Justification for High Speed Reenactments of the Ives-Stilwell Experiment

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1. Abstract

A Special Theory of Relativity (STR) test theory is a generalization of the Lorentz transforms using additional parameters. One can then analyze experiments using the test theory instead of STR and fit the parameters of the test theory to the experimental results. If the fitted parameter values differ significantly from the values corresponding to STR, then the experiment is inconsistent with STR. But more normally, such fits can show how well a given experiment confirms or disagrees with STR and what is its experimental accuracy. This gives a general and tractable method of analysis which can be common to multiple experiments.

Keywords: RMS, Standard Model, Ives Stilwell experiment, STR

2. Introduction

Different test theories differ in their assumptions about the form of the Lorentz transforms. There are at present three test theories of STR[1,2,3] . Zhang[4] discusses the interrelationships and presents a unified test theory encompassing the other three, with a better and more interpretable parameterization. His discussion implies that there will be no more test theories of STR that are not reducible to one of the first three. He named this theory RMS. Zhang also showed that modern experiments determine the Lorentz transforms coefficients to within a few parts per million. These test theories can also be used to examine potential alternate theories to STR - such alternate theories predict particular values of the parameters of the test theory, which can easily be compared to values determined by experiments analyzed with the test theory. The existing experiments put rather strong experimental constraints on any alternative theory. In particular, Zhang showed that these experimental limits essentially require that any theory based upon the existence of an ether be experimentally indistinguishable from STR and have an ether frame which is unobservable (the only alternative is for a theory to "live in the error bars" of the experiments, which is quite difficult given the high accuracies achieved by many of these experiments). RMS starts by admitting by reduction to absurd that there is a preferential inertial frame Σ in which the light propagates isotropically with the speed c_0. All other frames in motion with respect to Σ are considered non-preferential and the light speed is anisotropical. Let S be such a frame. If at t=0 a beam of light is emitted in Σ and if S moves with the speed v with respect to Σ and if v makes the angle $\theta<\pi/2$ with respect to the direction of the light beam then RMS assumes that S measures for light the speed c, where :

$$c(\theta,v)=c_0(1+A\frac{v^2}{c_0^{\ 2}}\sin^2\theta+B\frac{v^2}{c_0^{\ 2}}) \qquad [2.1]$$

Parameter A is a measure of light speed isotropy and is generally measured through a Michelson-Morley class of experiments. Parameter A has been tested by [5,6,7,8,9,10,11] to be less than $3*10^{-15}$ Parameter B is a measure of light speed invariance relative to the speed of the emitter/receiver and it is generally measured through Kennedy-Thorndike experiments. Parameter B has been tested by [12.13.14] to be less than $2*10^{-13}$. STR predicts A=B=0 and the experimental asymptotical limits for both A and B under STR are indeed zero. Non STR theories have non zero values for A and B. Any supporter of such theories needs to reconcile the theoretical values predicted for A and B with the experimental values. In other words, any proponent of a theory competing with STR must pass the RMS test. While traditionally the bracketing of the A and B parameters is done through the Michelson-Morley and Kennedy-Thorndike reenactments, in the following we will take the somewhat unorthodox step of using the Ives-Stilwell experiment for the purpose of setting the limits on A and B.

3. The Doppler Formulas of the Classical Theory

Let $\beta = v/c$, where v is the velocity of the source of light with respect to the observer, and c is the velocity of light relative to the reference frame in which the source is at rest. For the approaching source of light:

$$\omega_a = \omega(1-\beta\cos\theta) \qquad [3.1]$$

where ω_a is the frequency as measured by the observer, ω is the rest frequency, and θ is the angle made to the observer line of sight by the velocity vector of the source. For the receding source of light:

$$\omega_r = \omega(1 + \beta\cos\theta) \qquad [3.2]$$

4. The Doppler Formulas of Special Relativity as Applied to the Ives-Stilwell Experiment[16-17]

Let β, ω_a, ω_r, ω, and θ be as defined above.

and let $\gamma = [1 - v^2/c^2]^{-1/2}$.

For the approaching source of light, therefore,

$$\omega_a = \omega\gamma(1 - \beta\cos\theta) \qquad [4.1]$$

And for the receding source of light,

$$\omega_r = \omega\gamma(1 + \beta\cos\theta) \qquad [4.2]$$

From these two equations, we calculate the average wavelength, Ω,

$$\Omega = ½(\omega_a + \omega_r) = \omega\gamma \qquad [4.3].$$

Let $\Delta\omega = \Omega - \omega = \omega(\gamma - 1)$ [4.4]

which is the value predicted by the Theory of Special Relativity for the Ives-Stilwell[16] experiment.

5. The Doppler Formulas of the Test Theory:

Let c_a denote velocity of light from the approaching source. According to [2.1]:

$$c_a = c[1 + A(v^2/c^2)\sin^2\theta + B\, v^2/c^2] \qquad [5.1]$$

Let c_r denote velocity of light from the receding source. Since the speed of light is no longer isotropic within the framework of RMS:

$$c_r = c[1 - A(v^2/c^2)\sin^2\theta + B\, v^2/c^2] \qquad [5.2]$$

Let $D = 1 + B\beta^2$ [5.3]

$$c_a = c(D + A\,\beta^2\sin^2\theta) \qquad [5.4]$$

$c_r = c(D - A\beta^2 \sin^2\theta)$ [5.5]

Let $\beta_a = v/c_a$ and let $\beta_b = v/c_b$

For the approaching source of light, we compute the observed frequency[17]:

$$\omega_a = \omega\gamma(1 - \beta_a \cos\theta) \quad [5.6]$$

For the receding source of light, we compute the observed frequency[17]:

$$\omega_b = \omega\gamma(1 + \beta_b \cos\theta) \quad [5.7]$$

$$\Omega = ½(\omega_a + \omega_r) = \omega\gamma(1 + \frac{A\beta^3 \sin^2\theta\cos\theta}{D^2 - A^2\beta^4\sin^4\theta}) \quad [5.8]$$

$$\Delta\omega = \Omega - \omega = \omega(\gamma - 1) + \omega\gamma\frac{A\beta^3 \sin^2\theta\cos\theta}{D^2 - A^2\beta^4\sin^4\theta} \quad [5.9]$$

The term $\Delta_{err} = \omega\gamma\frac{A\beta^3 \sin^2\theta\cos\theta}{D^2 - A^2\beta^4\sin^4\theta}$ [5.10]

represents the STR violation as predicted by the RMS test theory. In order for the violation to be significant we would need to:

-have v as close to c as possible, hence the need for higher and higher speed reenactments of the Ives-Stilwell experiment (the current top limit is set[19] at β=0.28)

-have θ different from both 0 and $\pi/2$ (it can be shown that if we neglect the influence of the term in β^4 then the expression Δ_{err} is maximum when $\sin^2\theta\cos\theta$ reaches a maximum and this happens for $\theta=\arccos(\frac{1}{\sqrt{3}})$)

For the above conditions, the term $\Delta_{err} \approx \omega\gamma\frac{A\beta^3}{(1+B\beta^2)^2}\frac{2}{3\sqrt{3}}$ [5.11]

5. Conclusion

According to modern reenactments of the Ives Stilwell experiment STR agrees[18] with the experimental data within 10^{-9}. According to the authors of[18] the result confirms the relativistic Doppler formula and sets a new limit of 2.2×10^{-7} for deviations from the time dilation factor $\gamma_{SR} = (1-v^2/c^2)^{-1/2}$ at v=0.064c. Higher values of v are needed in order to better detect the RMS error term [5.11]

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