

The Coriolis Force in Maxwell's Equations

(A comparative study of Maxwell's 1865 paper 'A Dynamical Theory of the Electromagnetic Field' and his 1861 paper 'On Physical Lines of Force')

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Abstract. Maxwell's 1865 paper 'A Dynamical Theory of the Electromagnetic Field' appears to abandon the theory of molecular vortices that was a central feature of his 1861 paper 'On Physical Lines of Force'. After writing part I of his 1861 paper, Maxwell realized that a purely hydrodynamical approach to electromagnetic theory was insufficient and so he introduced electrical particles as idle wheels rolling around the outside of his molecular vortices. Maxwell was never clear about the details of the connecting mechanism between the electrical particles and the vortices and he gradually shifted towards a more elasticity based approach in which he emphasized the dielectric nature of the aether as opposed to the vortex nature.

This article investigates whether or not any physics was lost as a result of Maxwell apparently having abandoned his theory of molecular vortices by 1864. The focus of attention is centred on equation (5) of his 1861 paper as this equation contains the Coriolis force. Maxwell used the mathematical form of the Coriolis force to derive Ampère's Circuital Law and this paper will demonstrate that the Coriolis force can also be used to derive the $\mathbf{v} \times \mathbf{B}$ component of the Lorentz force. Since a rotating frame of reference is needed for a Coriolis force, it follows therefore that Ampère's Circuital Law and the $\mathbf{v} \times \mathbf{B}$ component of the Lorentz force must depend entirely on the fine-grain rotating aethereal substance within Maxwell's molecular vortices. The conclusion is that Maxwell need not have had any hesitation at all regarding his theory of molecular vortices, and that

the result of playing it down was that the physical explanation for $\nabla \times \mathbf{B}$ and Ampère's Circuital Law was lost to future generations.

Introduction

I. Maxwell's 1865 paper 'A Dynamical Theory of the Electromagnetic Field' [1],

http://www.zpenergy.com/downloads/Maxwell_1864_1.pdf
http://www.zpenergy.com/downloads/Maxwell_1864_2.pdf
http://www.zpenergy.com/downloads/Maxwell_1864_3.pdf
http://www.zpenergy.com/downloads/Maxwell_1864_4.pdf
http://www.zpenergy.com/downloads/Maxwell_1864_5.pdf
http://www.zpenergy.com/downloads/Maxwell_1864_6.pdf
<http://www.zpenergy.com/downloads/Diagram.pdf>

was so named because it adopted what he considered to be a more dynamical approach to electromagnetism as compared with his earlier approaches. His 1861 paper 'On Physical Lines of Force' [2]

http://vacuum-physics.com/Maxwell/maxwell_oplf.pdf

began with a purely hydrodynamical approach. Then in part II, Maxwell introduced electrical particles as idle wheels in order to give justification for the continuing existence of his sea of molecular vortices, but he was never clear about the precise details surrounding the interaction between these particles and the vortices themselves.

Maxwell said towards the end of part II of his 1861 paper '*The conception of a particle having its motion connected with that of a vortex by perfect rolling contact may appear somewhat awkward. I do not bring it forward as a mode of connexion existing in nature, or even as that which I would willingly assent to as an electrical hypothesis. It is, however, a mode of connexion which is mechanically conceivable, and easily investigated, and it serves to bring out the actual mechanical connexions between the known electro-magnetic phenomena; an that I venture to say that any one who understands the provisional and temporary character of this hypothesis, will find himself rather helped than hindered by it in his search after the true interpretation of the phenomena*'.

In his 1865 paper the vortex sea idea was not mentioned and Maxwell adopted a purely dynamical approach. It is clear from sections (24) and (25) entitled ‘Dynamical Illustration of Reduced Momentum’ that the vortex nature of electromagnetism was being alternatively illustrated dynamically in terms of a fly-wheel nature.

The Introduction of Electrical Particles

II. It was inevitable that Maxwell would have to introduce particles into his sea of aethereal vortices. The hydrodynamical equations that Maxwell used in part I of his 1861 paper are classical equations involving concepts such as density, velocity, pressure, and force, and they have no meaning outside of the context of particulate matter. If Maxwell’s vortex sea had been a pure dynamic aether without sinks or sources, then these classical equations of hydrodynamics could not have been meaningfully applied to it. Maxwell realized this fact indirectly when he realized that there could be no basis for a sea of aether vortices to continue to exist in the absence of surrounding particles. As such he introduced electrical particles as idle wheels to roll around the circumference of his aethereal vortices.

From part II of his 1861 paper onwards he began to pay more attention to the dynamics of the electrical particles. Having already derived Ampère’s Circuital Law (equation (9)) hydrodynamically in part I, Maxwell proceeded to derive it once again in part II, (see equation (33)), but this time using a more dynamical approach, after which he wrote, *“It appears therefore that, according to our hypothesis, an electric current is represented by the transference of the moveable particles interposed between the neighbouring vortices - - -”*.

In part III Maxwell used a purely elastic approach in which he concentrated on the tangential stresses on the electrical particles within his vortex cells.

The Purpose of Maxwell’s 1865 Paper

III. In sections **IV**, **V** and **VI** we will answer the three questions below.

(1) Was Maxwell's 1865 paper intended as a continuation of his 1861 paper?

(2) Was Maxwell's 1865 paper intended as a clarification of his 1861 paper?

(3) Is there anything that Maxwell explained hydrodynamically in part I of his 1861 paper which cannot be adequately explained dynamically in his 1865 paper?

Displacement and Displacement Current

IV. In Maxwell's 1865 paper part VI entitled 'Electromagnetic Theory of Light' we see a continuation of part III of his 1861 paper. In part III of his 1861 paper, Maxwell introduced the concept of electrical displacement in connection with tangential stress on the electrical particles in his vortex sea. He substituted the transverse elasticity and the density of his vortex sea into Newton's mechanical wave speed formula at equation (132) and he showed that electromagnetic waves propagate at the speed of light.

In the preamble of the same part and at equation (111) he introduced the concept of displacement current although he didn't do anything with it other than to add it to Ampère's Circuital Law at equation (112). In part VI of the 1865 paper, Maxwell used displacement current to derive the electromagnetic wave equation (see equation (69)) in conjunction with equations (B), (C), (D), and (E) of the same paper. This derivation of the electromagnetic wave equation is essentially a repeat of the physics that is in part III of the 1861 paper but this time Maxwell used displacement current instead of just simply displacement. It leads to exactly the same conclusion as in part III of the 1861 paper. In Maxwell's own words in his 1861 paper, his conclusion is, " - - - *we can scarcely avoid the inference that light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena* - - - " and in his own words in his 1865 paper, his conclusion is " - - - *that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws.*"

The Original Eight Maxwell's Equations

V. Maxwell's 1865 paper was a clarification of his 1861 paper in the respect that in part III entitled 'General Equations of the Electromagnetic Field', Maxwell chooses to highlight a set of eight equations most of which had previously been presented in his 1861 paper. In the listing below in which we use modern vector notation, we will use the symbol \mathbf{J} to denote electric current density, t to denote time, \mathbf{E} to denote electric field, \mathbf{v} to denote velocity, and \mathbf{D} to denote electric displacement.

The first one of 'Maxwell's Equations' is derived from the principle that the total electric current in any circuit must be the sum of the free current and the displacement current.

$$\mathbf{J} = \mathbf{J}_{\text{free}} + \epsilon d\mathbf{E}/dt \quad \text{(Total Electric Current)} \quad [\mathbf{A}]$$

The symbol ϵ is electric permittivity, which is inversely related to transverse elasticity.

The second Maxwell equation appears as equation (55) in part II of the 1861 paper. This equation tells us that the magnetic flux density vector \mathbf{B} can be derived from a vector \mathbf{A} such that,

$$\text{curl } \mathbf{A} = \mathbf{B} \quad \text{(The Fly-Wheel Equation)} \quad [\mathbf{B}]$$

Regarding the \mathbf{A} vector, Maxwell says "*It corresponds to the impulse which would act on the axle of a wheel in a machine if the actual velocity were suddenly given to the driving wheel, the machine being previously at rest.*" Maxwell expands upon the analogy between the mechanical fly-wheel and electromagnetism in his 1865 paper, in sections (24) and (25) entitled 'Dynamical Illustration of Reduced Momentum'.

The third Maxwell equation was derived hydrodynamically from the Coriolis force in part I of the 1861 paper (equation (9)), and dynamically in part II of the 1861 paper (equation (33)).

$$\text{curl } \mathbf{B} = \mu \mathbf{J} \quad \text{(Ampère's Circuital Law)} \quad [\mathbf{C}]$$

The symbol μ is magnetic permeability which is related to the inertial mass density of the sea of molecular vortices. Equation [C] should be

given a negative sign on the right hand side when it is being used in the context of electromagnetic induction whereas it should be given a positive sign on the right hand side when it is being used for situations in which the electric current creates the magnetic field. This equation demonstrates the inseparability of electric current and magnetic field as well as the linear to rotational relationship which exists between them. If we remove an electric current from a magnetic field abruptly, the magnetic field will burst and sparks will fly.

The fourth Maxwell equation appears as equation (77) in part II of Maxwell's 1861 paper. This equation is commonly known as the 'Lorentz Force'. It contains three terms. These three terms are respectively the force on a moving charged particle in a magnetic field, the torque equation (58) from the 1861 paper, and Gauss's law.

$$\mathbf{E} = \mathbf{v} \times \mathbf{B} - d\mathbf{A}/dt + \text{grad}\psi \quad \text{(The Lorentz Force)} \quad [\mathbf{D}]$$

where ψ refers to electrostatic potential energy. The torque equation is an electromagnetic induction equation that should more accurately contain a partial differential operator.

Using the name 'The Lorentz Force' is somewhat regrettable considering that Maxwell obtained the Coriolis $\mathbf{v} \times \mathbf{B}$ expression hydrodynamically when Lorentz was still a young boy. In 1884, Heaviside and Gibbs dropped the $\mathbf{v} \times \mathbf{B}$ term out of their reformulated versions of Maxwell's equations and in modern day textbooks the Lorentz force appears alongside Maxwell's equations as an electromagnetic equation that is additional to Maxwell's equations, as if it had never been one of Maxwell's equations in the first place. In the next paper 'Gravitational Induction and the Gyroscopic Force' at <http://www.wbabin.net/science/tombe5.pdf> it will be argued that a fourth centrifugal force term is missing from equation **[D]** above.

Maxwell's fifth equation is the electric displacement equation that appeared in part III of the 1861 paper near the beginning and again at equation (105) in the same part.

$$\mathbf{D} = \epsilon \mathbf{E} \quad \text{(Electric Elasticity Equation)} \quad [\mathbf{E}]$$

Maxwell's sixth equation is Ohm's law.

$$\mathbf{J}_{\text{free}} = \sigma \mathbf{E} \quad \text{(Ohm's Law)} \quad [\mathbf{F}]$$

The symbol σ refers to the electrical conductivity.

Maxwell's seventh equation appeared as equation (115) in his 1861 paper, and is simply Gauss's law,

$$\text{div } \mathbf{E} = \rho/\epsilon \quad \textbf{(Gauss's Law of Electrostatics)} \quad \textbf{[G]}$$

The symbol ρ refers to electric charge density.

Maxwell's eighth equation, number (113) in his 1861 paper, is the equation of continuity of electric charge, which was also used by Kirchhoff in his 1857 telegraphy equation paper [3].

$$\text{div } \mathbf{J} = -d\rho/dt \quad \textbf{(Equation of Continuity of Charge)} \quad \textbf{[H]}$$

Equations [A], [B], [D], [E], [F], and [H], would not normally appear in a modern day set of Maxwell's equations. A modern textbook would combine equation [A] with equation [C] as per equation (112) in part III of the 1861 paper, and the combination would be referred to as the Ampère/Maxwell equation. In a modern day textbook, the addition of Maxwell's displacement current to equation [C] would not be explained in terms of total electric current as per Maxwell's 1861 derivation, but rather in terms of adding on an extra term to Ampère's circuital law in order to retain the solenoidal nature of electric current in a capacitor circuit. Modern day displacement current is divorced from its Maxwellian and aethereal origins and it is explained as a time varying quantity that possesses some of the characteristics of electric current without actually constituting a real current.

Modern day sets of Maxwell's equations therefore only contain three of the original set with two of these having been amalgamated into one. Added to these two originals in modern textbooks are Faraday's law of electromagnetic induction and the equation stating that the divergence of \mathbf{B} is always zero. These two extra modern day Maxwell's equations are equations (54) and (57) respectively in part II of Maxwell's 1861 paper.

The Coriolis Force and the Aether

VI. Maxwell clearly failed to notice the physical significance of the two Coriolis force terms which appear as parts 3 and 4 on the right hand side of equation (5) in part I of his 1861 paper. The quantities α , β , and γ that appear in equation (5) are terms that depend on the magnitude of the circumferential velocity of the vortices. They are referred to as 'Magnetic Field Intensity'. From equation (5), we can see that,

$$\mathbf{F}/\text{volume} = \mu \mathbf{v} \times (\text{curl } \mathbf{v}) \quad [\mathbf{I}]$$

where

$$\text{curl } \mathbf{v} = \text{vorticity} \quad [\mathbf{J}]$$

The equation for the Coriolis force is,

$$\mathbf{F} = \mathbf{v} \times 2m\boldsymbol{\omega} \quad (\text{Coriolis Force}) \quad [\mathbf{K}]$$

where $\boldsymbol{\omega}$ is the angular velocity. For non-rigid dynamical space we can equate $2\boldsymbol{\omega}$ to vorticity, and so with μ being equal to inertial mass density we can see that equation [I] is also the Coriolis force.

We will now define the magnetic flux density \mathbf{B} as,

$$\mathbf{B} = \mu \mathbf{v} \quad [\mathbf{L}]$$

by analogy with the electric current density equation,

$$\mathbf{J} = \rho \mathbf{v} \quad [\mathbf{M}]$$

Let us now substitute equation [L] into equation [I]. This leads to,

$$\mathbf{F}/\text{volume} = \mathbf{B} \times (\text{curl } \mathbf{B}/\mu) \quad [\mathbf{N}]$$

Comparing with Maxwell's equation [C], and substituting equation [M], we obtain,

$$\mathbf{F}/\text{volume} = \mathbf{B} \times \rho \mathbf{v} \quad [\mathbf{O}]$$

This is exactly the same as,

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B} \quad [\mathbf{P}]$$

We have just shown that the $\mathbf{v} \times \mathbf{B}$ component of the Lorentz force is a form of the Coriolis force. The negative sign in equation **[P]** illustrates the reversal of cause and effect as compared to equation **[K]**. In equation **[P]**, the magnetic flux density \mathbf{B} has now come to refer to the fine-grain vorticity/angular momentum density. It is simply an issue of whether or not we use the right hand rule or the left hand rule. Whether or not we use a right hand rule or a left hand rule depends on identifying whether or not the electric current causes the magnetic field or whether the magnetic field induces an electric current.

Maxwell openly believed in the centrifugal force as a real force and he used fine-grain centrifugal force in his vortices to explain the magnetic force on a current carrying wire. He explained how this centrifugal force pushes the current carrying wire from behind. It would appear however that Maxwell never explicitly realized the hand of the Coriolis force in either Ampère's Circuital Law or in electromagnetic induction. The Coriolis force and the centrifugal force both require rotation in some form or other and the only way that this can be accounted for within the context of electromagnetism is by the rotation of the aethereal substance contained within the vortex cells. Maxwell's apparent failure to explicitly notice the role of the Coriolis force in shaping the theory of electromagnetism is almost certainly a major reason why he allowed the 1865 dynamical theory to take over from his 1861 theory of molecular vortices.

A magnetic field behaves like a rotating frame of reference. It differs from a normal rotating frame of reference in that it is not a large scale rotating frame. It is a fine-grain rotating frame filled with many microscopic vortices all aligned with each other in their axial planes as per the magnetic field lines of force so as to effectively render the magnetic field into one single rotating frame of reference perpendicular to the magnetic field lines.

In his 1865 paper, Maxwell retained the luminiferous aether in the form of a dielectric, but he abandoned the aethereal vortices. He elaborated on the fly-wheel nature of the magnetic field using a dynamical illustration of reduced momentum but he failed to realize that this analogy is not sufficient to account for the role of the Coriolis force in $\mathbf{v} \times \mathbf{B}$ and in Ampère's Circuital Law.

In the footnote of Einstein's addendum of 27th September 1905 entitled 'Does the Inertia of a Body depend on its Energy-Content?', Einstein claims that "*The principle of the constancy of the velocity of light is of course contained in Maxwell's equations*"

http://www.fourmilab.ch/etexts/einstein/E_mc2/www/

Einstein was of course referring to the 1884 Heaviside versions in which the \mathbf{vXB} factor had been removed. The \mathbf{vXB} factor actually has the effect of making Faraday's law into a total time derivative equation (see **Appendix A** in 'The Double Helix Theory of the Magnetic Field' <http://www.wbabin.net/science/tombe.pdf>). This in turn exposes the fact that \mathbf{vXB} and Faraday's law only apply in a specific reference frame with respect to which \mathbf{v} is measured relative to. Had Einstein paid heed to the original eight Maxwell's equations, he would not have been able to make his spurious claim regarding the constancy of the speed of light.

The Triple Vortex Theory of the Electric Dipole

VII. In 'The Double Helix Theory of the Magnetic Field' mentioned above, it was demonstrated how Maxwell's vortex cells might be more accurately replaced by rotating electron-positron dipoles in which each electron is undergoing a mutual circular orbit with a positron.

Let us consider electrons and positrons to be singularities in the mysterious aethereal space. If we consider an electron to be a sink and a positron to be a source, aether will flow between the two.

If the electron and positron orbit each other then the aether will become curled. The field lines will be spiralling into the electrons and out of the positrons making them each into vortices.

The rotating electron-positron dipole will effectively constitute an 'in vortex' and an 'out vortex' rolling around the outside of a central vortex that doesn't have a sink or source.

The rotating electron-positron dipole is the prototype miniature Ampère's Circuital Law.

The Telegraphy Equation

VIII. In 1857, Kirchhoff [3] used the equations of electromagnetism in order to derive the telegraphy equation. He did not use the concept of displacement current but instead he used other mathematical ingredients such as Poisson's equation and the equation of continuity of charge which in combination are mathematically equivalent to the displacement current equation. Kirchhoff believed that he had derived the speed of an electrical signal in a wire. In actual fact he had derived the electromagnetic wave equation without realizing it.

In 1861, Maxwell conceived of the idea of displacement current and added it to Ampère's Circuital Law [2]. This became of relevance in his 1864 paper [1] when he derived the electromagnetic wave equation. Maxwell's physical picture of displacement current was somewhat unclear. He first conceived of it in part III of his 1861 paper in relation to tangential stress on the electrical particles surrounding his molecular vortices. This has been interpreted nowadays as a linear polarization current in a dielectric. However Maxwell may well have been driving at something more subtle such as an angular acceleration of his molecular vortices.

The telegraphy equation almost certainly applies to a propagation of angular acceleration in Maxwell's vortex sea, and Maxwell's displacement current almost certainly refers to a non-divergent angular displacement current in which \mathbf{E} is equal to the electromagnetic induction force $-\partial\mathbf{A}/\partial t$. It is likely that Kirchhoff was completely wrong in believing that the telegraphy equation applies to the speed of an electrical signal in a wire.

References

[1] Clerk-Maxwell, J., "A Dynamical Theory of the Electromagnetic Field", *Philos. Trans. Roy. Soc.* 155, pp 459-512 (1865). Abstract: *Proceedings of the Royal Society of London* 13, pp. 531--536 (1864)

[2] Clerk-Maxwell, J., "On Physical Lines of Force", *Philosophical Magazine*, Volume 21, (1861)

[3] Kirchhoff, G., "On the motion of electricity in wires", *Philosophical Magazine*, Volume 13, pp. 393 -412 (1857)

An interesting interpretation of Kirchhoff's 1857 paper can be viewed on this web link,

[http://www.ifi.unicamp.br/~assis/Apeiron-V19-p19-25\(1994\).pdf](http://www.ifi.unicamp.br/~assis/Apeiron-V19-p19-25(1994).pdf)