

Theory of Electrical Resistivity

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Abstract

This paper presents the derivation of the equations for electrical resistivity of the metallic and semiconductor elements and compounds according to the Reciprocal System of physical theory developed by D. B. Larson. The factors involved include: the atomic rotational displacement (as modified by the passage of the current), the temperature, the interatomic distance, the volume of the crystal unit cell, the number of atoms in the crystal unit cell, atomic weight, and the diameter of the atoms.

keywords: electrical resistivity, metallic elements and compounds, semiconductor elements and compounds, Reciprocal System

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Introduction

The Reciprocal System of physical theory is described in the books by Dewey B. Larson, such as Ref. [1], [2], [3], and [4]. In those books, Larson derived some of the fundamental concepts of electrical resistivity from the Postulates of the Reciprocal System but did not complete the work (see, for example, pp. 234-238 of Ref. [3]). This paper, which is runnable as a *Mathcad* program, completes the work in deriving fully accurate equations for electrical resistivity of the metallic and semiconductor elements and compounds, alloys, and anisotropic and amorphous substances.

Nomenclature

A = cross-sectional area of specimen, cm^2

A_v = Avogadro's constant (number of atoms in a gram-mole)

a, b = constants for aggregate resistivity equation

C_{met} = number of impurity metallic atoms per cubic centimeter of a semiconductor

C_{semic} = number of semiconductor atoms per cubic centimeter

d_{atom} = atomic diameter ("nuclear diameter" in terms of conventional theory), fm

d_u = natural unit for atomic diameter, fm

I_R = interregional ratio (between time region and time-space region)

k = Boltzmann's constant, eV/K

k_1 = proportionality factor for temperature deviation of aggregate

L = length of specimen, cm

N = number of components of N-ary alloy

n = number of solutes in N-ary alloy

N_i = mass decimal fraction of atoms of substance i in disordered alloy

P = external pressure (dyne/ cm^2 , or N/m^2 , or atm, or kg/cm^2 (Bridgman's units))

P_0 = internal pressure (dyne/ cm^2 , or N/m^2 , or atm, or kg/cm^2 (Bridgman's units))

R = resistance of specimen, ohm or μ -ohm

R_p = resistance of specimen under pressure, ohm or μ -ohm

R_u = natural unit of resistance, ohm

s = space dimension

s_{aniso} = interatomic distance along specified a, b, or c principal crystal axis, angstrom

s_u = natural unit of space, cm

s_{t_u} = time region natural unit of space (for solids), angstrom

s_0 = chosen interatomic distance (usually smallest) in unit cell at 0 K and 0 external pressure, angstrom

T = temperature at which the resistivity is measured, K (usually 293.15 or 300)

T_{bc} = temperature of the beginning of conduction for a semiconductor, K

T_{sc} = temperature of superconduction for a metal, K

T_{t_u} = natural unit of temperature for solids (spatial vibration), K

$T_{t_inv_u}$ = inverse natural unit of temperatures for solids (temporal vibration), K

T_u = natural unit of temperature (for gas-like flow of electrons), K

t = time dimension

t_e = atomic electric displacement (integer, dimensionless; use alternate positive equivalent for most electronegative elements)

t_{e_mod} = atomic electric displacement as modified by passage of current (integer or half integer, dimensionless)

t_p = atomic primary magnetic rotational displacement of the element (integer, dimensionless)

t_s = atomic secondary magnetic rotational displacement of the element (integer, dimensionless)

V = volume of material at pressure P , cm^3

V_i = volume of i th phase of heterogenous mixture, cm^3

V_0 = volume of material at zero external pressure, cm^3

V_{uc} = volume of crystal unit cell at 0 K and 0 external pressure, angstrom^3

w = atomic weight of element (divided by unit atomic weight, so effectively dimensionless)

w_{met} = atomic weight of metallic impurity in semiconductor

w_{semic} = atomic weight of semiconductor

xx_i = decimal atomic concentration of the i th solute in a disordered alloy

Z = number of atoms in crystal unit cell

γ = density of amorphous solids, $\text{amu}/\text{angstrom}^3$

γ_u = natural unit of density for amorphous solids, $\text{amu}/\text{angstrom}^3$

ρ = molecular resistivity of crystal unit metallic cell at temperature T , $\mu\text{-ohm-cm}$

ρ_{semic} = molecular resistivity of crystal unit semiconductor cell at temperature T , ohm-cm

ρ_{aggr} = aggregate resistivity of bulk material at mean temperature T, $\mu\text{-ohm-cm}$

$\rho_{\text{dev_sc}}$ = deviation of resistivity at superconducting temperature due to higher temperature atoms or molecules

ρ_{u} = natural unit of resistivity, $\mu\text{-ohm-cm}$

σ_{met} = conductivity of metallic impurity in semiconductor, $\text{ohm}^{-1}\text{-cm}^{-1}$

σ_{mix} = conductivity of semiconductor with metallic impurity, $\text{ohm}^{-1}\text{-cm}^{-1}$

σ_{semic} = conductivity of semiconductor, $\text{ohm}^{-1}\text{-cm}^{-1}$

Φ_{T} = probability distribution function for deviation of molecular temperatures from the mean

$\phi_1, \phi_2, \dots, \phi_n$ = volume fraction of the individual phases of a heterogenous or multiphase mixture

ϕ_{met} = mass fraction of metallic atoms in semiconductor

Note: A black square in the upper right of an equation means that the equation is disabled from running in *Mathcad*. This is done because not all variables in the equation have, as yet, been given numerical values at that point in the program.

Reciprocal System Physical Constants

$$\begin{aligned}
 s_u &:= 4.558816 \cdot 10^{-6} \text{ cm} & (\text{Ref. [1], p. 160}) & & T_u &:= 7.20423 \cdot 10^{12} \text{ K} & (\text{Ref. [2], p. 59}) & & I_R &:= 156.4444 \\
 & & & & & & & & & & (\text{Ref. [1], p. 162}) \\
 s_{t_u} &:= \frac{s_u}{I_R \cdot 10^{-8}} & s_{t_u} &= 2.914 \text{ angstrom} & & d_u &:= 3.359 \text{ fm} & (\text{Ref. [5]}) & & & \\
 R_u &:= 8.83834 \cdot 10^{11} \text{ ohms} & (\text{Ref. [2], p. 110}) & & \rho_u &:= R_u \cdot s_{t_u} \cdot 10^{-8} \times 10^6 & & \rho_u &= 25755070686.736 \text{ } \mu\text{-ohm-cm} \\
 k &:= 8.6168 \cdot 10^{-5} \text{ eV/K} & T_{t_u} &:= 510.8 \text{ K} & T_{t_{\text{inv}_u}} &:= \frac{1}{T_{t_u}} & T_{t_{\text{inv}_u}} &= 0.00196 & \gamma_u &:= \frac{1}{2.914^3} & \gamma_u &= 0.04041 \\
 A_v &:= 6.02486 \cdot 10^{23} \text{ g-mole}^{-1} & & & & & & & & & \text{amu/angstrom}^3
 \end{aligned}$$

1. Derivation of the New Electrical Resistivity Equation for Metallic Elements

Electrical resistivity is defined in terms of the resistance of a specimen, its cross-sectional area, and its length:

$$\rho := R \cdot \frac{A}{L} \cdot 10^6 \quad \mu\text{-ohm-cm} \quad (1)$$

If we set A to 1 cm² and L to 1 cm, we have a unit cube of the material. So, resistivity can be equivalently defined as the resistance of a unit volume of material, the specific resistance, so to speak. It is thus an intrinsic material property. Ref. [2], p. 110, shows that electrical resistance has the dimensions of mass per unit time, (t³/s³)/t. So resistivity must have the dimensions

$$[t^2/s^2] = [t^2/s^3] \times s^2 / s \quad (2)$$

which is momentum. Resistance per unit volume, the equivalent definition of resistivity, would then be mass per unit time per unit volume.

The general equation, expressed in words, for matter properties in the Reciprocal System is as follows:

matter_property_for_specific_element_or_compound := natural_unit_value_for_that_property x
(geometrical_and_physical_factors_of_that_element_or_compound, expressed
non-dimensionally)

Here:

$$\rho := \rho_u \cdot \left(\frac{\text{factors}}{\text{natural_unit_value_of_factors}} \right) \quad (3)$$

In the Reciprocal System, ordinary electric current is comprised of massless, chargeless electrons. These travel *through* the atoms of matter, *not (simply) through the interstices*. They are *not scattered* by the atoms (as seen by conventional physics), and so Drude's equation does not apply at all to the Reciprocal System. There are no charged electrons or charged holes or excitons involved in ordinary electric current.

The mass in Eq. 2 is the mass of the atoms per crystal unit cell through which the electrons flow. The time is the rotational time displacement of these atoms. The volume is that of the crystal unit cell of the material. The temperature at which the resistivity is measured must be referred to the natural unit of temperature. Resistivity (for an atom) vanishes at the superconducting temperature; but for an aggregate, some atoms remain above superconducting temperature even though the bulk temperature is there or below, so aggregate resistivity doesn't go to zero at the superconducting temperature (as calculated by the Reciprocal System), rather it exponentially declines, as will be seen later. So it's the deviation from superconducting temperature that counts.

Two other factors are, of course, the diameter of the atoms through which the electrons travel, and the interatomic distance between the atoms. If this distance is not isotropic, then the crystal resistivity is anisotropic.

The time term involves the ratio of the (one-dimensional) electric rotational displacement to the one-dimensional equivalent of the mean magnetic rotational displacement. The electric rotational displacement may be modified by passage of the electric current, whereas the magnetic rotational displacement is not (if it were, the basic atomic structure would be altered). Atomic weight is a function of the magnetically charged neutrinos contained within an atom, and it's these neutrinos that modify the effective electric displacement of the atom in regards to electric current flow. The modification can be up or down (but usually down), because the neutrinos have both time and space displacements, and may be half integer because the neutrinos have only one rotating system, not two. Finally, it's necessary to square the time term because of the relationship of the time region to the time-space region ("The time region velocity and all quantities derived therefrom, which means all of the physical phenomena of the region, are therefore second power expressions of the corresponding time-space region quantities.", p. 19 of Ref. [4].)

Putting all this together we have

$$\rho := \rho_u \cdot \frac{(T - T_{sc})}{T_u} \cdot \frac{Z \cdot \frac{w}{1}}{V_{uc}} \cdot \frac{\frac{s_0}{s_{t_u}}}{\frac{d_{atom}}{d_u}} \cdot \left[\frac{1}{\left[\frac{t_{e_mod}}{\left(t_p^2 \cdot t_s \right)^{\frac{1}{3}}} \right]^2} \right] \quad \mu\text{-ohm-cm} \quad (4)$$

Sample Calculation for Copper

The following values apply:

$$T := 293.15 \text{ K} \quad (\text{room temperature})$$

$$T_{sc} := 47.88 \text{ K} \quad (\text{as calculated by the Reciprocal System, see Ref. [2], p. 84, pp. 116-117})$$

$$Z := 4$$

$$w := 63.546 \text{ amu}$$

$$V_{uc} := 46.653 \text{ angstrom}^3$$

$$s_0 := 2.542 \text{ angstrom}$$

$$d_{atom} := 8.366 \text{ fm}$$

$$t_p := 3$$

$$t_s := 2$$

$$t_{e_mod} := 13 \quad (\text{alternative equivalent plus 2, for this element})$$

$$\rho := \rho_u \cdot \frac{(T - T_{sc})}{T_u} \cdot \frac{Z \cdot \frac{w}{1}}{V_{uc}} \cdot \frac{\frac{s_0}{st_u}}{\frac{d_{atom}}{d_u}} \cdot \left[\frac{1}{\left[\frac{t_{e_mod}}{(t_p^2 \cdot t_s)^{\frac{1}{3}}} \right]^2} \right] \quad \rho = 1.68268 \text{ } \mu\text{-ohm-cm}$$

The experimental value, from Ref. [6], is 1.67 $\mu\text{-ohm-cm}$. Ref. [8] says it's 1.7 $\mu\text{-ohm-cm}$.

Table of Resistivity Values of the Metallic Elements

The Excel table below gives the values of the modified electric displacement, together with all the other input values, and the resultant value of the resistivity for each metallic element. Even mercury, Hg, which is liquid at room temperature is included, which shows that Eq. (4) can (apparently) be applied to liquids as well. Most electronegative elements use the alternative equivalent electropositive rotational displacements. The exceptions are noted with a "-" to the left of the element symbol.

Element	Atomic No.	Atomic Weight	Reg. Rot. Displ	Alt. Equiv. Rot.	Z	S0	d_atom	T - Ts	Vuc
Li	3	6.941	2-1-1	2-1-1	4	2.922	2	237.29	82.429
Be	4	9.012182	2-1-2	2-1-2	2	2.23	2	237.29	15.814
Na	11	22.98977	2-2-1	2-2-1	2	3.633	4.048	269.21	75.419
Mg	12	24.305	2-2-2	2-2-2	2	3.159	4.048	245.27	45.459
Al	13	26.981538	2-2-3	2-2-3	4	2.86	4.894	237.29	64.466
P--black	15	30.974	3-2-(3)	2-2-5	8	2.2	4.894	197.39	142.646
K	19	39.0983	3-2-1	3-2-1	2	4.516	8.366	277.19	170.227
Ca	20	40.078	3-2-2	3-2-2	4	3.928	8.366	261.23	49.611
Sc	21	44.95591	3-2-3	3-2-3	2	3.194	8.366	249.26	35.713
Ti	22	47.867	3-2-4	3-2-4	2	2.964	8.366	237.29	27.918
V	23	50.9415	3-2-5	3-2-5	2	2.638	8.366	233.3	28.245
Cr	24	51.9961	3-2-6	3-2-6	2	2.696	8.366	237.29	51.801
Mn	25	54.938049	3-2-7	3-2-7	4	2.608	8.366	241.28	23.258
Fe	26	55.845	3-2-8	3-2-8	2	2.478	8.366	229.31	42.637
Co	27	58.9332	3-2-9	3-2-9	4	2.412	8.366	237.29	43.135
Ni	28	58.6934	3-3-(8)	3-2-10	4	2.506	8.366	237.29	245.27
Cu	29	63.546	3-3-(7)	3-2-11	4	2.542	8.366	245.27	46.653
Zn	30	65.409	3-3-(6)	3-2-12	2	2.896	10.624	261.23	29.754
- Ga	31	69.723	3-3-(5)	3-2-13	8	2.809	8.366	277.19	137.122
- As	33	74.913	3-3-(3)	3-2-15	6	2.385	8.366	245.27	174.207
Se	34	78.96	3-3-(2)	3-2-16	32	3.481	8.366	261.23	220.335
Rb	37	85.4678	3-3-1	3-3-1	2	4.852	10.624	285.17	66.023
Sr	38	87.62	3-3-2	3-3-2	4	4.22	10.624	261.23	46.129
Y	39	88.90585	3-3-3	3-3-3	2	3.609	10.624	261.23	35.772
Zr	40	91.224	3-3-4	3-3-4	2	2.965	10.624	257.24	30.979
Nb	41	92.90638	3-3-5	3-3-5	2	2.844	10.624	237.29	28.670
Mo	42	95.94	3-3-6	3-3-6	2	2.738	11.66	237.29	26.966
Tc	43	98	3-3-7	3-3-7	2	2.738	11.66	237.29	54.641
Ru	44	101.07	3-3-8	3-3-8	2	2.738	11.66	237.29	58.344
Rh	45	102.9055	3-3-9	3-3-9	4	2.662	10.624	241.28	67.354
Pd	46	106.42	4-3-(8)	3-3-10	4	2.738	11.66	253.25	41.296
Ag	47	107.8682	4-3-(7)	3-3-11	4	2.877	12.766	265.22	52.333
Cd	48	112.411	4-3-(6)	3-3-12	2	3.207	14.922	273.2	
In	49	114.818	4-3-(5)	3-3-13	2	3.342	14.922	245.27	

Ru	44	101.07	3-3-8	3-3-8	2	2.738	11.66	237.29	54.641	
Rh	45	102.9055	3-3-9	3-3-9	4	2.662	10.624	241.28	58.344	
Pd	46	106.42	4-3-(8)	3-3-10	4	2.738	11.66	253.25	67.354	
Ag	47	107.8682	4-3-(7)	3-3-11	4	2.877	12.766	265.22	67.354 ₁₁	
Cd	48	112.411	4-3-(6)	3-3-12	2	3.207	14.922	273.2	41.296	
In	49	114.818	4-3-(5)	3-3-13	2	3.342	14.922	245.27	52.333	
-	Sn	50	117.71	4-3-(4)	3-3-14	8	2.813	12.766	265.22	264.281
-	Sb	51	121.75	4-3-(3)	3-3-15	6	2.838	14.922	261.23	220.897
	Cs	55	132.90545	4-3-1	4-3-1	2	5.195	13.942	285.17	125.000
	Ba	56	137.337	4-3-2	4-3-2	2	4.377	16.234	277.19	73.923
	La	57	138.9055	4-3-3	4-3-3	2	3.711	13.942	273.2	137.484
	Ce	58	140.116	4-3-4	4-3-4	4	3.616	16.234	273.2	137.468
	Pr	59	140.90765	4-3-5	4-3-5	4	3.616	16.234	281.18	68.356
	Nd	60	144.24	4-3-6	4-3-6	2	3.616	16.234	281.18	68.356
	Pm	61	145	4-3-7	4-3-7	2	3.616	16.234	281.18	67.381
	Sm	62	150.36	4-3-8	4-3-8	2	3.616	16.234	277.19	94.258
	Eu	63	151.964	4-3-9	4-3-9	2	3.982	15.058	277.19	66.778
	Gd	64	157.25	4-3-10	4-3-10	2	3.571	14.922	273.2	62.736
	Tb	65	158.92534	4-3-11	4-3-11	2	3.616	16.234	273.2	62.972
	Dy	66	162.5	4-3-12	4-3-12	2	3.526	14.922	273.2	60.720
	Ho	67	164.93032	4-3-13	4-3-13	2	3.535	15.058	277.19	60.848
	Er	68	167.259	4-3-14	4-3-14	2	3.535	15.058	281.18	60.848
	Tm	69	168.93421	4-3-15	4-3-15	2	3.526	14.922	281.18	58.161
	Yb	70	173.04	4-3-16	4-3-16	2	3.879	14.922	277.19	165.126
	Lu	71	174.967	4-4-(15)	4-3-17	2	3.444	14.922	277.19	57.554
	Hf	72	178.49	4-4-(14)	4-3-18	2	3.149	14.922	261.23	44.161
	Ta	73	180.9479	4-4-(13)	4-3-19	2	2.85	12.766	261.23	35.736
	W	74	183.84	4-4-(12)	4-3-20	2	2.738	11.66	245.27	31.497
	Re	75	186.207	4-4--(11)	4-3-21	2	2.738	11.66	253.25	28.986
	Os	76	190.23	4-4--(10)	4-3-22	2	2.738	11.66	237.29	27.847
	Ir	77	192.217	4-4-(9)	4-3-23	4	2.738	11.66	249.26	56.139
	Pt	78	195.078	4-4-(8)	4-3-24	4	2.738	11.66	261.23	59.773
	Au	79	196.96655	4-4-(7)	4-3-25	4	2.888	13.942	269.21	66.817
-	Hg	80	200.59	4-4-(6)	4-3-26	2	3.455	11.66	277.19	44.177
-	Tl	81	204.3833	4-4-(5)	4-3-27	2	3.455	13.942	277.19	56.074
-	Pb	82	207.2	4-4-(4)	4-3-28	4	3.455	13.942	277.19	115.212
-	Bi	83	208.98	4-4-(3)	4-3-29	6	3.159	13.942	273.2	136.432
-	Po	84	209	4-4-(2)	4-3-30	3	3.455	13.942	261.23	64.480
	Ra	88	207.2	4-4-2	4-4-2	2	4.383	15.058	277.19	49.883
	Th	90	232.0381	4-4-4	4-4-4	2	3.535	15.058	257.24	40.708
	Pa	91	231.03588	4-4-5	4-4-5	2	3.233	15.058	241.28	38.933
	U	92	238.02891	4-4-6	4-4-6	2	2.956	15.058	241.28	92.937
	Np	93	237	4-4-7	4-4-7	2	3.039	13.942	241.28	115.782
	Pu	94	244	4-4-8	4-4-8	4	3.159	13.942	241.28	
	Am	95	243	4-4-9	4-4-9	4	3.455	13.942	241.28	

Element	Atomic No.	te_mod	Calc. RHO	RHO Obs.	calc - obs	calc / obs	te - te_mod
Li	3	2.0	7.501	8.550	-1.049	0.877	-1.0
Be	4	4.5	3.826	4.000	-0.174	0.957	-2.5
Na	11	4.0	3.755	4.200	-0.445	0.894	-3.0
Mg	12	4.5	4.122	4.380	-0.258	0.941	-2.5
Al	13	6.0	2.630	2.660	-0.030	0.989	-3.0
K	19	3.0	6.383	6.150	0.233	1.038	-2.0
Ca	20	5.0	3.236	3.430	-0.194	0.943	-3.0
Sc	21	1.5	53.683	61.000	-7.317	0.880	1.5
Ti	22	2.0	39.458	42.000	-2.542	0.939	2.0
V	23	3.0	20.891	19.700	1.191	1.060	2.0
Cr	24	4.0	12.323	12.700	-0.377	0.970	2.0
Mn	25	1.0	223.470	185.000	38.470	1.208	6.0
Fe	26	5.0	9.137	9.710	-0.573	0.941	3.0
Co	27	6.5	6.270	6.240	0.030	1.005	2.5
Ni	28	6.5	6.413	6.840	-0.427	0.938	3.5
Cu	29	13.0	1.683	1.670	0.013	1.008	-2.0
Zn	30	6.0	6.091	5.920	0.171	1.029	6.0
Ga	31	3.0	29.461	27.000	2.461	1.091	10.0
- Rb	37	3.0	13.031	12.500	0.531	1.042	-2.0
- Sr	38	2.5	24.236	23.000	1.236	1.054	-0.5
Y	39	2.0	54.832	57.000	-2.168	0.962	1.0
Zr	40	2.5	41.693	42.100	-0.407	0.990	1.5
Nb	41	5.0	12.112	12.500	-0.388	0.969	0.0
Mo	42	8.0	4.949	5.200	-0.251	0.952	-2.0
Tc	43	4.0	21.849	22.600	-0.751	0.967	3.0
Ru	44	7.0	7.823	7.600	0.223	1.029	1.0
Rh	45	9.5	4.631	4.510	0.121	1.027	-0.5
Pd	46	6.0	11.061	10.800	0.261	1.024	4.0
Ag	47	15.0	1.562	1.590	-0.028	0.982	-4.0
Cd	48	6.5	6.943	6.830	0.113	1.017	5.5
In	49	5.0	8.848	8.370	0.478	1.057	8.0
Sn	50	4.0	11.944	11.000	0.944	1.086	10.0
Cs	55	3.0	19.134	20.000	-0.866	0.957	-2.0
- Ba	56	2.0	55.293	50.000	5.293	1.106	0.0
- La	57	2.5	58.888	57.000	1.888	1.033	0.5
- Ce	58	2.0	83.524	73.000	10.524	1.144	2.0
Pr	59	2.5	55.334	68.000	-12.666	0.814	2.5
Nd	60	2.5	56.956	64.000	-7.044	0.890	3.5
Pm	61	2.5	57.256	50.000	7.256	1.145	4.5
Sm	62	2.0	92.776	94.000	-1.224	0.987	6.0
Eu	63	2.0	79.579	90.000	-10.421	0.884	7.0
Gd	64	2.0	103.672	134.000	-30.328	0.774	8.0

Pr	59	2.5	55.334	68.000	-12.666	0.814	2.5
Nd	60	2.5	56.956	64.000	-7.044	0.890	3.5
Pm	61	2.5	57.256	50.000	7.256	1.145	4.5
Sm	62	2.0	92.776	94.000	-1.224	0.987	6.0
Eu	63	2.0	79.579	90.000	-10.421	0.884	7.0
Gd	64	2.0	103.672	134.000	-30.328	0.774	8.0
Tb	65	2.0	103.806	114.000	-10.194	0.911	9.0
Dy	66	3.0	49.856	57.000	-7.144	0.875	9.0
Ho	67	2.5	76.175	87.000	-10.825	0.876	10.5
Er	68	2.5	78.197	87.000	-8.803	0.899	11.5
Tm	69	2.5	83.170	79.000	4.170	1.053	12.5
Yb	70	2.5	32.542	29.000	3.542	1.122	13.5
Lu	71	2.5	83.818	79.000	4.818	1.061	14.5
Hf	72	4.0	37.510	35.100	2.410	1.069	14.0
Ta	73	8.0	12.428	12.500	-0.072	0.994	11.0
W	74	12.5	5.795	5.650	0.145	1.026	7.5
Re	75	7.5	18.293	19.300	-1.007	0.948	13.5
Os	76	11.0	8.473	8.120	0.353	1.044	11.0
Ir	77	14.5	5.135	5.300	-0.165	0.969	8.5
Pt	78	10.0	10.785	10.600	0.185	1.017	14.0
Au	79	19.5	2.329	2.350	-0.021	0.991	5.5
- Hg	80	3.5	82.002	94.100	-12.098	0.871	22.5
- Tl	81	6.0	18.733	18.000	0.733	1.041	21.0
- Pb	82	6.0	22.394	20.800	1.594	1.077	-2.0
- Ra	88	2.0	99.956	100.000	-0.044	1.000	0.0
- Th	90	7.5	12.606	13.000	-0.394	0.970	-3.5
Pa	91	6.5	18.530	17.700	0.830	1.047	-1.5
U	92	5.5	29.874	30.800	-0.926	0.970	0.5
Np	93	3.0	116.072	122.000	-5.928	0.951	4.0
Pu	94	2.5	149.869	146.000	3.869	1.026	5.5
Am	95	3.5	66.853	68.000	-1.147	0.983	5.5

elec_resistivity_final.mcd

correlation = .985 -.666 .991 4.388 < Averages

2. Low Temperature Resistivity of Aggregate

The values computed above are for individual crystal cells in which the atoms are all at the same temperature. For an aggregate of material, the temperature will not be the same throughout--there will be a normal probability distribution of temperatures up and down from the average. At values somewhat higher than the calculated superconducting temperature, the deviations up and down equal, so the bulk resistivity equals the molecular resistivity. However, near the superconducting temperature and below, the deviations below vanish and so do not counter the deviations above, so there will be resistivity due to the higher temperature atoms. The aggregate resistivity at these temperatures is calculated as follows.

Eq. (4) can be rewritten to separate the T term from the constant term.

$$a := \rho_u \cdot \frac{1}{T_u} \cdot \frac{Z \cdot \frac{w}{1}}{V_{uc}} \cdot \frac{\frac{s_0}{st_u}}{\frac{d_{atom}}{d_u}} \cdot \left[\frac{1}{\left[\frac{t_{e_mod}}{\left(t_p^2 \cdot t_s \right)^{\frac{1}{3}}} \right]^2} \right]$$

$$b := \rho_u \cdot \frac{T_{sc}}{T_u} \cdot \left[\frac{Z \cdot \frac{w}{1}}{V_{uc}} \cdot \frac{\frac{s_0}{st_u}}{\frac{d_{atom}}{d_u}} \cdot \left[\frac{1}{\left[\frac{t_{e_mod}}{\left(t_p^2 \cdot t_s \right)^{\frac{1}{3}}} \right]^2} \right] \right]$$

$$\rho := a \cdot T - b$$

(5)

In the Reciprocal System, the atoms of a solid or liquid follow a normal probability distribution of temperatures; atoms of a gas follow Maxwell's distribution, in agreement with conventional theory and the experimental data. The standard normal probability function for temperature T is defined as

$$\Phi_T := \frac{1}{\sqrt{2 \cdot \pi}} \cdot e^{-\frac{T^2}{2}} \quad (6)$$

To obtain the resistivity contributed by atoms having a temperature higher than the superconducting temperature (when the mean temperature is at the superconducting temperature or below as calculated by the Reciprocal System), we must multiply the above function by the temperature, T, and integrate from 0 to infinity (the number of atoms in a material aggregate being usually greater than 10^{23}). We must also multiply the integral by a factor k1, which is equal to .25, because we need only the positive deviation below the superconducting temperature (there being no negative deviation) and because only one-half of a thermal vibration (outward) is effective against the space-time progression (inward in the time region). So:

$$k1 := .5 \cdot .5 \quad k1 = 0.25 \quad (7)$$

$$\rho_{\text{dev_sc}} := a \cdot k1 \cdot \int_0^{\infty} T \cdot \frac{1}{\sqrt{2 \cdot \pi}} \cdot e^{-\frac{T^2}{2}} dT \quad (8)$$

$$\rho_{\text{dev_sc}} := 0.09974 \cdot a$$

Therefore:

$$\rho_{\text{aggr}} := \rho_{\text{dev_sc}} \cdot e^{\left[-.5 \cdot \left[(T - T_{\text{sc}})^2\right]\right]} \quad T \leq T_{\text{sc}} \quad (9)$$

$$\rho_{\text{aggr}} := \rho + \rho_{\text{dev_sc}} \cdot e^{\left[-.5 \cdot \left[(T - T_{\text{sc}})^2\right]\right]} \quad T \geq T_{\text{sc}} \quad \text{(for a few K beyond } T_{\text{sc}}, \text{ otherwise just } \rho) \quad (10)$$

The exponential factors in Eq. (9) and Eq. (10), come from setting the lower limit of the integral in Eq. (8) to $(T - T_{\text{sc}})$ rather than to 0.

Sample Low Temperature Calculations for Copper

$$a := \rho_u \cdot \frac{1}{T_u} \cdot \frac{Z \cdot \frac{w}{1}}{V_{uc}} \cdot \frac{\frac{s_0}{st_u}}{\frac{d_{atom}}{d_u}} \cdot \left[\frac{1}{\left[\frac{t_{e_mod}}{\left(t_p^2 \cdot t_s \right)^{\frac{1}{3}}} \right]^2} \right]$$

$$a = 0.00686$$

$$b := \rho_u \cdot \frac{T_{sc}}{T_u} \cdot \left[\frac{Z \cdot \frac{w}{1}}{V_{uc}} \cdot \frac{\frac{s_0}{st_u}}{\frac{d_{atom}}{d_u}} \cdot \left[\frac{1}{\left[\frac{t_{e_mod}}{\left(t_p^2 \cdot t_s \right)^{\frac{1}{3}}} \right]^2} \right] \right]$$

$$b = 0.32848$$

$$\rho_{dev_sc} := 0.09974 \cdot a$$

$$\rho_{dev_sc} = 0.00068 \quad \mu\text{-ohm-cm}$$

Make *Mathcad* user functions:

$$\rho(a, T, b) := a \cdot T - b \quad \rho_{\text{aggr}}(\rho, \rho_{\text{dev_sc}}, T, T_{\text{sc}}) := \rho + \rho_{\text{dev_sc}} \cdot e^{\left[-.5 \cdot \left[(T - T_{\text{sc}})^2\right]\right]} \quad T \geq T_{\text{sc}}$$

T := 50

$$\rho_{\text{cu}} := \rho(a, T, b) \quad \rho_{\text{cu}} = 0.01454 \quad \rho_{\text{aggr_cu}} := \rho_{\text{aggr}}(\rho_{\text{cu}}, \rho_{\text{dev_sc}}, T, T_{\text{sc}}) \quad \rho_{\text{aggr_cu}} = 0.01462$$

T := 49

$$\rho_{\text{cu}} := \rho(a, T, b) \quad \rho_{\text{cu}} = 0.00768 \quad \rho_{\text{aggr_cu}} := \rho_{\text{aggr}}(\rho_{\text{cu}}, \rho_{\text{dev_sc}}, T, T_{\text{sc}}) \quad \rho_{\text{aggr_cu}} = 0.00805$$

T := 48

$$\rho_{\text{cu}} := \rho(a, T, b) \quad \rho_{\text{cu}} = 0.00082 \quad \rho_{\text{aggr_cu}} := \rho_{\text{aggr}}(\rho_{\text{cu}}, \rho_{\text{dev_sc}}, T, T_{\text{sc}}) \quad \rho_{\text{aggr_cu}} = 0.0015$$

T := 47

$$\rho_{\text{aggr_cu}} := \rho_{\text{aggr}}(0, \rho_{\text{dev_sc}}, T, T_{\text{sc}}) \quad \rho_{\text{aggr_cu}} = 0.00068$$

T := 46

$$\rho_{\text{aggr_cu}} := \rho_{\text{aggr}}(0, \rho_{\text{dev_sc}}, T, T_{\text{sc}}) \quad \rho_{\text{aggr_cu}} = 0.00046$$

T := 45

$$\rho_{\text{aggr_cu}} := \rho_{\text{aggr}}(0, \rho_{\text{dev_sc}}, T, T_{\text{sc}}) \quad \rho_{\text{aggr_cu}} = 0.00012$$

So, as you can see, the exponential decline in aggregate resistivity is very steep. The observed experimental superconducting temperature for bulk Cu is 46 K, not quite 2 K lower than the calculated molecular superconducting temperature. Below is a graph of the aggregate resistivity (multiplied by 100000) versus mean temperature, K.

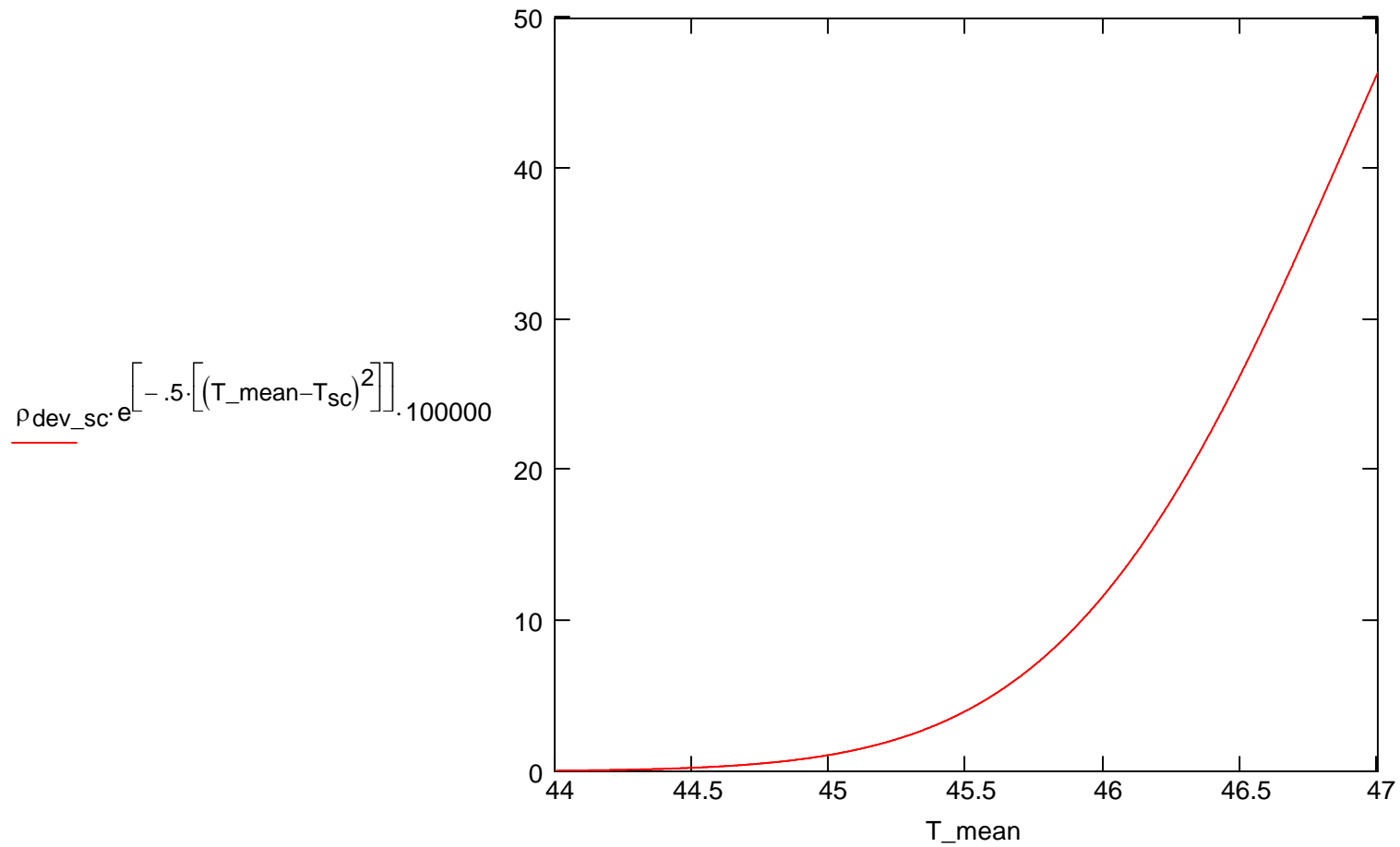


Figure 1. Low Temperature Resistivity of Copper

Similar such curves can be seen in the resistivity of other metallic elements; see Ref. [9] for the empirical graphs.

3. Effect of Pressure on Electrical Resistivity

The effect of pressure on volume is given by Eq. (4-8) of Ref. [2], p. 39:

$$V := V_0 \cdot \frac{P_0^{.5}}{(P_0 + P)^{.5}}$$

where V is the volume of material at external pressure P , V_0 is the volume at zero temperature and pressure, and P_0 is the internal pressure (Reciprocal System built-in pressure, without any external pressure).

Larson states, Ref. [2], p. 118: "Because the movement of electrons (space) through matter is the inverse of the movement of matter through space, the inter-regional relations applicable to the effect of pressure on resistance are the inverse of those that apply to the change in volume under pressure." Therefore, by analogy, the equation for resistance under pressure is

$$R_p := R \cdot \frac{P_0^2}{(P + P_0)^2} \quad (11)$$

where R is the resistance at zero (or close to zero) external pressure P . Note that we have to use resistance, rather than resistivity, because the volume of the specimen will decrease under pressure. The internal pressure values for use in Eq. (11) should be the same as those used in the compressibility calculations. There is some evidence, however, that the change in internal pressure, transitions as they are called, may occur at somewhat different levels of external pressure P for resistivity. The associated data base, included with Ref. [5], has notes on this for the relevant elements.

Sample Calculation for Copper

$R := 1.68 \text{ } \mu\text{-ohm}$ (at room temperature, atmospheric pressure, as calculated above for a unit cm cube of material)

$P_0 := .698 \cdot 10^6 \text{ atm}$ (from the associated data base, Ref. [5], using the same value as for the original one for compression)

$P := 4.9345 \cdot 10^4 \text{ atm}$ equal to 5 GPa, admittedly, a rather high external pressure (but below the calculated first transition pressure for P_0)

$R_p := R \cdot \frac{P_0^2}{(P + P_0)^2}$ $R_p = 1.46547 \text{ } \mu\text{-ohm}$, which is within the range of Bridgman's experiments. Remember: there is some uncertainty as to the value of P_0 to use.

% reduction: $100 \cdot \frac{R - R_p}{R} = 12.76946$

This is high compared with the value on p. 558 of Ref. [7], 8%. If that result were right, this would make the value of R equal to $1.5456 \text{ } \mu\text{-ohm}$. In that case, the correct value of P_0 to use here would be $1.159093 \times 10^6 \text{ atm}$. This is close to the second compression transition point, $1.1980 \times 10^6 \text{ atm}$. See pp. 118-121 of Ref. [2] for more discussion of these points.

4. Derivation of the New Electrical Resistivity Equation for Semiconductor Elements

The same definition of resistivity applies to semiconductor elements as for metallic elements: mass per unit time per unit volume. However, the high electronegative character of the semiconductor elements, including B, C, Si, S, Ge, and Te, causes them to vibrate thermally in time rather than in space. (Note that B, boron, is nominally an electropositive element, with rotational displacement 2-1-3, but it often assumes the equivalent 2-2-(5) electronegative rotational displacement; a single B radical may have the -3 valence, etc.) Sometimes gray-Sn and Se are counted as semiconductors, but they are really metalloids and so for them we use the metallic resistivity equation.

The temperature of the beginning of conduction for semiconductor elements, T_{bc} , is calculated the same way that the temperature of superconduction is calculated for the metallic elements, namely twice the value of T_0 , the temperature at which the specific heat curve of the atom (not the aggregate) crosses the zero point of specific heat (see Ref. [2]).

The thermal vibration in time, with dimensions t/s , equivalent to the normal dimensions of energy, starts at 0 and increases with T . The energy of the atom-electron ensemble is defined as $4k(T-T_{bc})$, where k is Boltzmann's constant (the atoms and the electrons of the solid are in thermal equilibrium). If we were dealing with electrical conductivity we would use this expression directly, but we are using its inverse here, resistivity, so the correct expression to use is $1/(4k(T-T_{bc}))$. Physically the resistivity must be infinite just prior to the beginning of conduction and must asymptotically decline with T ; thus an exponential expression is required.

Also because of the thermal vibration in time, the key variable, T , must be in the denominator of the expression, not the numerator. As with other Reciprocal System equations, whenever the key variable is in the denominator it must be squared. To balance the dimensions, the square of the appropriate natural unit of temperature must be placed in the numerator. In this case, the correct natural unit temperature is the inverse of the normal solid state unit temperature and is denoted by $T_{t_inv_u}$.

Putting all this together, and using ohm-cm, rather than μ -ohm-cm, we have

$$\rho_{\text{semic}} := \frac{\rho_u}{10^6} \cdot \frac{(T_{t_inv_u})^2}{(T - T_{bc})^2} \cdot \frac{Z \cdot \frac{W}{1}}{\frac{V_{uc}}{(s_{t_u})^3}} \cdot \frac{\frac{s_0}{s_{t_u}}}{\frac{d_{atom}}{d_u}} \cdot \left[\frac{e^{\left[\frac{1}{4k \cdot (T - T_{bc})} \right]}}{\left[\frac{t_{e_mod}}{(t_p^2 \cdot t_s)^{\frac{1}{3}}} \right]^2} \right]^2 \quad \text{ohm-cm} \quad (T > T_{bc}) \quad (12)$$

Undoubtedly there will be a probability distribution of atoms with temperature higher than T_{bc} when the bulk temperature is T_{bc} and below, and this will contribute some aggregate conduction before T_{bc} , but this can be neglected for the present.

We will now apply Eq. (12) to each of the semiconductor elements in turn. Ref. [10] contains the experimental data for comparison.

B, Boron Resistivity

$$w := 10.8110 \quad Z := 50 \quad s_0 := 1.5725 \quad V_{uc} := 391.422 \quad T := 300 \quad t_e := -5 \quad t_p := 2 \quad t_s := 2 \quad d_{atom} := 2$$

$$T_{bc} := 191.52 \quad T := 300$$

$$t_{e_mod} := -7$$

$$\rho_B := \frac{\rho_u}{10^6} \cdot \frac{(T_{t_inv_u})^2}{(T - T_{bc})^2} \cdot \frac{Z \cdot \frac{w}{1}}{\frac{V_{uc}}{(s_{t_u})^3}} \cdot \frac{\frac{s_0}{s_{t_u}}}{\frac{d_{atom}}{d_u}} \cdot \left[\frac{1}{e^{4k \cdot (T - T_{bc})}} \cdot \frac{\left[\frac{t_{e_mod}}{(t_p^2 \cdot t_s)^{\frac{1}{3}}} \right]^2}{1} \right]^2$$

$$\rho_B = 8.74425 \times 10^6 \text{ ohm-cm}$$

Ref. [10], p. 402, for β -rhombohedral, at 300 K, gives the intrinsic conductivity in the range 10^{-7} to 10^{-6} (ohm-cm) $^{-1}$. Converting to resistivity, we have

$$\rho_{B_obs1} := \frac{1}{10^{-7}} \quad \rho_{B_obs1} = 1 \times 10^7 \quad \rho_{B_obs2} := \frac{1}{10^{-6}} \quad \rho_{B_obs2} = 1 \times 10^6 \quad \text{Take your pick.}$$

The temperature graph follows.

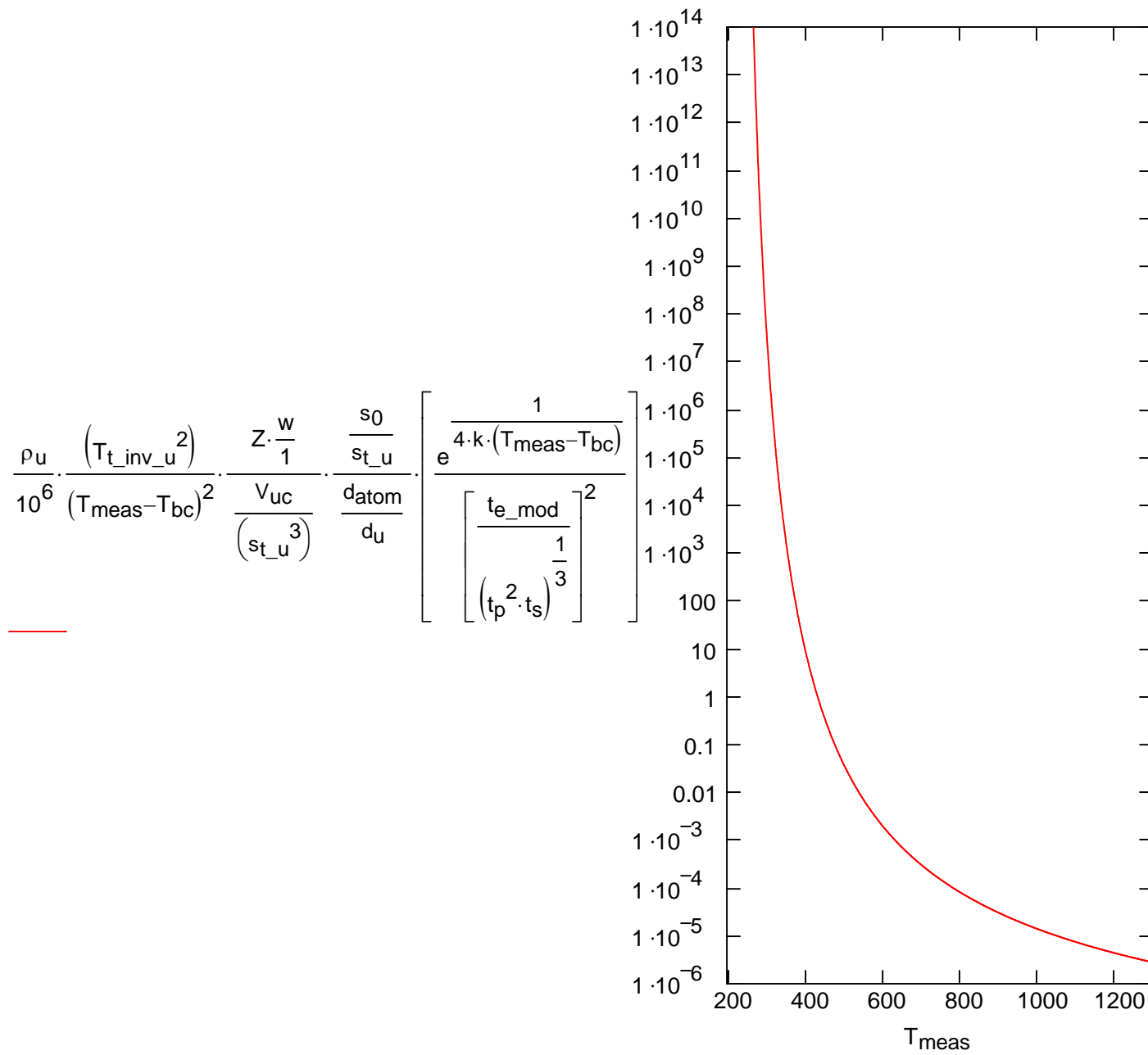


Figure 2. Resistivity of Boron

C, Carbon (Diamond) Resistivity

$$w := 12.0107 \quad Z := 8 \quad s_0 := 1.5382 \quad V_{uc} := 45.356 \quad T := 300 \quad t_e := -4 \quad t_p := 2 \quad t_s := 2 \quad d_{atom} := 2$$

$$T_{bc} := 287.28 \quad \text{so conduction begins just before room temperature} \quad t_{e_mod} := -4 \quad (\text{no change})$$

$$\rho_C := \frac{\rho_u}{10^6} \cdot \frac{(T_{t_inv_u})^2}{(T - T_{bc})^2} \cdot \frac{Z \cdot \frac{w}{1}}{V_{uc}} \cdot \frac{\frac{s_0}{s_{t_u}}}{\left(\frac{s_{t_u}}{3}\right)} \cdot \frac{d_{atom}}{d_u} \cdot \left[\frac{e^{\frac{1}{4 \cdot k \cdot (T - T_{bc})}}}{\left[\frac{t_{e_mod}}{\left(\frac{1}{(t_p^2 \cdot t_s)^3}\right)} \right]^2} \right]^2 \quad \rho_C = 8.10777 \times 10^{96} \text{ ohm-cm}$$

Ref. [10], p. 14, states: "Due to the large band gap (5.48 eV) and the high ionization energy of donors, most diamonds are insulators at room temperature." And on p. 15: "Most electrical, optical and thermal properties of diamond are extrinsic; i.e., strongly dependent on the impurity content, the most important impurities being nitrogen and boron." The very high resistivity calculated here is thus verified to the extent it can be.

The temperature graph follows.

$$\frac{\rho_u}{10^6} \cdot \frac{(T_{t_inv_u}^2)}{(T_{meas}-T_{bc})^2} \cdot \frac{Z \cdot \frac{w}{1}}{V_{uc}} \cdot \frac{\frac{s_0}{s_{t_u}}}{\frac{d_{atom}}{d_u}} \cdot \left[\frac{1}{e^{4 \cdot k \cdot (T_{meas}-T_{bc})}} \cdot \left[\frac{t_{e_mod}}{\left(\frac{1}{t_p^2 \cdot t_s} \right)^3} \right]^2 \right]$$

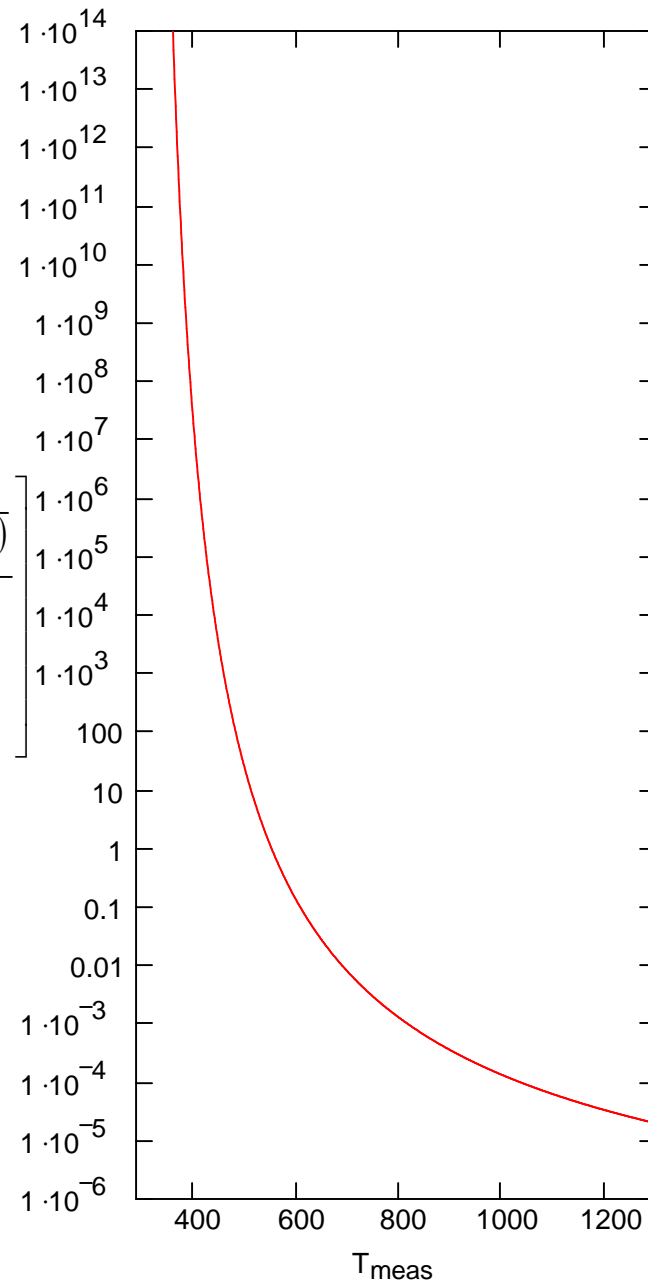


Figure 3. Resistivity of Carbon

Si, Silicon Resistivity

$$w := 28.0855 \quad Z := 8 \quad s_0 := 2.3287 \quad V_{uc} := 158.569 \quad T := 300 \quad t_e := -4 \quad t_p := 3 \quad t_s := 2$$

$$T_{bc} := 95.76 \quad d_{atom} := 4.984 \quad t_{e_mod} := -4 \quad (\text{no change})$$

$$\rho_{Si} := \frac{\rho_u}{10^6} \cdot \frac{(T_{t_inv_u})^2}{(T - T_{bc})^2} \cdot \frac{Z \cdot \frac{w}{1}}{V_{uc}} \cdot \frac{\frac{s_0}{s_{t_u}}}{\frac{d_{atom}}{d_u}} \cdot \left[\frac{1}{e^{4 \cdot k \cdot (T - T_{bc})}} \cdot \frac{t_{e_mod}}{\left(\frac{t_p^2 \cdot t_s}{1} \right)^{\frac{1}{3}}} \right]^2 \quad \rho_{Si} = 28.32788 \quad \text{ohm-cm}$$

Ref. [10], p. 21, gives the intrinsic electrical conductivity of Si as $3.16 \times 10^{-2} \text{ (ohm-cm)}^{-1}$. Thus,

$$\rho_{Si_obs} := \frac{1}{3.16 \cdot 10^{-2}} \quad \rho_{Si_obs} = 31.64557 \quad \text{ohm-cm}$$

It's amazing that other references give very different values:

Lange's Handbook of Chemistry: $\rho = .1 \text{ ohm-cm}$

Handbook of Physical Quantities: $\rho = 230000 \text{ ohm-cm}$

Wolf, *Semiconductors:* $\rho = 113636 \text{ ohm-cm}$

Podesta, *Understanding Properties of Matter:* $\rho = .1 \text{ ohm-cm}$

Madelung's work is the most authoritative, and it comes closest to the value calculated by the Reciprocal System.

The temperature graph follows.

$$\frac{\rho_u}{10^6} \cdot \frac{(T_{t_inv_u})^2}{(T_{meas}-T_{bc})^2} \cdot \frac{Z \cdot \frac{w}{1}}{V_{uc}} \cdot \frac{\frac{s_0}{s_{t_u}}}{\frac{d_{atom}}{d_u}} \cdot \left[\frac{1}{e^{4 \cdot k \cdot (T_{meas}-T_{bc})}} \cdot \left[\frac{t_{e_mod}}{\left(\frac{1}{(t_p^2 \cdot t_s)^3} \right)^2} \right]^2 \right]$$

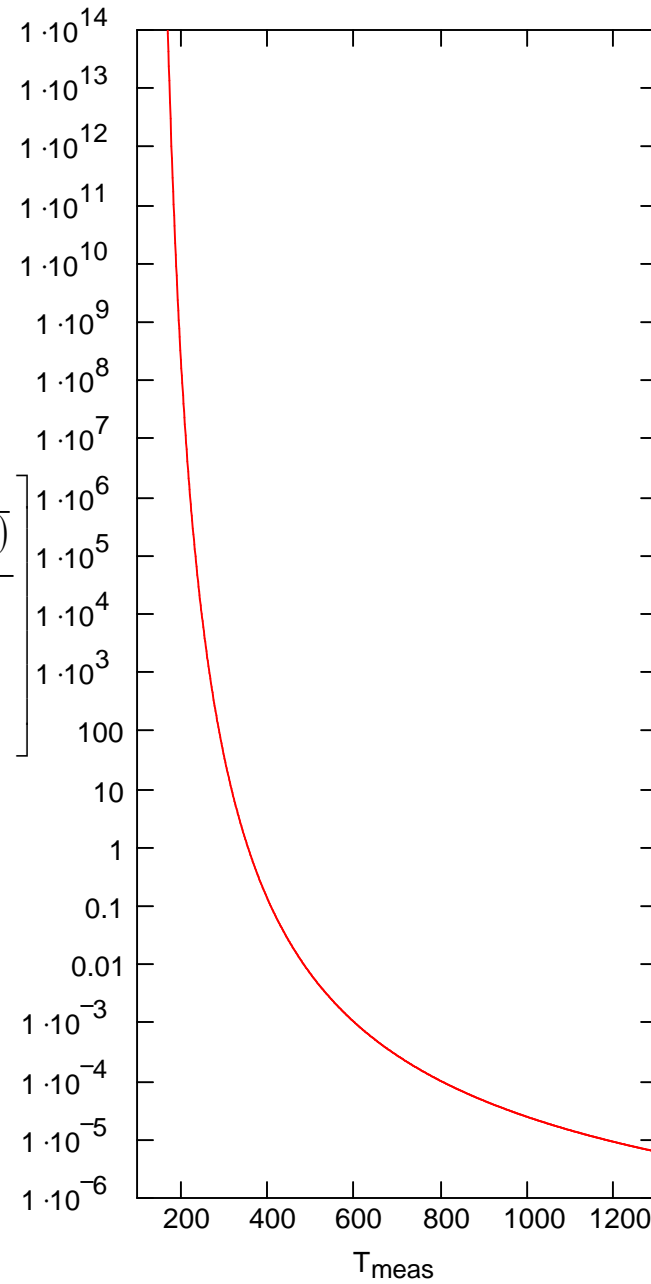


Figure 4. Resistivity of Silicon

S, Sulfur Resistivity

Unfortunately, we have to make an adjustment in T_{bc} to fit the resistivity data at room temperature; the value is not 2 or 8 but 6.377 times the regular value of $2 \times T_0$. One, therefore, wonders about the specific heat calculations for sulfur....maybe T_0 is multiplied by a mixture of 2 and 8. Anyhow, it's an insulator!

$$w := 32.0650 \quad Z := 18 \quad s_0 := 2.1097 \quad V_{uc} := 433.779 \quad T := 300 \quad t_e := -2 \quad t_p := 3 \quad t_s := 2 \quad d_{atom} := 4.984$$

$$T_{bc} := 229 \quad T := 295 \quad t_{e_mod} := -2 \quad (\text{no change})$$

$$\rho_S := \frac{\rho_u}{10^6} \cdot \frac{(T_{t_inv_u})^2}{(T - T_{bc})^2} \cdot \frac{Z \cdot \frac{w}{1}}{\left(\frac{V_{uc}}{s_{t_u}}\right)^3} \cdot \frac{\frac{s_0}{s_{t_u}}}{\frac{d_{atom}}{d_u}} \cdot \left[\frac{1}{e^{4 \cdot k \cdot (T - T_{bc})}} \cdot \frac{\left[\frac{t_{e_mod}}{\left(t_p^2 \cdot t_s\right)^{\frac{1}{3}}} \right]^2} \right]^2 \quad \rho_S = 7.7123 \times 10^{15} \quad \text{ohm-cm}$$

Ref. [10], p. 423, gives the intrinsic electrical conductivity of S at $T = 295 \text{ K}$ as $11 \times 10^{-17} (\text{ohm-cm})^{-1}$. Thus,

$$\rho_{S_obs} := \frac{1}{11 \cdot 10^{-17}} \quad \rho_{S_obs} = 9.09091 \times 10^{15} \quad \text{ohm-cm}$$

The temperature graph follows.

$$\frac{\rho_u}{10^6} \cdot \frac{(T_{t_inv_u})^2}{(T_{meas}-T_{bc})^2} \cdot \frac{Z \cdot \frac{w}{1}}{V_{uc}} \cdot \frac{\frac{s_0}{s_{t_u}}}{\frac{d_{atom}}{d_u}} \cdot \left[\frac{1}{e^{4 \cdot k \cdot (T_{meas}-T_{bc})}} \cdot \left[\frac{t_{e_mod}}{(t_p^2 \cdot t_s)^{\frac{1}{3}}} \right]^2 \right]$$

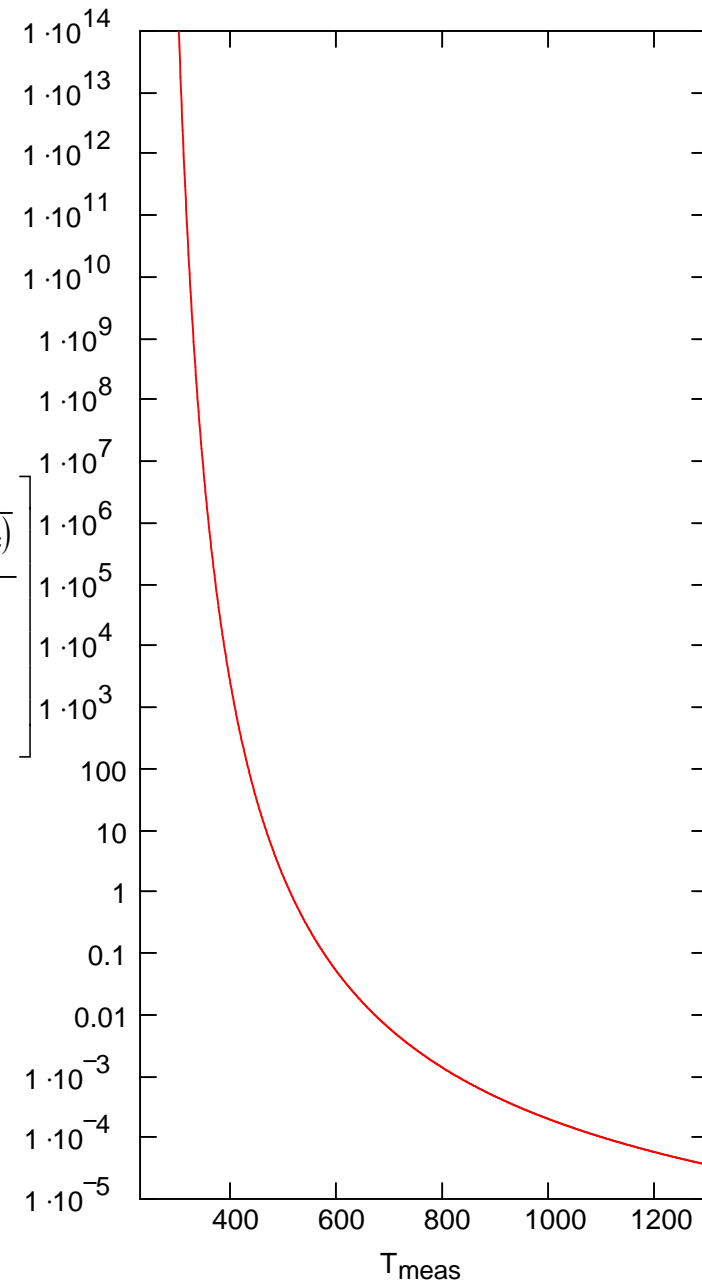


Figure 5. Resistivity of Sulfur

Ge, Germanium Resistivity

$$w := 72.6400 \quad Z := 32 \quad s_0 := 2.478 \quad V_{uc} := 179.071 \quad T := 300 \quad t_e := -4 \quad t_p := 3 \quad t_s := 3$$

$$T_{bc} := 51.87 \quad d_{atom} := 8.366 \quad t_{e_mod} := -2$$

$$\rho_{Ge} := \frac{\rho_u}{10^6} \cdot \frac{(T_{t_inv_u})^2}{(T - T_{bc})^2} \cdot \frac{Z \cdot \frac{w}{1}}{V_{uc}} \cdot \frac{\frac{s_0}{s_{t_u}}}{\frac{d_{atom}}{d_u}} \cdot \frac{e^{\frac{1}{4 \cdot k \cdot (T - T_{bc})}}}{\left[\frac{t_{e_mod}}{(t_p^2 \cdot t_s)^{\frac{1}{3}}} \right]^2} \quad \rho_{Ge} = 47.35187 \quad \text{ohm-cm}$$

Ref. [10], p. 49, gives the intrinsic electrical conductivity of Ge as $2.1 \times 10^{-2} \text{ (ohm-cm)}^{-1}$. Thus,

$$\rho_{Ge_obs} := \frac{1}{2.1 \cdot 10^{-2}} \quad \rho_{Ge_obs} = 47.61905 \quad \text{ohm-cm}$$

The temperature graph follows.

$$\frac{\rho_u}{10^6} \cdot \frac{(T_{t_inv_u})^2}{(T_{meas}-T_{bc})^2} \cdot \frac{Z \cdot \frac{w}{1}}{V_{uc}} \cdot \frac{\frac{s_0}{s_{t_u}}}{\frac{d_{atom}}{d_u}} \cdot \left[\frac{1}{e^{4 \cdot k \cdot (T_{meas}-T_{bc})}} \cdot \left[\frac{t_{e_mod}}{\left(\frac{1}{t_p^2 \cdot t_s} \right)^{\frac{1}{3}}} \right]^2 \right]$$

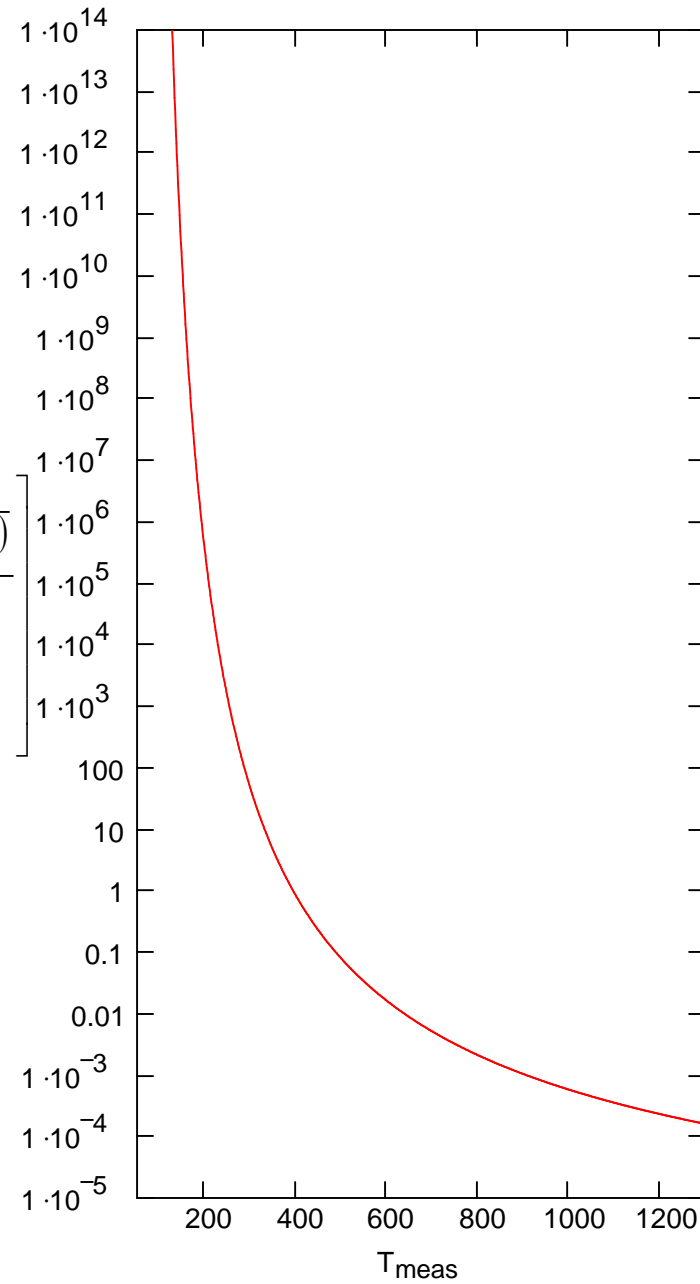


Figure 6. Resistivity of Germanium

Te, Tellurium Resistivity

$$w := 127.6 \quad Z := 3 \quad s_0 := 2.28239 \quad V_{uc} := 98.732 \quad T := 300 \quad t_e := -2 \quad t_p := 4 \quad t_s := 3$$

$$T_{bc} := 27.93 \quad d_{atom} := 16.234 \quad T := 293 \quad t_{e_mod} := -7$$

$$\rho_{Te} := \frac{\rho_u}{10^6} \cdot \frac{(T_{t_inv_u})^2}{(T - T_{bc})^2} \cdot \frac{Z \cdot \frac{w}{1}}{V_{uc}} \cdot \frac{\frac{s_0}{s_{t_u}}}{\frac{d_{atom}}{d_u}} \cdot \left[\frac{1}{e^{4 \cdot k \cdot (T - T_{bc})}} \cdot \frac{t_{e_mod}}{\left(\frac{1}{(t_p^2 \cdot t_s)^3} \right)^2} \right]^2 \quad \rho_{Te} = 0.3338 \quad \text{ohm-cm}$$

Ref. [10], p. 431, gives two values for the intrinsic electrical conductivity of Te: $1.56 \text{ (ohm-cm)}^{-1}$ and $3.04 \text{ (ohm-cm)}^{-1}$. Thus,

$$\rho_{Te_obs1} := \frac{1}{1.56} \quad \rho_{Te_obs1} = 0.64103 \quad \text{ohm-cm}$$

$$\rho_{Te_obs2} := \frac{1}{3.04} \quad \rho_{Te_obs2} = 0.32895 \quad \text{ohm-cm}$$

Presumably, the smaller resistivity is in the crystal direction with the smaller interatomic distance.

The temperature graph follows.

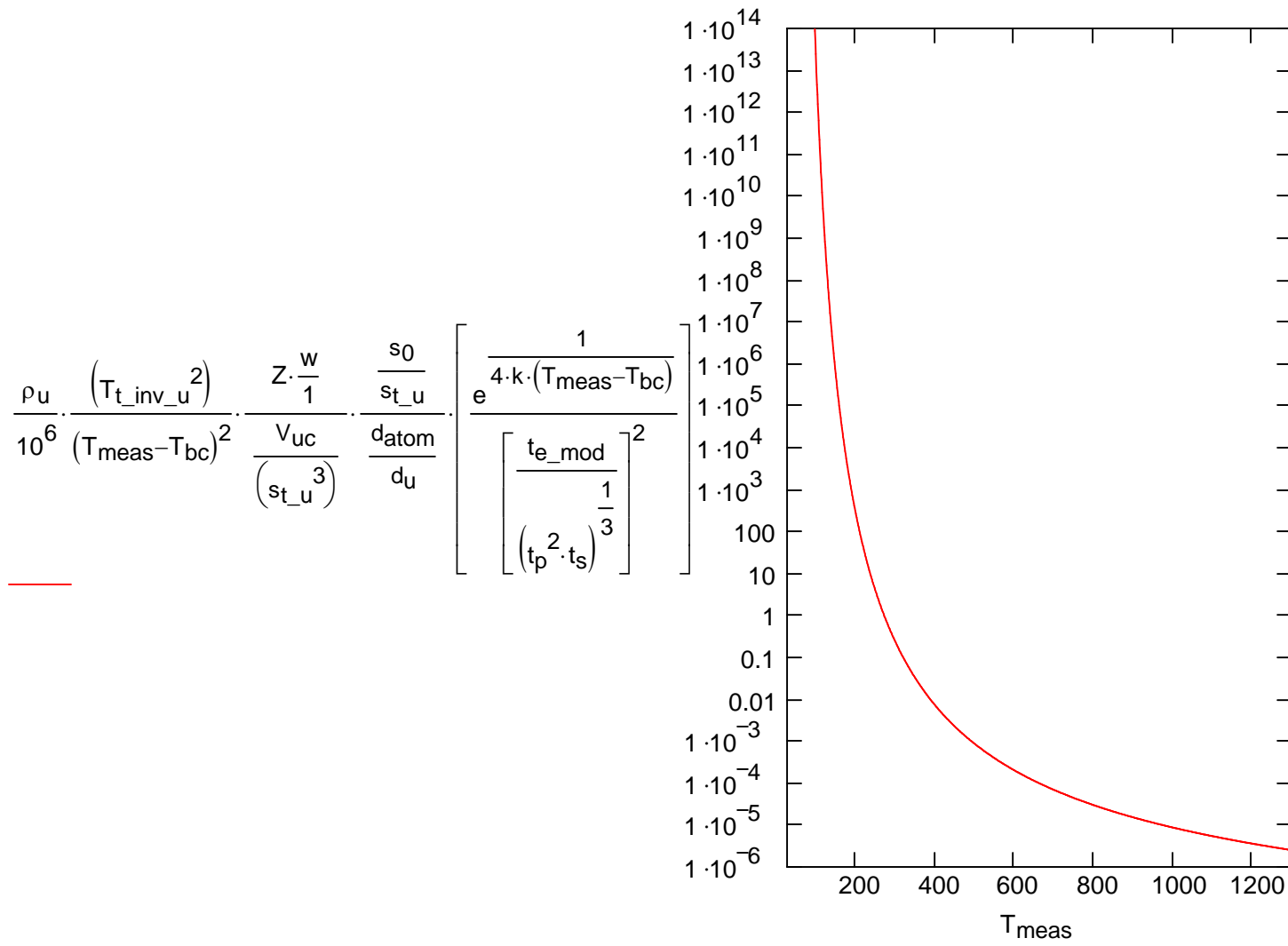


Figure 7. Resistivity of Tellurium

5. Resistivity of Anisotropic Elements

For metallic elements, we simply replace s_0 , the shortest interatomic distance in Eq. (4), with the interatomic distance in the desired resistivity direction, s_{aniso} .

$$\rho_{\text{aniso}} := \rho_u \cdot \frac{(T - T_{\text{sc}})}{T_u} \cdot \frac{Z \cdot \frac{W}{1}}{\frac{V_{\text{uc}}}{(s_{\text{t}_u})^3}} \cdot \frac{\frac{s_{\text{aniso}}}{s_{\text{t}_u}}}{\frac{d_{\text{atom}}}{d_u}} \cdot \left[\frac{1}{\left[\frac{t_{\text{e_mod}}}{(t_p^2 \cdot t_s)^{\frac{1}{3}}} \right]^2} \right] \quad \mu\text{-ohm-cm} \quad (13)$$

Therefore the ratio of resistivities in different crystal directions should be equal to the ratio of the interatomic distances in those directions. Nye, on p. 205 in Ref. [11], which is based on the 1926 *International Critical Tables*, gives the resistivity values, ρ_a , ρ_b , ρ_c , for the three principal crystal directions (a, b, c) of Sn, Bi, Cd, and Zn. In each case, $\rho_a = \rho_b$, and $\rho_c > \rho_a$ and $\rho_c > \rho_b$. The interatomic distances in the different directions are not given, but c is always greater than a or b so we can conclude that we have at least qualitative agreement. Unfortunately, there does not seem to exist a comprehensive compilation of resistivity data along the three principal crystal axes of each metallic element.

Similarly for semiconductor elements:

$$\rho_{\text{aniso_semic}} := \frac{\rho_u}{10^6} \cdot \frac{(T_{\text{t_inv}_u})^2}{(T - T_{\text{bc}})^2} \cdot \frac{Z \cdot \frac{W}{1}}{\frac{V_{\text{uc}}}{(s_{\text{t}_u})^3}} \cdot \frac{\frac{s_{\text{aniso}}}{s_{\text{t}_u}}}{\frac{d_{\text{atom}}}{d_u}} \cdot \left[\frac{e^{\left[\frac{1}{4k \cdot (T - T_{\text{bc}})} \right]}}{\left[\frac{t_{\text{e_mod}}}{(t_p^2 \cdot t_s)^{\frac{1}{3}}} \right]^2} \right] \quad \text{ohm-cm} \quad (T > T_{\text{bc}}) \quad (14)$$

Again, there is insufficient data for us to set up a comparison between theory and experiment, but the resistivities in the different crystal directions should be equal to the ratio of the interatomic distances in those directions.

6. Resistivity of Amorphous Elements

For metallic elements we replace the crystal mass per volume term, $Z \cdot \frac{w}{1} / \frac{V_{uc}}{(s_{t_u})^3}$, in Eq. (4), with the

term γ/γ_u where γ is the measured density of the amorphous specimen, in amu / angstrom³, and γ_u is the natural unit of density, here $1 \text{ amu}/2.914^3 = .04041 \text{ amu}/\text{angstrom}^3$. We also must replace the s_0 term by s_{amorph} because in an amorphous solid the controlling interatomic distance is the longest, not the shortest, distance. So, Eq. (4) is modified to

$$\rho_{\text{amorph}} := \rho_u \cdot \frac{(T - T_{sc})}{T_u} \cdot \frac{\gamma}{\gamma_u} \cdot \frac{s_{\text{amorph}}}{s_{t_u}} \cdot \frac{d_{\text{atom}}}{d_u} \cdot \left[\frac{1}{\left[\frac{t_{e_mod}}{(t_p^2 \cdot t_s)^{\frac{1}{3}}} \right]^2} \right] \quad \mu\text{-ohm-cm} \quad (15)$$

Similarly, Eq. (12) for semiconductor elements is modified to

$$\rho_{\text{amorph_semic}} := \frac{\rho_u}{10^6} \cdot \frac{(T_{t_inv_u})^2}{(T - T_{bc})^2} \cdot \frac{\gamma}{\gamma_u} \cdot \frac{s_{\text{amorph}}}{s_{t_u}} \cdot \frac{d_{\text{atom}}}{d_u} \cdot \left[\frac{e^{\left[\frac{1}{4k \cdot (T - T_{bc})} \right]}}{\left[\frac{t_{e_mod}}{(t_p^2 \cdot t_s)^{\frac{1}{3}}} \right]^2} \right] \quad \text{ohm-cm} \quad (T > T_{bc}) \quad (16)$$

Unfortunately none of the References listed have a compilation of experimental resistivity values for amorphous metallic and semiconductor elements. There are various isolated values given in the physics literature, but there is too much uncertainty here to set up a comparison between theory and experiment. Ref. [12], p. 74, says "...the difference in resistivity between the crystalline and amorphous states for dielectrics and metals is always less than an order of magnitude and is generally less than a factor of 3. For semiconductors, however, resistivity changes of 10 orders of magnitude between the crystalline and amorphous states are not uncommon..." It's fairly clear, by inspection, that the reduction in density of an amorphous solid is more than offset by the increase in the characteristic interatomic distance.

7. Resistivity of Heterogenous or Multiphase Mixtures

Our treatment for this case is the same as that described on pp. 85-88 of Ref. [13]. Let ρ_i be the resistivity of the i th phase of a heterogenous metallic or semiconductor mixture and ϕ_i be its volume fraction. Then, by inspection, the resultant mixture resistivity is computed from

$$\phi_i = V_i / \Sigma V_i$$

$$\rho_{\text{mix}} := \sum_i (\rho_i \cdot \phi_i) \quad \mu\text{-ohm-cm or ohm-cm, as appropriate} \quad (17)$$

8. Resistivity of Metallic Alloys--Disordered Solutions

Binary Alloys

Ref. [4], p. 46, states: "Any of the electropositive elements can ... mix in any proportions except to the extent that geometrical considerations intervene." Consider binary alloys first. Let the decimal atomic fraction of the solute element be xx_1 , and therefore that of the solvent element be $1-xx_1$. The resistivity curve, $\rho_{0,1}$, is parabolic as concentration of the solute component increases from 0 to 1. This can be seen by studying Eq. 11: the solute atoms create a *negative* pressure ($-P$ in the equation) because the atomic rotational force is *repulsive*; this negative pressure reduces the denominator and thus increases the resistance of the solid solution in a quadratic manner.

The concentration factor, c , is the focal length of the parabola, the sum of the resistivity per unit mass of the atoms in the unit cell of each element (prior to mixing) divided by the average resistivity:

$$c := \left(\frac{\rho_0}{Z_0 \cdot w_0} + \frac{\rho_1}{Z_1 \cdot w_1} \right) \cdot \frac{2}{(\rho_0 + \rho_1)} \quad (18)$$

The subscript 0 is for the base (or solvent or initial) component. A parabolic shaping factor, aa , can be defined in terms of c :

$$aa := \frac{1}{4 \cdot c} \quad (19)$$

By geometry, the value of xx at which resistivity is maximized is

$$xx_{1_max} := \frac{\frac{1}{2} \cdot (\rho_1 - \rho_0 + aa)}{aa} \quad \text{(the minimum value of } aa \text{ is } |\rho_1 - \rho_0|, \text{ so that } xx_{max} \text{ will not exceed 1)} \quad (20)$$

The maximum value of resistivity is then

$$\rho_{max} := \rho_0 + aa \cdot xx_{1_max}^2$$

The resistivity of the binary alloy, ρ_{1_2} , can then be simply expressed as

$$\rho_{0_1} := \rho_{\max} - aa \cdot (xx_1 - xx_{1_{\max}})^2 \quad \mu\text{-ohm-cm} \quad (21)$$

Note that if the components are interchanged, that is, the original component becomes the solute component and the solute component becomes the solvent component, the value of ρ_{0_1} at the new xx computes to be the same as the value of ρ_{0_1} at the old $1-xx$.

Sample Calculation for Gold Silver

$$\rho_{Au} := 2.329 \quad \rho_{Ag} := 1.562 \quad w_{Au} := 196.96655 \quad w_{Ag} := 107.8682 \quad Z_{Au} := 4 \quad Z_{Ag} := 4$$

$$\rho_0 := \rho_{Au} \quad \rho_1 := \rho_{Ag} \quad w_0 := w_{Au} \quad w_1 := w_{Ag} \quad Z_0 := Z_{Au} \quad Z_1 := Z_{Ag}$$

$$c := \left(\frac{\rho_0}{Z_0 \cdot w_0} + \frac{\rho_1}{Z_1 \cdot w_1} \right) \cdot \frac{2}{(\rho_0 + \rho_1)} \quad c = 0.00338 \quad aa := \frac{1}{4 \cdot c} \quad aa = 73.95939$$

(aa is in units of resistivity)

$$xx_{1_max} := \frac{.5 \cdot (\rho_1 - \rho_0 + aa)}{aa} \quad xx_{1_max} = 0.49481$$

$$\rho_{max} := \rho_0 + aa \cdot xx_{1_max}^2 \quad \rho_{max} = 20.43734 \quad \mu\text{-ohm-cm}$$

$$\rho_{max} - aa \cdot (xx - xx_{1_max})^2$$

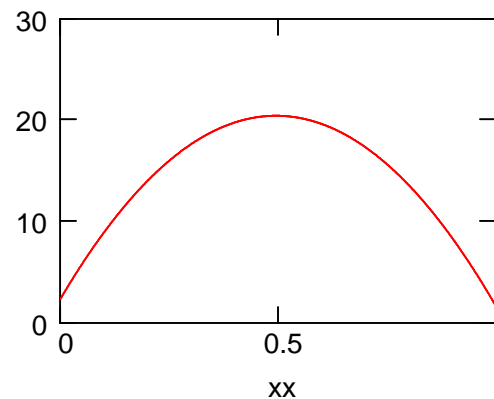


Figure 8. Resistivity of Au-Ag

But the resulting graph for ρ_{0_1} is *not* in close agreement with the curve shown on p. 4-76 of Ref. [14]; the maximum value of ρ there is approx. 11. However, Ref. [15], pp. 47-48, reports a study by Linde indicating that a 1% atomic increase of Ag in Au increases resistivity by .38 $\mu\text{-ohm-cm}$. Extrapolated to 50%, this implies a maximum resistivity of the alloy of 19 $\mu\text{-ohm-cm}$, which is close to our calculation. Therefore the experimental curve may be for a more or less ordered, not disordered, solution.

Sample Calculation for Platinum Palladium

$$\begin{aligned} \rho_{Pt} &:= 10.785 & \rho_{Pd} &:= 11.061 & w_{Pt} &:= 195.078 & w_{Pd} &:= 106.42 & Z_{Pt} &:= 4 & Z_{Pd} &:= 4 \\ \rho_0 &:= \rho_{Pt} & \rho_1 &:= \rho_{Pd} & w_0 &:= w_{Pt} & w_1 &:= w_{Pd} & Z_0 &:= Z_{Pt} & Z_1 &:= Z_{Pd} \\ c &:= \left[\left(\frac{\rho_{Pt}}{Z_0 \cdot w_0} \right) + \left(\frac{\rho_{Pd}}{Z_1 \cdot w_1} \right) \right] \cdot \frac{2}{(\rho_0 + \rho_1)} & c &= 0.00364 & aa &:= \frac{1}{4 \cdot c} & aa &= 68.60198 \\ xx1_{max} &:= \frac{.5 \cdot (\rho_1 - \rho_0 + aa)}{aa} & xx1_{max} &= 0.50201 \\ \rho_{max} &:= \rho_0 + aa \cdot xx1_{max}^2 & \rho_{max} &= 28.07377 & & & & \mu\text{-ohm-cm} \end{aligned}$$

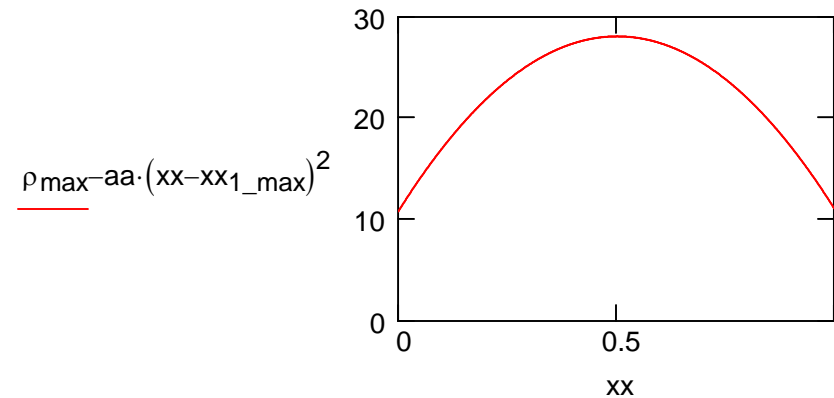


Figure 9. Resistivity of Pt-Pd

The resulting graph for ρ_{0_1} is in close agreement with the curve shown on p. 4-76 of Ref. [14].

Sample Calculation for Potassium Rubidium

$$\rho_K := 6.383 \quad \rho_{Rb} := 13.031 \quad w_K := 39.0983 \quad w_{Rb} := 85.4678 \quad Z_K := 2 \quad Z_{Rb} := 2$$

$$\rho_0 := \rho_K \quad \rho_1 := \rho_{Rb} \quad w_0 := w_K \quad w_1 := w_{Rb} \quad Z_0 := Z_K \quad Z_1 := Z_{Rb}$$

$$c := \left(\frac{\rho_0}{w_0 \cdot Z_0} + \frac{\rho_1}{w_1 \cdot Z_1} \right) \cdot \frac{2}{(\rho_0 + \rho_1)} \quad c = 0.01015 \quad aa := \frac{1}{4 \cdot c} \quad aa = 24.62057$$

$$xx1_{max} := \frac{.5 \cdot (\rho_1 - \rho_0 + aa)}{aa} \quad xx1_{max} = 0.63501$$

$$\rho_{max} := \rho_0 + aa \cdot xx1_{max}^2 \quad \rho_{max} = 16.31091 \quad \mu\text{-ohm-cm}$$

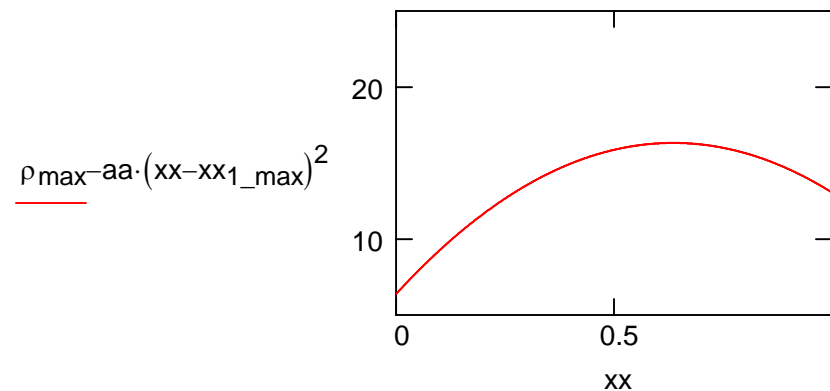


Figure 10. Resistivity of K-Rb

The resulting graph for $\rho_{0,1}$ is in close agreement with the curve shown on p. 4-76 of Ref. [14].

Sample Calculation for Indium Lead

$$\rho_{\text{In}} := 8.848 \quad \rho_{\text{Pb}} := 22.394 \quad w_{\text{In}} := 114.818 \quad w_{\text{Pb}} := 207.2 \quad Z_{\text{In}} := 2 \quad Z_{\text{Pb}} := 4$$

$$\rho_0 := \rho_{\text{In}} \quad \rho_1 := \rho_{\text{Pb}} \quad w_0 := w_{\text{In}} \quad w_1 := w_{\text{Pb}} \quad Z_0 := Z_{\text{In}} \quad Z_1 := Z_{\text{Pb}}$$

$$c := \left(\frac{\rho_0}{w_0 \cdot Z_0} + \frac{\rho_1}{w_1 \cdot Z_1} \right) \cdot \frac{2}{(\rho_0 + \rho_1)} \quad c = 0.0042 \quad aa := \frac{1}{4 \cdot c} \quad aa = 59.57636$$

$$xx1_{\text{max}} := \frac{.5 \cdot (\rho_1 - \rho_0 + aa)}{aa} \quad xx1_{\text{max}} = 0.61369$$

$$\rho_{\text{max}} := \rho_0 + aa \cdot xx1_{\text{max}}^2 \quad \rho_{\text{max}} = 31.28508 \quad \mu\text{-ohm-cm}$$

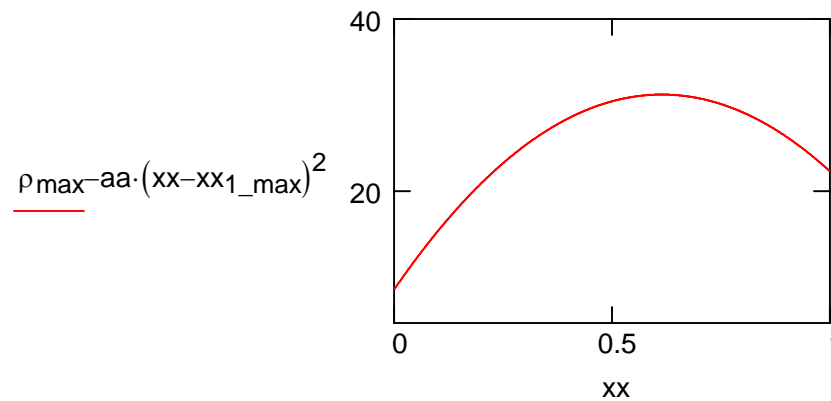


Figure 11. Resistivity of In-Pb

The resulting graph for $\rho_{0,1}$ is in close agreement with the curve shown on p. 4-76 of Ref. [14].

Sample Calculation for Copper Zinc

$$\rho_{\text{Cu}} := 1.683 \quad \rho_{\text{Zn}} := 6.091 \quad w_{\text{Cu}} := 63.546 \quad w_{\text{Zn}} := 65.409 \quad Z_{\text{Cu}} := 4 \quad Z_{\text{Zn}} := 2$$

$$\rho_0 := \rho_{\text{Cu}} \quad \rho_1 := \rho_{\text{Zn}} \quad w_0 := w_{\text{Cu}} \quad w_1 := w_{\text{Zn}} \quad Z_0 := Z_{\text{Cu}} \quad Z_1 := Z_{\text{Zn}}$$

$$c := \left(\frac{\rho_0}{w_0 \cdot Z_0} + \frac{\rho_1}{w_1 \cdot Z_1} \right) \cdot \frac{2}{(\rho_0 + \rho_1)} \quad c = 0.01368 \quad aa := \frac{1}{4 \cdot c} \quad aa = 18.27214$$

$$xx1_{\text{max}} := \frac{.5 \cdot (\rho_1 - \rho_0 + aa)}{aa} \quad xx1_{\text{max}} = 0.62062$$

$$\rho_{\text{max}} := \rho_0 + aa \cdot xx1_{\text{max}}^2 \quad \rho_{\text{max}} = 8.72088 \quad \mu\text{-ohm-cm}$$

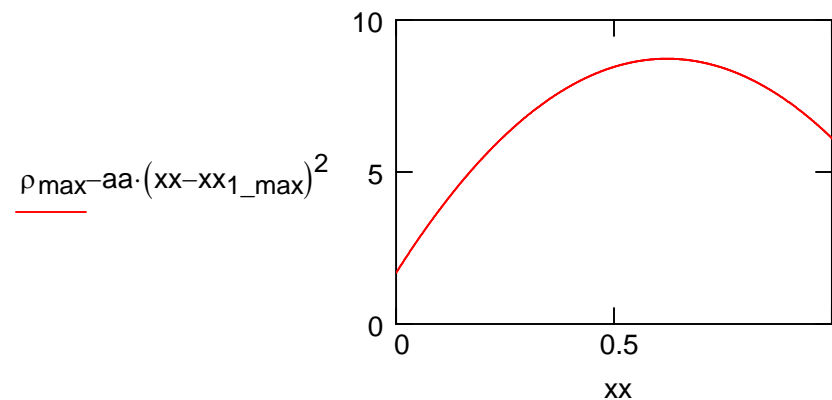


Figure 12. Resistivity of Cu-Zn

At $xx = .5$, the resistivity is $\rho_{\text{max}} - aa \cdot (.5 - xx1_{\text{max}})^2 = 8.45503 \mu\text{-ohm-cm}$. Ref. [6], p. 219, says it's 6.3.

Sample Calculation for Platinum Iridium

$$\rho_{Pt} := 10.785 \quad \rho_{Ir} := 5.135 \quad w_{Pt} := 195.078 \quad w_{Ir} := 192.217 \quad Z_{Pt} := 4 \quad Z_{Ir} := 4$$

$$\rho_0 := \rho_{Pt} \quad \rho_1 := \rho_{Ir} \quad w_0 := w_{Pt} \quad w_1 := w_{Ir} \quad Z_0 := Z_{Pt} \quad Z_1 := Z_{Ir}$$

$$c := \left(\frac{\rho_0}{w_0 \cdot Z_0} + \frac{\rho_1}{w_1 \cdot Z_1} \right) \cdot \frac{2}{(\rho_0 + \rho_1)} \quad c = 0.00258 \quad aa := \frac{1}{4 \cdot c} \quad aa = 97.07296$$

$$xx1_{max} := \frac{.5 \cdot (\rho_1 - \rho_0 + aa)}{aa} \quad xx1_{max} = 0.4709$$

$$\rho_{max} := \rho_0 + aa \cdot xx1_{max}^2 \quad \rho_{max} = 32.31045 \quad \mu\text{-ohm-cm}$$

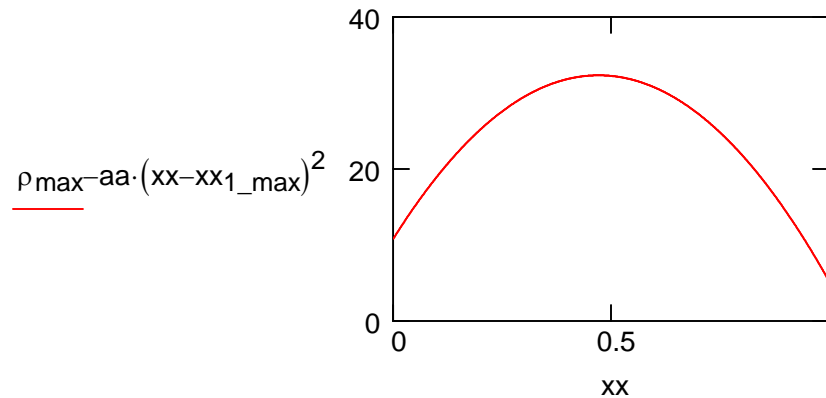


Figure 13. Resistivity of Pt-Ir

At $xx = .1$, the resistivity is $\rho_{max} - aa \cdot (.1 - xx1_{max})^2 = 18.95657 \quad \mu\text{-ohm-cm}$. Ref. [6], p. 219, says it's 24.8.

Sample Calculation for Platinum Rhodium

$$\rho_{Pt} := 10.785 \quad \rho_{Rh} := 4.631 \quad w_{Pt} := 195.078 \quad w_{Rh} := 102.9055 \quad Z_{Pt} := 4 \quad Z_{Rh} := 4$$

$$\rho_0 := \rho_{Pt} \quad \rho_1 := \rho_{Rh} \quad w_0 := w_{Pt} \quad w_1 := w_{Rh} \quad Z_0 := Z_{Pt} \quad Z_1 := Z_{Rh}$$

$$c := \left(\frac{\rho_0}{w_0 \cdot Z_0} + \frac{\rho_1}{w_1 \cdot Z_1} \right) \cdot \frac{2}{(\rho_0 + \rho_1)} \quad c = 0.00325 \quad aa := \frac{1}{4 \cdot c}$$

$$xx1_{max} := \frac{.5 \cdot (\rho_1 - \rho_0 + aa)}{aa} \quad xx1_{max} = 0.45997$$

$$\rho_{max} := \rho_0 + aa \cdot xx1_{max}^2 \quad \rho_{max} = 27.04584 \quad \mu\text{-ohm-cm}$$

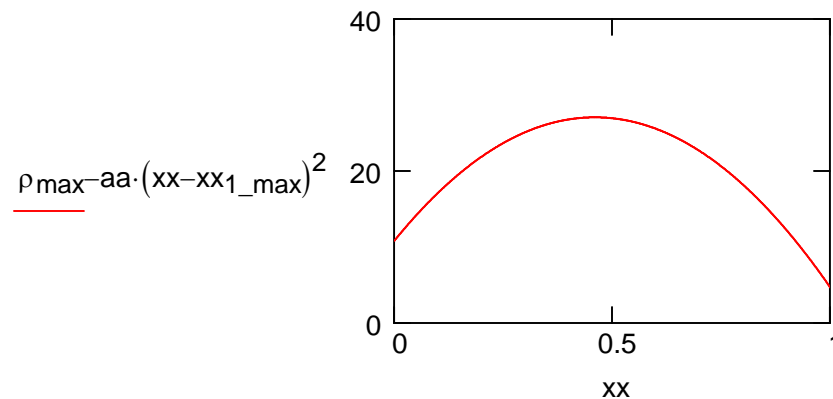


Figure 14. Resistivity of Pt-Rh

At $xx = .1$, the resistivity is $\rho_{max} - aa \cdot (.1 - xx1_{max})^2 = 17.08688 \text{ } \mu\text{-ohm-cm}$. Ref. [6], p. 219, says it's 18.7.

Sample Calculation for Copper Gold

$$\rho_{\text{Cu}} := 1.683 \quad \rho_{\text{Au}} := 2.329 \quad w_{\text{Cu}} := 63.546 \quad w_{\text{Au}} := 196.96655 \quad Z_{\text{Cu}} := 4 \quad Z_{\text{Au}} := 4$$

$$\rho_0 := \rho_{\text{Cu}} \quad \rho_1 := \rho_{\text{Au}} \quad w_0 := w_{\text{Cu}} \quad w_1 := w_{\text{Au}} \quad Z_0 := Z_{\text{Cu}} \quad Z_1 := Z_{\text{Au}}$$

$$c := \left(\frac{\rho_0}{w_0 \cdot Z_0} + \frac{\rho_1}{w_1 \cdot Z_1} \right) \cdot \frac{2}{(\rho_0 + \rho_1)} \quad c = 0.00477 \quad aa := \frac{1}{4 \cdot c}$$

$$xx1_{\text{max}} := \frac{.5 \cdot (\rho_1 - \rho_0 + aa)}{aa} \quad xx1_{\text{max}} = 0.50617$$

$$\rho_{\text{max}} := \rho_0 + aa \cdot xx1_{\text{max}}^2 \quad \rho_{\text{max}} = 15.09888 \text{ } \mu\text{-ohm-cm}$$

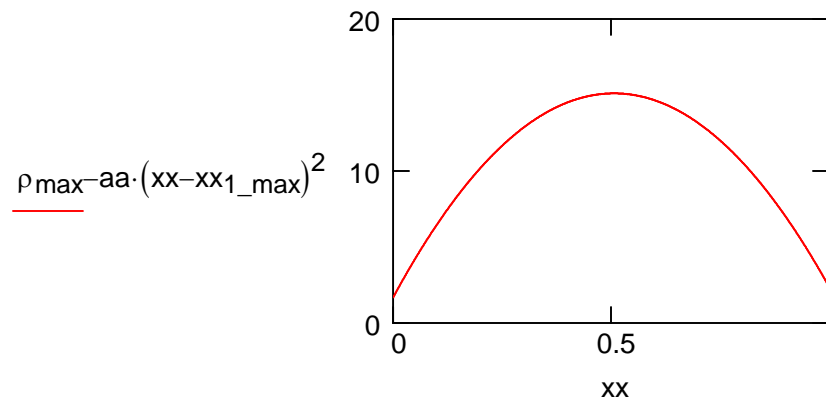


Figure 15. Resistivity of Cu-Au

The resulting graph is close to that shown on p. 26 of Ref. [15].

Many other binary alloys are covered in Ref. [15], but their resistivity curves are similar to the ones already covered and so there should no difficult in applying the above equations to these other alloys.

N-ary Alloys

The above equations can be generalized to alloys with more than two components, as follows ($n = N-1$).

$$\begin{aligned}
 c_1 &:= \left(\frac{\rho_0}{w_0 \cdot Z_0} + \frac{\rho_1}{w_1 \cdot Z_1} \right) \cdot \frac{2}{(\rho_0 + \rho_1)} \quad \blacksquare \\
 c_2 &:= \left(\frac{\rho_0}{w_0 \cdot Z_0} + \frac{\rho_2}{w_2 \cdot Z_2} \right) \cdot \frac{2}{(\rho_0 + \rho_2)} \quad \blacksquare \\
 &\cdot \\
 &\cdot \\
 &\cdot \\
 c_n &:= \left(\frac{\rho_0}{w_0 \cdot Z_0} + \frac{\rho_n}{w_n \cdot Z_n} \right) \cdot \frac{2}{(\rho_0 + \rho_n)} \quad \blacksquare
 \end{aligned} \tag{22}$$

The value of resistivity when the solutes have fractional concentrations of xx_1, xx_2, \dots, xx_n , is

$$(xx_1 - xx_{1_max})^2 + (xx_2 - xx_{2_max})^2 + \dots + (xx_n - xx_{n_max})^2 := -4 \cdot c_1 \cdot c_2 \cdot \dots \cdot c_n \cdot (\rho - \rho_{max}) \quad \blacksquare \tag{23}$$

A parabolic shaping factor, aa , can be defined in terms of the c_i , as before:

$$aa := \frac{1}{4 \cdot (c_1 \cdot c_2 \cdot \dots \cdot c_n)} \quad \blacksquare \tag{24}$$

The individual maximum coordinates are computed from

$$xx_{1_max} := \frac{\frac{1}{2 \cdot n} \cdot (\rho_1 - \rho_0 + aa)^2}{aa} \quad (25a)$$

$$xx_{2_max} := \frac{\frac{1}{2 \cdot n} \cdot (\rho_2 - \rho_0 + aa)^2}{aa} \quad (25b)$$

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$$xx_{n_max} := \frac{\frac{1}{2 \cdot n} \cdot (\rho_n - \rho_0 + aa)^2}{aa} \quad (25c)$$

The maximum value of resistivity is then

$$\rho_{max} := \rho_0 + aa \cdot \left(xx_{1_max}^2 + xx_{2_max}^2 + \dots + xx_{n_max}^2 \right) \quad \mu\text{-ohm-cm} \quad (26)$$

The resistivity of the N-ary alloy, ρ_{0_n} , can then be simply expressed as

$$\rho_{0_n} := \rho_{max} - aa \cdot \left[(xx_1 - xx_{1_max})^2 + (xx_2 - xx_{2_max})^2 + \dots + (xx_n - xx_{n_max})^2 \right] \quad \mu\text{-ohm-cm} \quad (27)$$

Sample Calculation for Constantan

Constantan is an alloy comprised of Cu, Ni, and Mn. It's commonly used in thermocouples, so let's calculate its resistivity.

$$N := 3 \quad n := N - 1 \quad (\text{number of solutes})$$

$$\rho_{\text{Cu}} := 1.683 \quad \rho_{\text{Ni}} := 6.413 \quad \rho_{\text{Mn}} := 223.47 \quad (\text{all in } \mu\text{-ohm-cm})$$

$$Z_{\text{Cu}} := 4 \quad Z_{\text{Ni}} := 4 \quad Z_{\text{Mn}} := 4$$

$$w_{\text{Cu}} := 63.546 \quad w_{\text{Ni}} := 58.6934 \quad w_{\text{Mn}} := 54.938049$$

$$c_1 := \left(\frac{\rho_{\text{Cu}}}{w_{\text{Cu}} \cdot Z_{\text{Cu}}} + \frac{\rho_{\text{Ni}}}{w_{\text{Ni}} \cdot Z_{\text{Ni}}} \right) \cdot \frac{2}{(\rho_{\text{Cu}} + \rho_{\text{Ni}})} \quad c_1 = 0.00838$$

$$c_2 := \left(\frac{\rho_{\text{Cu}}}{w_{\text{Cu}} \cdot Z_{\text{Cu}}} + \frac{\rho_{\text{Mn}}}{w_{\text{Mn}} \cdot Z_{\text{Mn}}} \right) \cdot \frac{2}{(\rho_{\text{Cu}} + \rho_{\text{Mn}})} \quad c_2 = 0.00909$$

$$aa := \frac{1}{4 \cdot c_1 \cdot c_2} \quad aa = 3279.83489$$

$$xx_{\text{Ni_max}} := \frac{\frac{1}{2 \cdot 2} \cdot (\rho_{\text{Ni}} - \rho_{\text{Cu}} + aa)}{aa} \quad xx_{\text{Ni_max}} = 0.25036$$

$$xx_{\text{Mn_max}} := \frac{\frac{1}{2 \cdot 2} \cdot (\rho_{\text{Mn}} - \rho_{\text{Cu}} + aa)}{aa} \quad xx_{\text{Mn_max}} = 0.26691$$

$$\rho_{\max} := \rho_{\text{Cu}} + aa \cdot (xx_{\text{Ni}_{\max}}^2 + xx_{\text{Mn}_{\max}}^2) \quad \rho_{\max} = 440.91476 \quad \mu\text{-ohm-cm}$$

$$xx_{\text{Ni}} := .469 \quad xx_{\text{Mn}} := .0112 \quad (\text{known data for Constantan, from Ref. [7], p. 559, converted to at. \%})$$

$$\rho_{0_n} := \rho_{\max} - aa \cdot \left[(xx_{\text{Ni}} - xx_{\text{Ni}_{\max}})^2 + (xx_{\text{Mn}} - xx_{\text{Mn}_{\max}})^2 \right]$$

$$\rho_{0_n} = 69.67536 \quad \mu\text{-ohm-cm}$$

Ref. [7], p. 559, says the value is 50 $\mu\text{-ohm-cm}$. In order for that to be the case, the fraction of the Ni would actually have to be .482--this is only 2.77% greater, so we can probably claim success for the calculation. Here's the 3D plot:

$$\rho_{0_n}(xx_{\text{Ni}}, xx_{\text{Mn}}) := \rho_{\max} - aa \cdot \left[(xx_{\text{Ni}} - xx_{\text{Ni}_{\max}})^2 + (xx_{\text{Mn}} - xx_{\text{Mn}_{\max}})^2 \right]$$

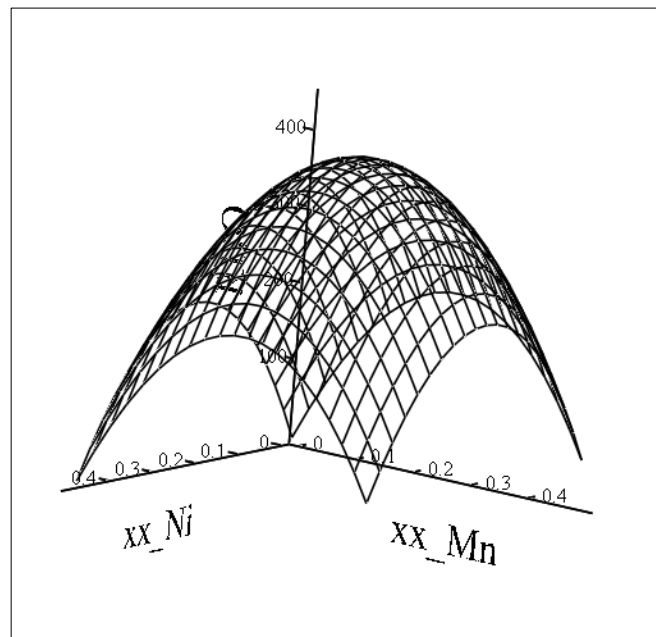


Figure 16. Resistivity of Cu-Ni-Mn

As expected, planar cuts of the surface give parabolic segments.

ρ_{0_n}

Sample Calculation for Iron Nickel Chromium

$$N := 3 \quad n := N - 1 \quad n = 2 \quad (\text{number of solutes})$$

$$\rho_{\text{Fe}} := 9.137 \quad \rho_{\text{Ni}} := 6.413 \quad \rho_{\text{Cr}} := 12.323 \quad (\text{all in } \mu\text{-ohm-cm})$$

$$Z_{\text{Fe}} := 2 \quad Z_{\text{Ni}} := 4 \quad Z_{\text{Cr}} := 2$$

$$w_{\text{Fe}} := 55.845 \quad w_{\text{Ni}} := 58.6934 \quad w_{\text{Cr}} := 51.9961$$

$$c_1 := \left(\frac{\rho_{\text{Fe}}}{w_{\text{Fe}} \cdot Z_{\text{Fe}}} + \frac{\rho_{\text{Ni}}}{w_{\text{Ni}} \cdot Z_{\text{Ni}}} \right) \cdot \frac{2}{(\rho_{\text{Fe}} + \rho_{\text{Ni}})} \quad c_1 = 0.01404$$

$$c_2 := \left(\frac{\rho_{\text{Fe}}}{w_{\text{Fe}} \cdot Z_{\text{Fe}}} + \frac{\rho_{\text{Cr}}}{w_{\text{Cr}} \cdot Z_{\text{Cr}}} \right) \cdot \frac{2}{(\rho_{\text{Fe}} + \rho_{\text{Cr}})} \quad c_2 = 0.01867$$

$$aa := \frac{1}{4 \cdot c_1 \cdot c_2} \quad aa = 954.18344$$

$$xx_{\text{Ni_max}} := \frac{\frac{1}{2 \cdot 2} \cdot (\rho_{\text{Ni}} - \rho_{\text{Fe}} + aa)}{aa} \quad xx_{\text{Ni_max}} = 0.24929$$

$$xx_{\text{Cr_max}} := \frac{\frac{1}{2 \cdot 2} \cdot (\rho_{\text{Cr}} - \rho_{\text{Fe}} + aa)}{aa} \quad xx_{\text{Cr_max}} = 0.25083$$

$$\rho_{\max} := \rho_{\text{Fe}} + aa \cdot (xx_{\text{Ni}_{\max}}^2 + xx_{\text{Cr}_{\max}}^2) \quad \rho_{\max} = 128.46883 \quad \mu\text{-ohm-cm}$$

$$xx_{\text{Ni}} := .285 \quad xx_{\text{Cr}} := .215$$

$$\rho_{0_n} := \rho_{\max} - aa \cdot \left[(xx_{\text{Ni}} - xx_{\text{Ni}_{\max}})^2 + (xx_{\text{Cr}} - xx_{\text{Cr}_{\max}})^2 \right] \quad \rho_{0_n} = 126.02651 \quad \mu\text{-ohm-cm}$$

Ref. [9], p. 329, shows graphs of resistivity for Iron Nickel Chromium for various concentrations. For the concentrations used above, the experimental value from the appropriate graph is 100 $\mu\text{-ohm-cm}$. Therefore the calculation is within the "ballpark" but the spread is greater than we would like. Again, the formulae are very sensitive to concentration, so a mistake in the given solute concentrations could possible explain the spread.

$$\rho_{0_n}(xx_{Ni}, xx_{Cr}) := \rho_{max} - aa \cdot \left[(xx_{Ni} - xx_{Ni_max})^2 + (xx_{Cr} - xx_{Cr_max})^2 \right]$$

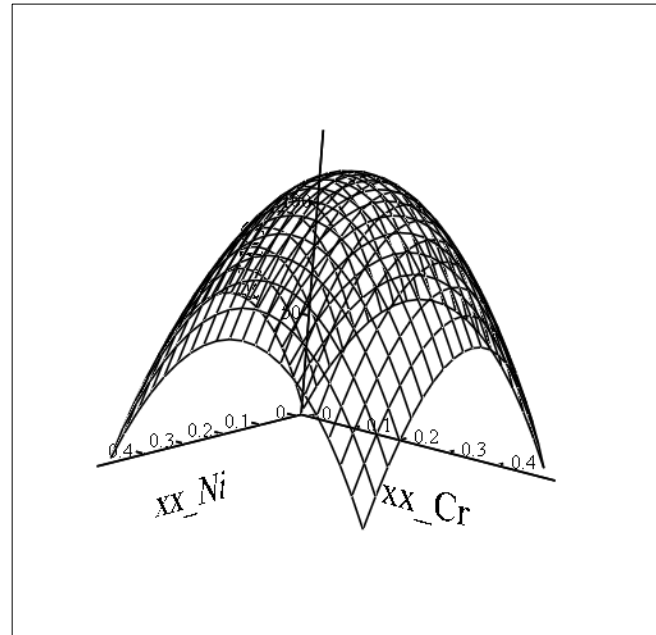


Figure 17. Resistivity of Fe-Ni-Cr

As expected, planar cuts of the surface give parabolic segments.

ρ_{0_n}

9. Resistivity of Metallic Alloys--Ordered Solutions, Intermetallic Compounds

Unlike the disordered alloys considered above, intermetallic compounds have fixed proportions of the elements. The resistivity can be calculated using the same equation as that for an individual element but substituting the data for the compound. Let subscript 1 apply to the first element and subscript 2 apply to the second element. Then Eq. 4 can be rewritten as

$$\rho_{1_2} := \frac{\rho_u \cdot d_u}{\left(\frac{V_{uc}}{s_{t_u}^3}\right)} \cdot \frac{s_0}{s_{t_u} \cdot T_u} \cdot \left[\frac{Z_1 \cdot w_1 \cdot (T - T_{sc_1})}{d_{atom_1}} \cdot \frac{1}{\left[\frac{t_{e_mod_1}}{\left(t_{p_1}^2 \cdot t_{s_1}\right)^{\frac{1}{3}}}\right]^2} + \frac{Z_2 \cdot w_2 \cdot (T - T_{sc_2})}{d_{atom_2}} \cdot \frac{1}{\left[\frac{t_{e_mod_2}}{\left(t_{p_2}^2 \cdot t_{s_2}\right)^{\frac{1}{3}}}\right]^2} \right]^{\frac{1}{2}} \quad (28)$$

It's easy to generalize Eq. 28 to apply to any number of elements for a compound.

$$\rho_{1_n} := \frac{\rho_u \cdot d_u}{\left(\frac{V_{uc}}{s_{t_u}^3}\right)} \cdot \frac{s_0}{s_{t_u} \cdot T_u} \cdot \sum_{i=1}^n \left[\frac{Z_i \cdot w_i \cdot (T - T_{sc_i})}{d_{atom_i}} \cdot \frac{1}{\left[\frac{t_{e_mod_i}}{\left(t_{p_i}^2 \cdot t_{s_i}\right)^{\frac{1}{3}}}\right]^2} \right]^{\frac{1}{2}} \quad (29)$$

(Of course, $\sum_{i=1}^n Z_i := Z$ for the unit cell; the resistivity is in the same direction as s_0 .)

Sample Calculation for Gold Copper, AuCu₃

Here:

$$V_{uc} := 52.642 \quad s_0 := 2.6365 \quad T := 473.15 \quad (\text{based on graph on p. 119 of Ref. [15] for 200 deg. C})$$

$$Z_1 := 1 \quad w_1 := 196.96655 \quad T_{sc_1} := 23.94 \quad d_{atom_1} := 13.942 \quad t_{e_mod_1} := 19.5 \quad t_{p_1} := 4 \quad t_{s_1} := 3$$

$$Z_2 := 3 \quad w_2 := 63.546 \quad T_{sc_2} := 47.88 \quad d_{atom_2} := 8.366 \quad t_{e_mod_2} := 13 \quad t_{p_2} := 3 \quad t_{s_2} := 2$$

$$\rho_{1_2} := \frac{\rho_u \cdot d_u}{V_{uc}} \cdot \frac{s_0}{s_{t_u} \cdot T_u} \cdot \left[\frac{Z_1 \cdot w_1 \cdot (T - T_{sc_1})}{d_{atom_1}} \cdot \frac{1}{\left[\frac{t_{e_mod_1}}{\left(t_{p_1}^2 \cdot t_{s_1} \right)^{\frac{1}{3}}} \right]^2} + \frac{Z_2 \cdot w_2 \cdot (T - T_{sc_2})}{d_{atom_2}} \cdot \frac{1}{\left[\frac{t_{e_mod_2}}{\left(t_{p_2}^2 \cdot t_{s_2} \right)^{\frac{1}{3}}} \right]^2} \right]$$

$$\rho_{1_2} = 3.13707 \quad \mu\text{-ohm-cm}$$

As expected, this agrees with the experimental value reported in Ref. [15], for 25% (atomic) Au. Resistivity calculations for many other compounds are provided in the Reciprocal System Database, Ref. [5].

10. Resistivity of Semiconductor Elements with Metallic Impurities

Typical metallic impurity concentrations in semiconductors range from 10^{15} to 10^{20} atoms per cubic centimeter, which is very dilute, so the fractional volume method and the pressure solubility method given above do not apply to this situation. This leaves the mass fraction method as the proper way to go.

The added metallic atoms increase conductivity. Let σ_{mix} be the conductivity of the mixture, σ_{met} be the conductivity of the metal element, ϕ_{met} be the mass fraction of metallic element, w_{met} be the atomic weight of the metal element, w_{semic} be the atomic weight of the semiconductor element, C_{met} be the number of metallic atoms per cubic centimeter, and C_{semic} be the number of semiconductor atoms per cubic centimeter. Then, for a binary mix we have

$$\phi_{\text{met}} := \frac{C_{\text{met}} \cdot w_{\text{met}}}{C_{\text{met}} \cdot w_{\text{met}} + C_{\text{semic}} \cdot w_{\text{semic}}} \quad (30)$$

$$\sigma_{\text{mix}} := \sigma_{\text{met}} \cdot \phi_{\text{met}} + \sigma_{\text{semic}} \cdot (1 - \phi_{\text{met}}) \quad (31)$$

Sample Calculation for Silicon and Aluminum

$$C_{\text{Al}} := 1 \cdot 10^{15} \quad w_{\text{Al}} := 26.981538 \quad \sigma_{\text{Al}} := \frac{1}{2.630 \cdot 10^{-6}} \quad \sigma_{\text{Al}} = 3.80228 \times 10^5 \text{ ohm}^{-1}\text{-cm}^{-1}$$

$$w_{\text{Si}} := 28.0855 \quad \gamma_{\text{Si}} := 2.33 \quad C_{\text{Si}} := \frac{A_V}{w_{\text{Si}}} \cdot \gamma_{\text{Si}} \quad C_{\text{Si}} = 4.99828 \times 10^{22}$$

$$\sigma_{\text{Si}} := \frac{1}{28.32788} \quad \sigma_{\text{Si}} = 0.0353 \text{ ohm}^{-1}\text{-cm}^{-1} \quad \phi_{\text{Al}} := \frac{C_{\text{Al}} \cdot w_{\text{Al}}}{C_{\text{Al}} \cdot w_{\text{Al}} + C_{\text{Si}} \cdot w_{\text{Si}}} \quad \phi_{\text{Al}} = 1.922046 \times 10^{-8}$$

$$\sigma_{\text{mix}} := \sigma_{\text{Al}} \cdot \phi_{\text{Al}} + \sigma_{\text{Si}} \cdot (1 - \phi_{\text{Al}}) \quad \sigma_{\text{mix}} = 0.04261 \quad \rho_{\text{mix}} := \frac{1}{\sigma_{\text{mix}}} \quad \rho_{\text{mix}} = 23.46918 \text{ ohm-cm}$$

Sample Calculation for Germanium and Aluminum

$$w_{\text{Ge}} := 72.64 \quad \gamma_{\text{Ge}} := 5.321 \quad C_{\text{Ge}} := \frac{A_V}{w_{\text{Ge}}} \cdot \gamma_{\text{Ge}} \quad C_{\text{Ge}} = 4.41331 \times 10^{22}$$

$$\phi_{\text{Al}} := \frac{C_{\text{Al}} \cdot w_{\text{Al}}}{C_{\text{Al}} \cdot w_{\text{Al}} + C_{\text{Ge}} \cdot w_{\text{Ge}}} \quad \phi_{\text{Al}} = 0 \quad \sigma_{\text{Ge}} := \frac{1}{47.35187} \quad \sigma_{\text{Ge}} = 0.02112 \quad \text{ohm}^{-1}\text{-cm}^{-1}$$

$$\sigma_{\text{mix}} := \sigma_{\text{Al}} \cdot \phi_{\text{Al}} + \sigma_{\text{Ge}} \cdot (1 - \phi_{\text{Al}}) \quad \sigma_{\text{mix}} = 0.02432 \quad \rho_{\text{mix}} := \frac{1}{\sigma_{\text{mix}}} \quad \rho_{\text{mix}} = 41.12072 \quad \text{ohm-cm}$$

Let's plot resistivity of as a function of bulk concentration (C_B) of Al for both Si and Ge.

$$\sigma_{\text{Al}} \cdot \left(\frac{C_B \cdot w_{\text{Al}}}{C_B \cdot w_{\text{Al}} + C_{\text{Si}} \cdot w_{\text{Si}}} \right) + \sigma_{\text{Si}} \cdot \left(1 - \frac{C_B \cdot w_{\text{Al}}}{C_B \cdot w_{\text{Al}} + C_{\text{Si}} \cdot w_{\text{Si}}} \right)$$

$$\sigma_{\text{Al}} \cdot \left(\frac{C_B \cdot w_{\text{Al}}}{C_B \cdot w_{\text{Al}} + C_{\text{Ge}} \cdot w_{\text{Ge}}} \right) + \sigma_{\text{Ge}} \cdot \left(1 - \frac{C_B \cdot w_{\text{Al}}}{C_B \cdot w_{\text{Al}} + C_{\text{Ge}} \cdot w_{\text{Ge}}} \right)$$

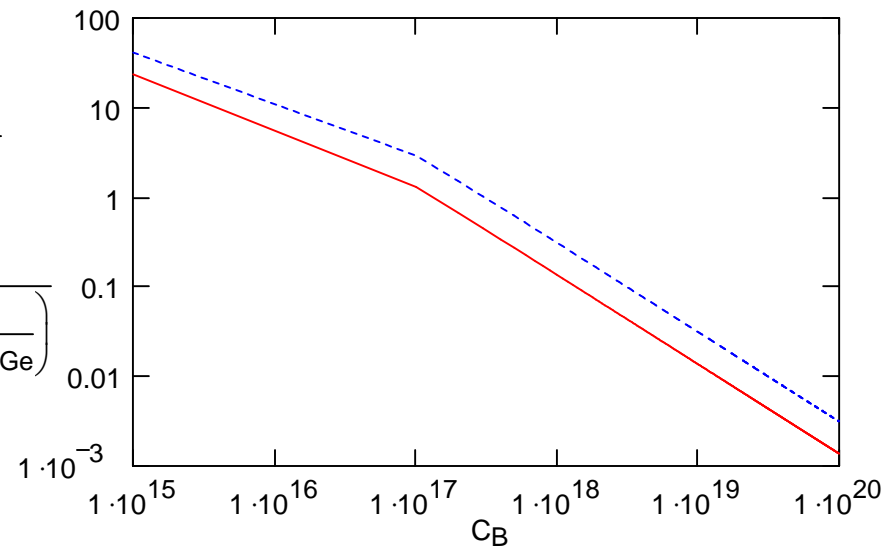


Figure 18. Resistivity of Si Doped with Al, Ge

The resulting graphs agree in a general way with those on p. 309 of Ref. [16], except that in ours the resistivity is higher for Ge than with Si; some of the empirical data reported in Ref. [16] is now obsolete and has been superseded by that in Ref. [10].

11. Resistivity of Semiconductor Compounds

Semiconductor compounds may consist entirely of semiconductor elements or a combination of metallic and semiconductor elements. For a semiconductor compound with only semiconductor elements, the general equation for resistivity is

$$\rho_{\text{semic_compound}} := \rho_u \cdot \frac{(T_{t_inv_u})^2}{V_{uc}} \cdot \frac{s_0 \cdot d_u}{(s_{t_u})^3} \cdot \sum_i \left[\frac{Z_i \cdot \frac{w_i}{1}}{d_{\text{atom}_i} \cdot (T - T_{bc_i})^2} \cdot \frac{e^{\left[\frac{1}{4k \cdot (T - T_{bc_i})} \right]}}{\left[\frac{t_{e_mod_i}}{(t_{p_i})^2 \cdot t_{s_i}} \right]^{\frac{1}{3}}} \right]^2 \quad (T > T_{bc_i}) \quad (32)$$

ohm-cm

For a mixed compound:

$$\rho_{\text{semic_met}} := \frac{\rho_u \cdot d_u \cdot \frac{s_0}{s_{t_u}}}{10^6 \cdot \frac{V_{uc}}{(s_{t_u})^3}} \cdot \left[\frac{(T_{t_inv_u})^2}{(T - T_{bc_1})^2} \cdot \frac{Z_1 \cdot w_1}{d_{\text{atom}_1}} \cdot \frac{e^{\left[\frac{1}{4 \cdot k \cdot (T - T_{bc_1})} \right]}}{\left[\frac{t_{e_mod_1}}{(t_{p_1})^2 \cdot t_{s_1}} \right]^{\frac{1}{3}}} \right]^2 \dots + \frac{T - T_{sc_2}}{T_u} \cdot \frac{Z_2 \cdot w_1}{d_{\text{atom}_2}} \cdot \left[\frac{1}{\left[\frac{t_{e_mod_2}}{(t_{p_2})^2 \cdot t_{s_2}} \right]^{\frac{1}{3}}} \right]^2 \quad (33)$$

ohm-cm

(All temperatures are in K, as usual.)

Sample Calculation for Gallium Arsenide

Whereas the individual elements of Gallium and Arsenic are metallic and have metallic resistivity, when they are combined in GaAs (with an electronegative-type bond) their electronegative natures dominate and so the compound has semiconductor resistivity.

$$V_{uc} := 180.717 \quad s_0 := 2.4712 \quad Z_{Ga} := 4 \quad Z_{As} := 4 \quad w_{Ga} := 69.723 \quad w_{As} := 74.913$$

$$T_{sc_Ga} := 15.96 \quad T_{bc_As} := 47.88 \quad d_{atom_Ga} := 8.366 \quad d_{atom_As} := 8.366 \quad T := 293.15 \text{ K}$$

$$t_{p_Ga} := 3 \quad t_{p_As} := 3 \quad t_{s_Ga} := 3 \quad t_{s_As} := 3 \quad t_{e_mod_Ga} := 3.5 \quad t_{e_mod_As} := 2.5$$

$$T_{sc_As} := 47.88 \quad T_{bc_Ga} := 15.96$$

$$\rho_{GaAs} := \frac{\rho_u \cdot d_u \cdot \frac{s_0}{s_{t_u}}}{10^6 \cdot \frac{V_{uc}}{(s_{t_u})^3}} \left[\frac{\left(T_{t_inv_u}^2 \right) \cdot Z_{Ga} \cdot w_{Ga}}{\left(T - T_{bc_Ga} \right)^2 \cdot d_{atom_Ga}} \cdot \frac{e^{\left[\frac{1}{4 \cdot k \cdot (T - T_{bc_Ga})} \right]}}{\left[\frac{t_{e_mod_Ga}}{\left(t_{p_Ga}^2 \cdot t_{s_Ga} \right)^{\frac{1}{3}}} \right]^2} \dots \right. \\ \left. + \frac{\left(T_{t_inv_u}^2 \right) \cdot Z_{As} \cdot w_{As}}{\left(T - T_{bc_As} \right)^2 \cdot d_{atom_As}} \cdot \frac{e^{\left[\frac{1}{4 \cdot k \cdot (T - T_{bc_As})} \right]}}{\left[\frac{t_{e_mod_As}}{\left(t_{p_As}^2 \cdot t_{s_As} \right)^{\frac{1}{3}}} \right]^2} \right]$$

$$\rho_{GaAs} = 4.95934 \text{ ohm-cm}$$

This is somewhat lower than the value read from Fig. 2.11.12 of Ref. [10]. But note that our calculation is for resistivity along the line from a Ga atom to an As atom--in which the interatomic distance is s_0 .

Here's the graph, but plotting σ instead of ρ so as to be able to compare with Fig. 2.11.12 of the CD of Ref. [10].:

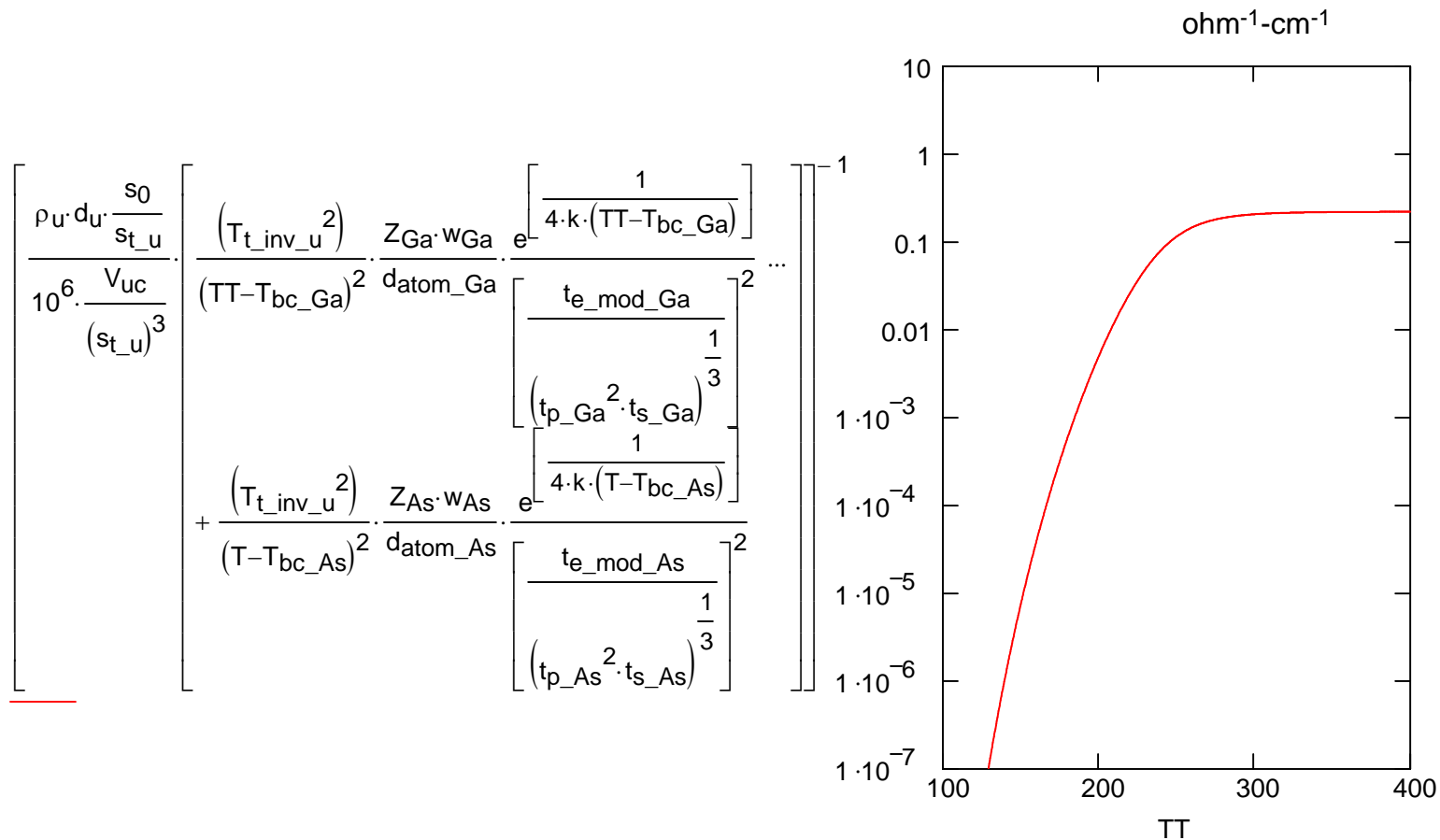


Figure 19. Resistivity of GaAs

The curve is identical in shape, but somewhat offset from, the curve referenced above, for vacuum-annealed GaAs. To secure closer agreement, one could modify the values of te_mod_Ga and te_mod_As , but this would then lessen the predictive power of the equations--and would go against the Reciprocal System's prohibition of the use of adjustable parameters.

There are numerous other semiconductor compounds, many of which are presented in Ref. [7] and Ref. [10], but it should be obvious how to apply Eqs. (32) and (33) to these.

Conclusion

In the Reciprocal System, ordinary (non-static) electrical current is comprised of massless, chargeless electrons. Solid state band theory is therefore not used, and so the Reciprocal System has no need for electrons with "effective mass," "holes" of any kind ("light" or "heavy") with any "effective mass," phonons, excitons, energy gaps, acceptors, or donors. The electrons have no charge here and so there is no need for "charge balancing." Equations for the resistivity of metallic and semiconductor elements and compounds and alloys have been derived from the principles of the Reciprocal System. There are no adjustable parameters used; all structural constants come from the theory. The electric displacement of an element is, of course, modified somewhat by the passage of electric current; but this is to be expected on the basis of the theory. The agreement between the resistivity data and the calculations is, on the whole, very good, and is within the level of uncertainty of the experiments.

Acknowledgements

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