The "Twins Paradox" - Demystified

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There is a famous series of paintings by the surrealist painter Salvador Dali, entitled "The Persistence of Memory". In the following we will discuss the persistency of an error introduced by H. Dingle several years after Einstein's death.

The error stems from a misinterpretation of chapter 4 "Physical Meaning of the Equations Obtained in Respect to Moving Rigid Bodies and Moving Clocks"[1]. In the following we will maintain Einstein's notation in order to allow for an easier comparison with the original paper. In this chapter Einstein starts from the well known Lorentz transformation:

$$\tau=\beta(t-vx/c^2) \qquad (1.1)$$

He assumes that there is a system (k) that has a clock situated in the origin $\xi=0$ that measures the time τ. The system (or inertial frame) (k) is characterized by the coordinates ξ,τ. The system (k) moves with speed v with respect to another system (K), characterized by the coordinates x,t.

The **fixed point** $\xi=0$ in (k) corresponds to the **variable point** x=vt in (K). Substituting in (1.1) Einstein obtained[1] :

$$\tau=\beta(1-v^2/c^2)t=t/\beta \qquad (1.2)$$

Dingle and quite a few others after him proposed the following "paradox": they considered the inverse Lorentz transformation:

$$t=\beta(\tau+v\xi/c^2) \qquad (1.3)$$

Then, Dingle made the error of assuming that, from the perspective of frame (k)

$$\xi= - v\tau \qquad (1.4)$$

Substituting (1.4) into (1.3) Dingle obtained :

$$t=\beta\tau(1-v^2/c^2)= \tau/\beta \qquad (1.5)$$

Obviously, (1.5) contradicts (1.2). Dingle's error is easily corrected by observing that **variable point** x=vt in frame (K) corresponds to the **fixed point** $\xi=0$ in frame (k) and not to the **variable point** $\xi= - v\tau$. Now, by correctly substituting $\xi=0$ into (1.3) one obtains the correct expression for t:

$$t=\beta\tau \qquad (1.5a)$$

In other words, the relativistic event **a**($\xi=0, t=\beta\tau$) in (k) corresponds to the event **A**($x=vt$, $\tau=t/\beta$) in (K). Both events measure the **same** passage of time.
Considering now the **fixed point** $x=0$ in (K) corresponding to the **variable point** $\xi= - v\tau$ in (k) and considering the inverse Lorentz transformation:

$$t=\beta(\tau+v\xi/c^2) \qquad (1.6)$$

we obtain:

$$t=\beta\tau(1-v^2/c^2)= \tau/\beta \qquad (1.7)$$

Substituting $x=0$ in (1.1) we obtain :

$$\tau=\beta t \qquad (1.8)$$

In other words the event **B**($x=0,\tau=\beta t$) in(K) corresponds to the event **b**($\xi=-v\tau, t=\tau/\beta$) in (k). Both events measure the **same** passage of time.
The error made by Dingle and his followers is the unjustified attempt to compare events **A** and **B** in (K) or **a** and **b** in (k).
It is clear that (1.5a) is in agreement with (1.2) and so are (1.7) and (1.8), they represent the **same** expression and there is no "paradox". Over time the "Dingle paradox" has evolved into "the twins' paradox", the same persistent error. Dingle waged a long correspondence with the editors of "Nature" on this subject.

In chapter 4[1] Einstein wrote: "From this there ensues the following peculiar consequence. If at the points A and B of K there are stationary clocks which, viewed in the stationary system, are synchronous; and if the clock at A is moved with the velocity *v* along the line AB to B, then on its arrival at B the two clocks no longer synchronize, but the clock moved from A to B lags behind the other which has remained at B by $tv^2/2c^2$ (up to magnitudes of fourth and higher order), *t* being the time occupied in the journey from A to B. …..".

Dingle and his followers thought that they have proved that: "If at the points A and B of K there are stationary clocks which, viewed in the stationary system, are synchronous; and if the clock at A is moved with the velocity *v* along the line AB to B, then on its arrival at B the two clocks no longer synchronize, but the clock moved from A to B lags behind the other which has remained at B by $tv^2/2c^2$ *AND the clock in B is* ***slower*** *than the other one at A by* $tv^2/2c^2$……."

As we have just shown above, from (1.2) and (1.5a) the correct conclusion is that : "If at the points A and B of K there are stationary clocks which, viewed in the stationary system, are synchronous; and if the clock at A is moved with the velocity *v* along the line AB to B, then on its arrival at B the two clocks no longer synchronize, but the clock moved from A to B lags behind the other which has remained at B by $tv^2/2c^2$ *AND the clock in B is* ***faster*** *than the other one at A by* $tv^2/2c^2$…….." No contradiction.

1. A. Einstein “On the Electrodynamics of Moving Bodies”,Annalen der Physik 17, 1905, http://www.fourmilab.ch/etexts/einstein/specrel/www/