 1968.Ch.2&3.

[4] Faraj, A., The Ives-Stilwell experiment, The General Science Journal, (2005) .