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Fifth Experimental Test of the Theory of General Relativity
The Slowing Down Effect of Moving Clocks

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Abstract
The formula which expresses the difference of the trajectory period of planets with their proper period is deduced. According to this formula , the slowing down effect of clocks situated at rest at the surface of the Earth is 4.665 s/(10 year). If one can made up a clock whose extent of error is smaller than 0.4983 s / each year, then the mythology of the Einstein's theory of general relativity will automatically evaporate.

According to Einstein's theory of general relativity, the motion of Earth in the gravitational field of the Sun must lead to the slowing down effect of standard clocks situated at rest at the surface of Earth. Now the calculation of the effect will be an important criteria of the validity of general relativity since there are atomic clocks with high acuracy which may be considered as standard clocks.

Ⅰ. Kepler's third law in the theory of general relativity

The equations of the motion of planets in the Sun gravitational field can be described by three equations of first integral as follws [1]

$$
\begin{aligned}
&e^{\mu}\left(\frac{dt}{d\tau}\right)^2 - e^{-\mu}\left(\frac{dr}{d\tau}\right)^2 - r^2\left(\frac{d\phi}{d\tau}\right)^2 = 1,\\
&e^{\mu}\frac{dt}{d\tau} = k, \qquad\qquad (1)\\
&r^2\frac{d\phi}{d\tau} = h,
\end{aligned}
$$

where

$$e^{\mu} = 1 - \frac{2m}{r},$$

and m to be the gravitation radius of the Sun: m= 1.48 (km)

One obtains from these equations the following equation of the trajectory of planets [1]

$$r = \frac{p}{1 + e\cos\rho\phi},$$

where

$$\rho = 1 - \frac{3m^2}{h^2}, \; h = \sqrt{mp}.$$

The trajector periods Tc of planets calculated in the sun's frame of reference can be obtained by eliminated the variable r in the equations (1) as follows

$$\mathrm{Tc} = \frac{p^2 k}{h} \int_0^{2\pi/\rho} \frac{d\phi}{(1+ e\cos\rho\phi)^2 \left[1 - 2m(1+ e\cos\rho\phi)/p\right]}. \tag{2}$$

Introduce a complex variable z = exp (i $\rho\phi$), we obtain

$$\mathrm{Tc} = \frac{p^2 k}{ih\rho} \oint_{|z|=1} \frac{zdz}{(\frac{1}{2}ez^2 + z + \frac{1}{2}e)^2 \left\{1 - \alpha\left[1 + \frac{1}{2}e(z + z^{-1})\right]\right\}}$$

$$= \frac{p^2 k}{ih\alpha\rho} \left(\frac{-2}{e}\right)^3 \oint_{|z|=1} F(z)dz, \tag{3}$$

where $\alpha = 2m/p << 1$ and

$$F(z) = \frac{z^2}{(z^2 + \frac{2}{e}z + 1)^2 (z^2 + \frac{2}{e^*} + 1)},$$

where

$$e^* = \frac{\alpha}{1-\alpha}e.$$

Put

$$z^2 + \frac{2}{e}z + 1 = 0, \tag{4}$$

we obtain two roots

$$z_1 = \frac{1}{e}\left(\sqrt{1-e^2} - 1\right), \; z_2 = -\frac{1}{e}\left(\sqrt{1-e^2} + 1\right). \tag{5}$$

where $|z_1| < 1$.

Put

$$z^2 + \frac{2}{e^*}z + 1 = 0, \tag{6}$$

we obtain two roots

$$z_3 = \frac{1}{e^*}\left(\sqrt{1+e^{*2}} + 1\right), \; z_4 = \frac{1}{e^*}\left(1 - \sqrt{1-e^{*2}}\right)., \tag{7}$$

where $|z_4| < 1$.

Now the function F(z) can be written in the form

$$F(z) = \frac{z^2}{(z - z_1)^2 (z - z_2)^2 (z - z_3)(z - z_4)} . \tag{8}$$

Applied the techniques of complex function integral,we have

$$\oint_{|z|=1} F(z)dz = 2\pi i\{res(F(z_1)) + res(F(z_4))\} ,$$

where

$$res(F(z_1)) = \lim_{z\to z_1} \frac{d}{dz}\left[(z - z_1)^2 F(z)\right] == f(z_1)(2 - g(z_1)),$$

$$f(z_1) = \frac{z_1}{(z_1 - z_2)^2 (z_1 - z_3)(z_1 - z_4)}$$

$$g(z_1) = z_1\left(\frac{2}{z_1 - z_2} + \frac{1}{z_1 - z_3} + \frac{1}{z_1 - z_4}\right),$$

By means of Eqs.(4) and (6) we obtain

$$\begin{aligned}(z_1 - z_3)(z_1 - z_4) &= z_1^2 - \frac{2}{e^*} z_1 + 1 \\ &= z_1(z_1 + \frac{1}{z_1} + \frac{2}{e} - (\frac{2}{e} + \frac{2}{e^*})) = - z_1\left(\frac{2}{e} + \frac{2}{e^*}\right) \\ &= \frac{2}{e^2\alpha}\left(1 - \sqrt{1 - e^2}\right)\end{aligned}$$

$$\begin{aligned}\frac{1}{z_1 - z_3} + \frac{1}{z_1 - z_4} &= \frac{2z_1 - (z_3 + z_4)}{(z_1 - z_3)(z_1 - z_4)} = \\ &= \frac{1}{e}\left(\alpha\sqrt{1 - e^2} - 1\right)\left(\sqrt{1 - e^2} + 1\right)\end{aligned}$$

and then we have

$$f(z_1) = - \frac{e^3\alpha}{8(1 - e^2)},$$

$$g(z_1) = 2 - \frac{1}{\sqrt{1 - e^2}} - \alpha\sqrt{1 - e^2},$$

$$res(F(z_1)) = - \frac{e^3\alpha}{8(1 - e^2)^{3/2}}\left[1 + \alpha(1 - e^2)\right]. \tag{9}$$

Since z_4 to be first order singular point we have

$$res\left(F(z_4)\right) = \lim_{z \to z_4}\left[(z - z_4)F(z)\right] = \frac{z_4^2}{(z_4 - z_1)^2 (z_4 - z_2)^2 (z_4 - z_3)}.$$

Considering z_4 to be the root of Eq.(6) we have

$$z_4 + \frac{1}{z_4} - \frac{2}{e^*} = 0,$$

$$\left(z_4 - z_1\right)^2 \left(z_4 - z_2\right)^2 = z_4^2 \left(z_4 + \frac{1}{z_4} + \frac{2}{e} \right)^2$$

$$= z_4^2 \left(\frac{2}{e^*} + \frac{2}{e} \right)^2 = \frac{4z_4^2}{(\alpha\, e)^2},$$

$$z_4 - z_3 = -\frac{2}{e^*}\sqrt{1 - e^2},$$

and then

$$res\left(F(z_4)\right) = -\frac{1}{8}\frac{\alpha^2 e^* e^2}{\sqrt{1 - e^{*2}}} = -\frac{1}{8}\frac{\alpha^3 e^3}{\sqrt{(1-\alpha)^2 - \alpha^2 e^2}}. \qquad (10)$$

From (3), (9) and (10) we obtain

$$T_c = \frac{2\pi p^2 k}{h\rho \left(1 - e^2\right)^{3/2}}\left[1 + \left(1 - e^2\right)\alpha\right] + o(\alpha), \qquad (11)$$

which can be expressed in the form

$$\mathrm{Tc} = \mathrm{Tn}\, \mathrm{k}\, \rho^{-1}\left(1 + \alpha\,(1 - e^2)\right) + \mathrm{o}(\alpha), \qquad (12)$$

where

$$\mathrm{Tn} \overset{def}{=} 2\pi p^2 h^{-1} \Big/ \sqrt{(1 - e^2)^3}. \qquad (13)$$

By considered the second equation in (1), the periods of the proper time of planets can be determined as follows

$$\mathrm{Tp} = \frac{p^2}{h}\int_0^{2\pi/\rho} \frac{d\phi}{(1 + e\cos\rho\,\phi)^2} = \mathrm{Tn}/\rho, \qquad (14)$$

which is the recorded time by the clocks situated at rest at the surfaces of the planets which move in a cicle around the Sun.

In order to determine the constant k in the formula (12) , we consider the following equation

$$\left(\frac{du}{d\phi}\right)^2 = -\left(1 - k^2\right)\frac{1}{h^2} + \frac{2m}{h^2}u - u^2 + 2mu^2, \qquad (15)$$

which comes from (1)[1] and where $u = 1/r$ .We then have

$$\frac{du}{d\phi} = -\frac{m}{h^2} ep \sin\rho\phi \ .$$

Putting $\phi = 0$, we obtain

$$\frac{du}{d\phi} = 0,\ u = \frac{1+e}{p}, \qquad (16)$$

and then we obtain from (15)

$$k = \sqrt{1-\frac{1}{2}\alpha\left(1-e^2\right)-\frac{1}{2}\alpha^2\left(1+e\right)^3} = 1-\frac{1}{4}\alpha\left(1-e^2\right)+o(\alpha). \qquad (17)$$

If we ignore α in (12), (14), we then have $\rho = k = 1$. The formula (12) reduces to

$$\text{Tc} = 2p^2\pi h^{-1}\Big/\sqrt{(1-e^2)^3}\ .$$

Replacing h by $c\sqrt{mp}$, we obtain from above formula

$$\frac{T_c^{\ 2}}{a^3} = \frac{4\pi^2}{mc^2}, \qquad (18)$$

where $a = p/\left(1-e^2\right)$. The formula (18) is just the Kepler's third law. Therefore (12) can be expressed as follows

$$\frac{T_c^{\ 2}}{a^3} = \frac{4\pi^2}{mc^2}\eta\ , \qquad (19)$$

where

$$\eta = k\rho\left\{1+\alpha\left(1-e^2\right)+\frac{\alpha^2(1-e^2)^{3/2}}{\sqrt{1-2\alpha+\left(1-e^2\right)\alpha^2}}\right\} =$$

$$= \sqrt{1-\frac{1}{2}\alpha\left(1-e^2\right)-\frac{1}{2}\alpha^2\left(1+e\right)^3}\left(1-\frac{3}{2}\alpha\right)$$

$$\times\left\{1+\alpha\left(1-e^2\right)+\frac{\alpha^2(1-e^2)^{3/2}}{\sqrt{1-2\alpha+\left(1-e^2\right)\alpha^2}}\right\},$$

which can be reduced into the form

$$\eta = 1-\frac{3}{4}\alpha\left(1+e^2\right)+o(\alpha) \qquad (20)$$

Since $\alpha = 2m/p = 2m/\left(a\left(1 - e^2\right)\right)$, we have

$$\eta \approx 1 - \frac{3}{2}\left(\frac{m}{a}\right)\left(\frac{1 + e^2}{1 - e^2}\right). \qquad (10)$$

Ⅱ. The slowing down effect of the clocks situated at rest at the surface of the Earth

By considered the second equation in (1), the period of the proper time of planets can be determined as follows

$$\mathrm{Tp} = \frac{p^2}{h}\int_0^{2\pi/\rho} \frac{d\phi}{(1 + e\cos\rho\phi)^2} = \frac{T_n}{\rho} \quad , \qquad (11)$$

which is the recorded time by the clocks situated at rest at the surfaces of the planets when which move a cicle around the Sun..

Therefore we obtain, from the Eqs.(4),(6) and (7)

$$\Delta T = T_c - T_p = \frac{3}{4}\alpha\left(1 - e^2\right)T_c + o(\alpha). \qquad (12)$$

We then have

$$\Delta T \approx \frac{3}{2}\frac{m}{a}T_c. \qquad (13)$$

Take m = 1.48 (km) (the gravitation radius of the Sun), for the Earth ,Tc = $0.316 \cdot 10^8$ s , and a = $1.495 \cdot 10^8$ km [(3)], we then obtain

$$\Delta T \approx \frac{3 \times 1.48km}{2 \times 1.495 \times 10^8 km} \times 3.16 \times 10^7 s = 0.4893s \qquad (14)$$

The clocks situated at rest at the surface of the Earth clock will slowing down 4.893 s every ten years.

If one can made up a clock whose extent of the error is smaller than 0.4983 s / every year, then the mythology of the Einstein 's theory of general relativity will automatically evaporate.

From (12) , (14) and (17) we obtain

$$\Delta T = T_c - T_p = \frac{T_n}{\rho}\left\{\left[1 - \frac{1}{4}\alpha\left(1 - e^2\right)\right] \times \left[1 + \alpha\left(1 - e^2\right)\right] - 1\right\} + o(\alpha)$$

$$= \frac{3}{4}T_p\frac{2m}{p}\left(1 - e^2\right) + o(\alpha) = \frac{3}{2}\left(\frac{m}{a}\right)T_p + o(\alpha)$$

where

$$a = \frac{p}{1-e^2}.$$

We have

$$T_p = T_c - \Delta T,$$

$$\Delta T = \frac{3}{2}\left(\frac{m}{a}\right)(T_c - \Delta T) + o(\alpha)$$

$$= \frac{3}{2}\left(\frac{m}{a}\right)T_c - o(\alpha)\Delta T + o(\alpha),$$

and then

$$\Delta T = \frac{3}{2}\left(\frac{m}{a}\right)T_c. \qquad (18)$$

Take m = 1.48 (km) (the gravitation radius of the Sun), for the Earth ,Tc = 0.316•10^8 s , and a = 1.495•10^8 km[3] , we then obtain

$$\Delta T \approx \frac{3\times 1.48km}{2\times 1.495\times 10^8 km}\times 3.16\times 10^7 s = 0.4893s$$

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