

Unification between Gravity and Electricity

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Abstract

Much has been said about the unification between gravity and electricity without giving any numerical values that could be validated by experiments.

A moving charged particle creates an electric current which in turn induces a magnetic field and in the same way a moving masse body creates a masse current which in turn induces a gravitomagnetic field. By using the *Force Transformation*, this exposé shows the reader how to derive the magnetic field directly from the electric field without using formulae memorized in school, it also shows the reader how to derive the gravitomagnetic field directly from the gravitational field without using Einstein equation of gravity. By this way the coupling coefficient between gravitomagnetic field and the magnetic field has been calculated, thereby unifying gravity and electricity.

We know that a rotating charged body induces a magnetic field, likewise a rotating masse body induces a gravitomagnetic field due to circular masse currents, in that connection since the Earth is in rotation, it induces a gravitomagnetic field whose numerical value at the North Pole has been calculated in this exposé, it is equal to $1 \times 10^{-14} \text{ s}^{-1}$. As we shall see later in this exposé, the rotation of the accretion disc of the galaxies induces a gravitomagnetic field, this field tends to compress the galaxies into thin discs and this explains the shape of the galaxies. It also explains the cone like shape of the bipolar matter jets by the disc of accretion. By knowing the speed of rotation and the masse of the galaxy accretion disc, one can calculate the gravitomagnetic field. The flat shape of the rings of Saturn, the fact that the orbit of the Moon is on the axis of rotation of the Earth, the fact that the tails of comets are always pointed away from the Sun and the fact that the orbits of the planets of the solar system are almost in the same plane is due to the gravitomagnetic field that tends to flatten rotating a masse system.

This rapport also gives numerical values of gravitomagnetism permeability μ_g and permittivity ϵ_g which are equal to $\approx 9.4 \times 10^{-27} \text{ m/kg}$ and $\approx 1.18 \times 10^9 \text{ kg.s}^2/\text{m}^3$ respectively. Assuming that $\epsilon_g = 1/4\pi G$ and $c^2\epsilon_g\mu_g = 1$ where G is the constant of gravity and c the speed of light.

It has been shown, in this rapport, that light is composed of electromagnetic waves and gravitomagnetic waves and that the electromagnetic wave can not exist without the gravitomagnetic wave but the gravitomagnetic wave can exist without the electromagnetic wave. A new wave equation of energy has been derived thereby putting into light part of the missing energy of the universe.

And finally it has been shown that the gravitomagnetic and magnetic fields are due to time and space delay. There is no literature in this exposé; we have confined ourselves strictly to the *Force Transformation* without asking ourselves whether the gravity is a tensor or a field. Thus we have confined ourselves to the forces exerted on relative moving particles and transforming these forces from a reference frame at rest to a moving reference frame.

Before devising gravitomagnetic, we have to know what we are looking for; this rapport will guide engineers to devising sensors to measure gravitomagnetic quantities.

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Unification between gravity and electricity

Objective

Hypothesis

Method

EXISTENCE OF MAGNETIC FIELD

Electric conductor of infinite length

EXISTENCE OF GRAVITOMAGNETIC FIELD

Mass conductor of infinite length

Attractive gravitomagnetic force

Repulsive gravitomagnetic force

Relationship between magnetic and gravitomagnetic field

Circular masse ring in rotation

Masse disc in rotation

Gravitomagnetic effect on the shape of galaxies

Bipolar jets of matter

Spiral aspiration of matter

Full sphere in rotation

Gravitomagnetic field in Earths poles

Masse and charged particle

PROPAGATION OF GRAVITOMAGNETIC WAVES

Electromagnetism and gravitomagnetism

Gravitomagnetic induction

Electrogravitomagnetic wave energy

Electrogravitomagnetic equation wave of energy

PHYSICAL CONSTANTS

PHYSICAL QUANTITIES

TIME AND SPACE DELAY

Unification between gravity and electricity

Objective

Show the existence of gravitomagnetic field, we will not propose any theory in this exposé; we will only use mathematical tools that are accepted and experimented by scientists.

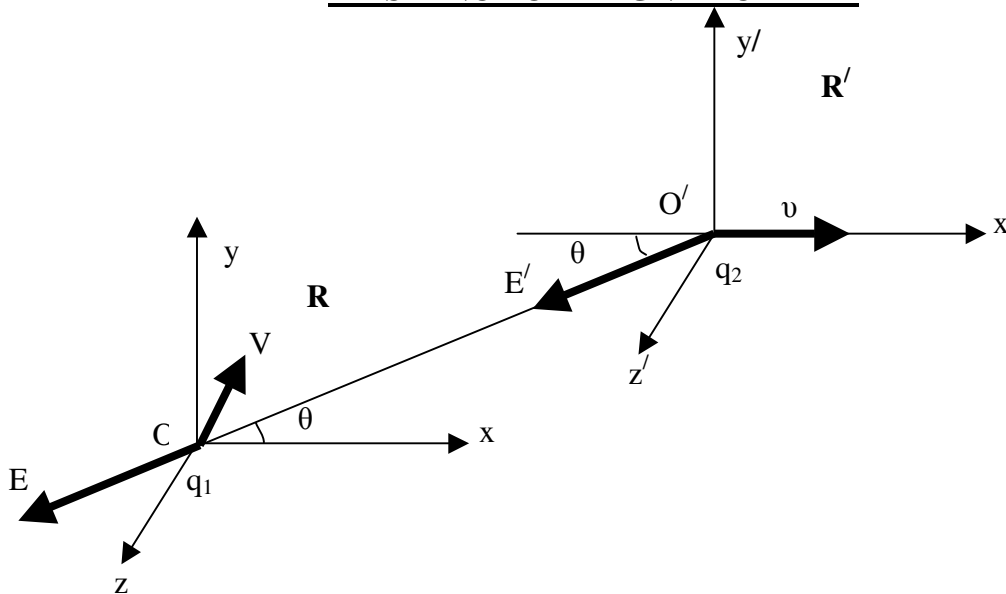
Hypothesis

The gravitation interaction in vacuum is propagated at a finite speed; let this speed be equal to the speed of light.

Method

In accordance with the reference frame to reference frame forces transformation principle, we will apply the Lorentz transformation to the electric field in order to show the existence of magnetic field. Analogically, we will apply the Lorentz transformation to the gravitational field in order to show the existence of gravitomagnetic field. And finally, we will show that light is composed of electromagnetic and gravitomagnetic waves. That wave, thus the light, will be known as electrogravitomagnetic wave.

EXISTENCE OF MAGNETIC FIELD



Consider a reference frame R' in translation along Ox axis at a velocity of v with respect to a reference frame R . Let q_2 be a charge at rest in the reference frame R' , consequently this charge q_2 is also moving at a velocity of v with respect to the reference frame R . We are going to calculate the force that charge q_2 exerts on the charge q_1 , the later q_1 is moving at velocity of V with respect to reference frame R . E' denotes the electric field build by q_2 with respect to reference frame R' at a given radial distance r in space, E denotes the electric field build by q_2 with respect to reference frame R at a given radial distance r in space. The yOx and $y'O'x'$ plains are in the same plain and Ox and $O'x'$ axis are parallel.

Let us find the force exerted by charge q_2 on the charge q_1 with respect to reference frame R . Applying Lorentz transformation to a force permits us to determine how a force is

transformed from the reference frame R' to the reference frame R using the following equations.

$$F_x = \frac{1}{(1 + \beta \frac{V'_x}{c})} (F'_x + \beta \frac{F'_y V'_y}{c})$$

$$F_y = \frac{1}{\gamma(1 + \beta \frac{V'_x}{c})} F'_y$$

$$F_z = \frac{1}{\gamma(1 + \beta \frac{V'_x}{c})} F'_z$$

With ;

$$V'_x = \frac{(V_x - v)}{(1 - \beta \frac{V_x}{c})}$$

$$V'_y = \frac{V_y}{\gamma(1 - \beta \frac{V_x}{c})}$$

$$V'_z = \frac{V_z}{\gamma(1 - \beta \frac{V_x}{c})}$$

$$\text{With } \beta = \frac{v}{c} \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

c is the speed of light:

$$v \geq 0$$

We deduce that:

$$F_x = q_1 E'_x + \frac{\gamma \beta q_1}{c} (E'_y V_y + E'_z V_z)$$

$$F_y = \gamma q_1 E'_y - \frac{\gamma \beta q_1}{c} E'_y V_x$$

$$F_z = \gamma q_1 E'_z - \frac{\gamma \beta q_1}{c} E'_z V_x$$

To simplify the demonstration let the velocity V be in the $y'O'x'$ plain:

\Leftrightarrow

$$V_z = 0 \text{ and } E_z = 0$$

The equations before are simplified down to:

$$F_x = q_1 E'_x + \frac{\gamma \beta q_1}{c} E'_y V_y$$

$$F_y = \gamma q_1 E'_y - \frac{\gamma \beta q_1}{c} E'_y V_x$$

We note the 4 dimensions pseudo norm or quadrivector:

$r^2 = c^2 t^2 - \mathbf{r}^2 = r'^2 = c^2 t'^2 - \mathbf{r}'^2$, where \mathbf{r}^2 et \mathbf{r}'^2 are radial vectors that join q_1 and q_2 , r is the distance between the two charges q_1 and q_2 . This quantity $r^2 = c^2 t^2 - \mathbf{r}^2 = c^2 t'^2 - \mathbf{r}'^2$ does not depend on the reference frame and constitutes an invariant.

In accordance with the invariance of this quantity ;

With $k = 1/4\pi\epsilon_0$, ϵ_0 being the electric field permittivity in vacuum, then E'_x and E'_y are given by:

$$E'_x = -\frac{kq_2 \cos(\theta)}{r^2}$$

$$E'_y = -\frac{kq_2 \sin(\theta)}{r^2}$$

$$F_x = -\frac{kq_1q_2 \cos(\theta)}{r^2} - \frac{\gamma\beta kq_1q_2 \sin(\theta)V_y}{r^2c}$$

$$F_y = -\frac{\gamma kq_1q_2 \sin(\theta)}{r^2} + \frac{\gamma\beta kq_1q_2 \sin(\theta)V_x}{r^2c}$$

Fundamental equations

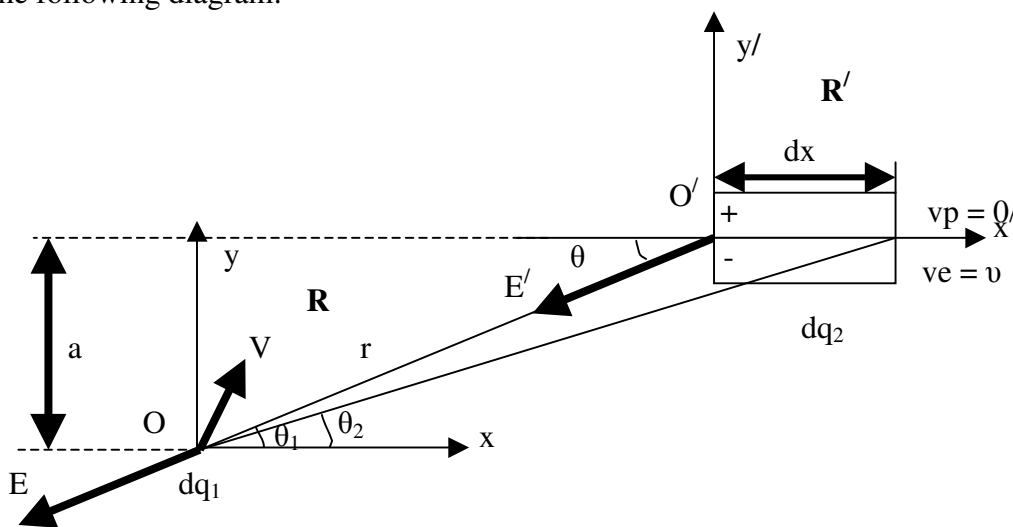
If $c \rightarrow \infty$, we get back the electrostatic equations, thus:

$$F_x = -\frac{kq_1q_2 \cos(\theta)}{r^2}$$

$$F_y = -\frac{kq_1q_2 \sin(\theta)}{r^2}$$

To show the validity of the fundamental equations before, we are going to determine the force exerted, on q_1 moving at a velocity of V with respect to the reference frame R , by a neutral rectiline conductor parallel to the Ox axis, passed by an electric current of I_2 .

This conductor is composed of positive charges that are immobile with respect to the reference frame R and negatives charges moving at a velocity of v along the Ox axis with respect to the reference frame R , but a rest with the respect to reference frame R' , as shown in the following diagram:



1) We will determine the force due to the positive charge dq_2 on dq_1 . Since the reference frame R' associated to the positive charge is at rest with respect to reference frame R , with $dq_2 > 0$, then $v_p = 0 \Rightarrow \beta = 0$ et $\gamma = 1$

\Rightarrow

$$dF_{xp} = - \frac{k \cdot dq_1 \cdot dq_2 \cdot \cos(\theta)}{r^2}$$

$$dF_{yp} = - \frac{k \cdot dq_1 \cdot dq_2 \cdot \sin(\theta)}{r^2}$$

2) We will determine the force due to the negative charge dq_2 on dq_1 . Since the reference frame R' associated is moving at a velocity of v with respect to the reference frame R , $v_e = v$ and $dq_2 < 0$:

$$dF_{xe} = + \frac{k \cdot dq_1 \cdot dq_2 \cdot \cos(\theta)}{r^2} + \frac{\gamma \beta k \cdot dq_1 \cdot dq_2 \cdot \sin(\theta) V_y}{r^2 c}$$

$$dF_{ye} = + \frac{\gamma k \cdot dq_1 \cdot dq_2 \cdot \sin(\theta)}{r^2} - \frac{\gamma \beta k \cdot dq_1 \cdot dq_2 \cdot \sin(\theta) V_x}{r^2 c}$$

Supposing that the system is linear:

$$dF_x = dF_{xp} + dF_{xe}$$

And

$$dF_y = dF_{yp} + dF_{ye}$$

\Rightarrow

$$dF_x = 0 + \frac{\gamma \beta k \cdot dq_1 \cdot dq_2 \cdot \sin(\theta) V_y}{r^2 c}$$

$$dF_y = \frac{(\gamma - 1) k \cdot dq_1 \cdot dq_2 \cdot \sin(\theta)}{r^2} - \frac{\gamma \beta k \cdot dq_1 \cdot dq_2 \cdot \sin(\theta) V_x}{r^2 c}$$

Fundamental equations for a neutral electric conductor

But $\beta = \frac{v}{c}$,

$$dF_x = + \frac{\gamma \cdot k \cdot dq_1 \cdot dq_2 \cdot v \sin(\theta) V_y}{r^2 c^2}$$

$$dF_y = + \frac{(\gamma - 1) k \cdot dq_1 \cdot dq_2 \cdot \sin(\theta)}{r^2} - \frac{\gamma \cdot k \cdot dq_1 \cdot dq_2 \cdot v \sin(\theta) V_x}{r^2 c^2}$$

But

$$v = \frac{dx}{dt}$$

v being the velocity of the charge.

$$dq_2 \cdot v = d q_2 \cdot \frac{dx}{dt} = \frac{dq_2 dx}{dt} = I_2 dx, \text{ where } I_2 = \text{the electric current}$$

By replacing $dq_2 v$ in the two functions dF_x and dF_y , we get ;

$$dF_x = + \frac{\gamma \cdot k \cdot dq_1 \cdot I_2 dx \cdot \sin(\theta) V_y}{r^2 c^2}$$

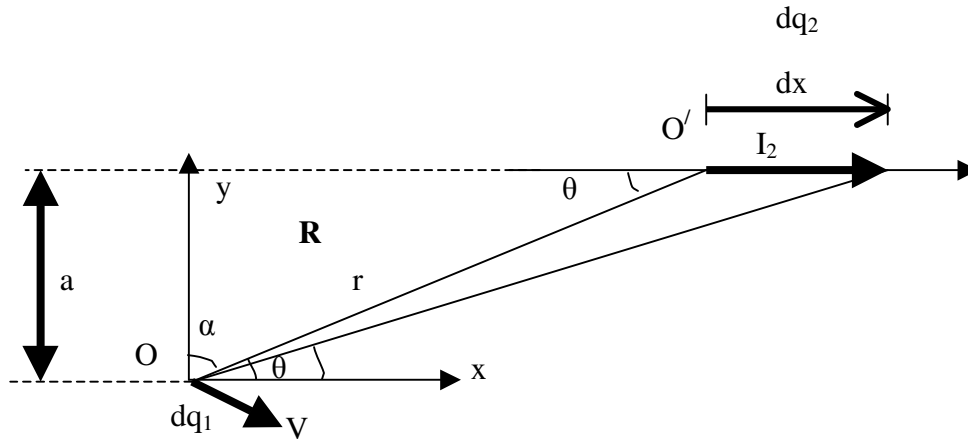
$$dF_y = + \frac{(\gamma - 1) k \cdot dq_1 \cdot dq_2 \cdot \sin(\theta)}{r^2} - \frac{\gamma \cdot k \cdot dq_1 \cdot I_2 dx \cdot \sin(\theta) V_x}{r^2 c^2}$$

If $v \ll c \Rightarrow \gamma \approx 1$, we get

$$dF_x = + \frac{k dq_1 I_2 dx \sin(\theta) V_y}{r^2 c^2}$$

$$dF_y = - \frac{k dq_1 I_2 dx \sin(\theta) V_x}{r^2 c^2}$$

Changing the variable, if $\theta = \pi/2 - \alpha$, θ varies from π to 0 and α varies from $-\pi/2$ to $\pi/2$, as shown in the following diagram.



$$dF_x = + \frac{k dq_1 I_2 dx \cos(\alpha) V_y}{r^2 c^2}$$

$$dF_y = - \frac{k dq_1 I_2 dx \cos(\alpha) V_x}{r^2 c^2}$$

But

$k = 1/4\pi\epsilon_0$, ϵ_0 being the electric field permittivity, let μ_0 be a variable such that, $\epsilon_0\mu_0 = 1/c^2$, by replacing c and k in the equations about, we get;

$$dF_x = + \frac{dq_1 \mu_0 I_2 dx \cos(\alpha) V_y}{4\pi r^2}$$

$$dF_y = - \frac{dq_1 \mu_0 I_2 dx \cos(\alpha) V_x}{4\pi r^2}$$

But we chose the the negative charge sense (direction of negative charge drift), it is opposite to the conventional sense (direction of the positive charge drift), we will invert the sign of the velocity v , thus, invert current I_2 sign in order to be in conformity with the conventional electric current sense.

Where I_2 denotes the conventional current sense, thus opposite to negative charge direction of movement.

Let,

$$dF_x = - \frac{dq_1 \mu_0 I_2 dx \cos(\alpha) V_y}{4\pi r^2}$$

$$dF_y = + \frac{dq_1 \mu_0 I_2 dx \cos(\alpha) V_x}{4\pi r^2}$$

We notice that a vector product appears between the velocity V and a field that we shall call B , supposing that a field B exists such that;

$$dF = \begin{pmatrix} dF_x \\ dF_y \\ 0 \end{pmatrix} = qV \times B = q \begin{pmatrix} V_x \\ V_y \\ 0 \end{pmatrix} \times \begin{pmatrix} dB_x \\ dB_y \\ dB_z \end{pmatrix}$$

After resolution and knowing that V_x can not be expressed as a function of V_y since the associated vectors are orthogonal, we find:

$$dB_x = 0$$

$$dB_y = 0$$

$$dB_z = - \frac{\mu_0 I_2 dx \cos(\alpha)}{4\pi r^2}$$

Written in another way:

$$dF_x = + dq_1 V_y dB_z$$

$$dF_y = - dq_1 V_x dB_z$$

The force exerted on q_1 due to the magnetic field B build by the electric current I_2 is given by:

$$F = q_1 V \times B$$

We have now shown the existence of the magnetic field by applying the Lorenz transformation to the electric field.

By using the Biot and Savart law to determine B , we get:

$$dB_x = 0$$

$$dB_y = 0$$

$$dB_z = - \frac{\mu_0 I_2 dx \cos(\alpha)}{4\pi r^2}$$

μ_0 being the permeability of magnetic field.

In that way, by changing the reference frame, the electric field components orthogonal to the movement were slightly modified. The magnetic field is just a relativity effect due to movement and reference frame change.

We deduce that if B is the magnetic field and E_2 the electric field build by a charge q_2 moving at a velocity of v along the Ox axis, the force to which a charged particle q_1 moving at a velocity of V is exerted is given by:

$$F = q_1 E_2 + q_1 V \times B$$

The components of that force are :

$$F_x = q(V_y B_z - V_z B_y)$$

$$F_y = q(V_z B_x - V_x B_z)$$

$$F_x = q(V_x B_y - V_y B_x)$$

Let:

$$dF_x = - \frac{dq_1 \mu_0 I_2 dx \cos(\alpha) V_y}{4\pi r^2}$$

$$dF_y = + \frac{dq_1 \mu_0 I_2 dx \cos(\alpha) V_x}{4\pi r^2}$$

We will replace dx and r^2 in the equations before to have α as the only variable, we get:

$$x = a \tan(\alpha)$$

$$dx = \frac{a \cdot d\alpha}{\cos^2(\alpha)}$$

$$r^2 = \frac{a^2}{\cos^2(\alpha)}$$

$$dF_x = - \frac{dq_1 \mu_0 I_2 dx \cos(\alpha) V_y d\alpha}{4\pi a}$$

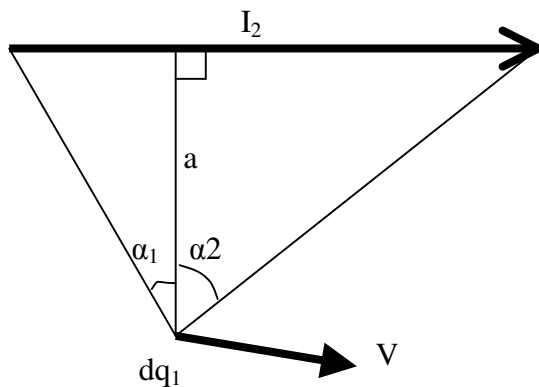
$$dF_y = + \frac{dq_1 \mu_0 I_2 dx \cos(\alpha) V_x d\alpha}{4\pi a}$$

By integration from α_1 to α_2 where α varies from $-\pi/2$ to $\pi/2$, we find:

$$dF_x = - \frac{dq_1 \mu_0 I_2 dx V_y [\sin(\alpha_2) - \sin(\alpha_1)]}{4\pi a}$$

$$dF_y = + \frac{dq_1 \mu_0 I_2 dx V_x [\sin(\alpha_2) - \sin(\alpha_1)]}{4\pi a}$$

Schematically, we have:



1) Electric conductor of infinite length

This implies that: $\alpha_1 = -\pi/2$ and $\alpha_2 = \pi/2$, the F_x and F_y are given by:

$$F_x = - \frac{\mu_0 I_2 dq_1 V_y}{2\pi a}$$

$$F_y = + \frac{\mu_0 I_2 dq_1 V_x}{2\pi a}$$

If the charge dq_1 is moving along the Ox axis, then $V_y = 0$

But:

$$dq_1 V_x = dq_1 (dx/dt) = dx (dq_1/dt) = dx \cdot I_1, \text{ where } I_1 = \text{electric current}$$

Let:

$$F_y = + \frac{\mu_0 I_1 I_2 dx}{2\pi a}$$

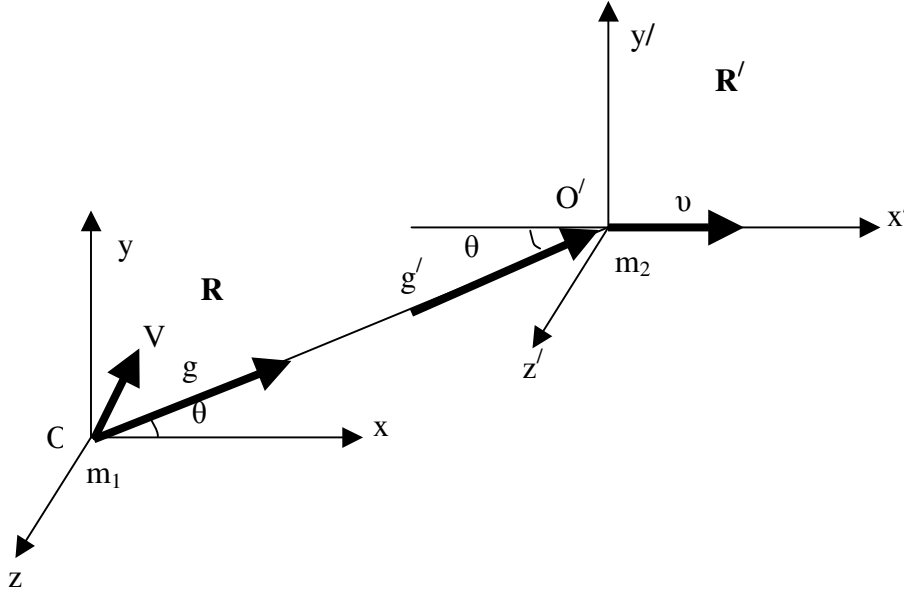
$$\frac{F_y}{dx} = + \frac{\mu_0 I_1 I_2}{2\pi a}$$

This verifies the force between two electric conductors per unit length.

First conclusion

In accordance with the principle of reference frame to reference frame force transformation, by applying the Lorentz transformation to the electric field, we did show the existence of magnetic field. Since the gravity is also a field, We will thereby apply the Lorentz transformation the the gravitionl field in order to show the existnce of gravitomagnetic field.

EXISTENCE OF GRAVITOMAGNETIC FIELD



Consider a reference frame R' in translation along Ox axis at a velocity of v with respect to a reference frame R . Let m_2 be a masse at rest in the reference frame R' , consequently this masse m_2 is also moving at a velocity of v with respect to the reference frame R . We are going to calculate the force that masse m_2 exerts on the masse m_1 , the later m_1 is moving at velocity of V with respect to the reference frame R . g' denotes the gravitational field build by m_2 with respect to the reference frame R' at a given radial distance r in space, g denotes the gravitational field build by m_2 with respect to the reference frame R at a given radial distance r in space. The yOx and $y'O'x'$ plains are in the same plain and Ox and $O'x'$ axis are parallel.

Let us find the force exerted by the masse m_2 on the masse m_1 with respect to the reference frame R . Applying Lorenz transformation to a force permits us to determine how a force is transformed from the reference frame R' to the reference frame R using the following equations:

$$F_x = \frac{1}{(1 + \frac{\beta V'_x}{c})} (F'_x + \beta \frac{F'_y V'_x}{c})$$

$$F_y = \frac{1}{\gamma(1 + \frac{\beta V'_x}{c})} .F'_y$$

$$F_z = \frac{1}{\gamma(1 + \frac{\beta V'_x}{c})} .F'_z$$

With;

$$V'_x = \frac{(V_x - v)}{(1 - \frac{\beta V_x}{c})}$$

$$V'_y = \frac{V_y}{\gamma(1 - \frac{\beta V_x}{c})}$$

$$V'_z = \frac{V_z}{\gamma(1 - \beta \frac{V_x}{c})}$$

$$\text{With } \beta = \frac{v}{c} \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

c is the speed of light.

$$v \geq 0$$

We deduce that:

$$F_x = m_1 g'_x + \frac{\gamma \beta m_1 (g'_y V_y + g'_z V_z)}{c}$$

$$F_y = \gamma m_1 g'_y - \frac{\gamma \beta m_1 g'_y V_x}{c}$$

$$F_z = \gamma m_1 g'_z - \frac{\gamma \beta m_1 g'_z V_x}{c}$$

To simplify the demonstration let the velocity V be in the y'O'x' plain.

$$V_z = 0 \text{ and } E_z = 0$$

The equations before are simplified down to:

$$F_x = m_1 g'_x + \frac{\gamma \beta m_1 g'_y V_y}{c}$$

$$F_y = \gamma m_1 g'_y - \frac{\gamma \beta m_1 g'_y V_x}{c}$$

We note the 4 dimensions pseudo norm or quadrivector:

$r^2 = c^2 t^2 - \mathbf{r}^2 = r'^2 = c^2 t'^2 - \mathbf{r}'^2$ where \mathbf{r}^2 and \mathbf{r}'^2 are radial vectors that join q1 and q2, r is the distance between the two charges q1 and q2. This quantity $r^2 = c^2 t^2 - \mathbf{r}^2 = c^2 t'^2 - \mathbf{r}'^2$ does not depend on the reference frame and constitutes an invariant.

In accordance with the invariance of this quantity;

Let $k = G = 1/4\pi\epsilon_g$, G being the constant of gravity and ϵ_g being the gravitational field permittivity in vacuum, g'_x and g'_y are given by:

$$g'_x = + \frac{k m_2 \cos(\theta)}{r^2}$$

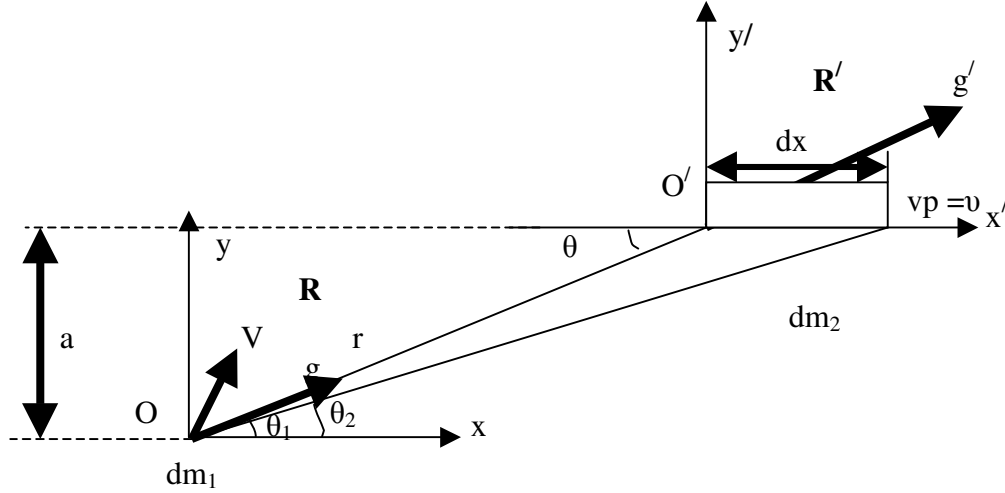
$$g'_y = + \frac{k m_2 \sin(\theta)}{r^2}$$

$F_x = + \frac{k m_1 m_2 \cos(\theta)}{r^2} + \frac{\gamma \beta k m_1 m_2 \sin(\theta) V_y}{r^2 c}$ $F_y = + \frac{\gamma k m_1 m_2 \sin(\theta)}{r^2} - \frac{\gamma \beta k m_1 m_2 \sin(\theta) V_x}{r^2 c}$
--

If $c \rightarrow \infty$, we get back Newton's equation, thus:

$$F_x = + \frac{k.m_1.m_2 \cos(\theta)}{r^2}$$

$$F_y = + \frac{k.m_1.m_2 \sin(\theta)}{r^2}$$



Consider a rectilinear pipe masse conductor without masse, of a finite length, parallel to the vector $O'x'$, passed by a constant masse current of I_2 and dx a given point elementary displacement along the $O'x'$. We will determine the force due to the masse dm_2 on dm_1 . Since the reference frame R' associated is moving at a velocity of v with respect to reference frame R , dF_x and dF_y are given by:

$$dF_x = + \frac{k.dm_1.dm_2 \cos(\theta)}{r^2} + \frac{\gamma\beta k.dm_1.dm_2 \sin(\theta)V_y}{r^2 c}$$

$$dF_y = + \frac{\gamma k.dm_1.dm_2 \sin(\theta)}{r^2} - \frac{\gamma\beta k.dm_1.dm_2 \sin(\theta)V_x}{r^2 c}$$

But $\beta = v/c$:

$$dF_x = + \frac{k.dm_1.dm_2 \cos(\theta)}{r^2} + \frac{\gamma dm_1.dm_2 v \sin(\theta)V_y}{r^2 c^2}$$

$$dF_y = + \frac{\gamma k.dm_1.dm_2 \sin(\theta)}{r^2} - \frac{\gamma dm_1.dm_2 v \sin(\theta)V_x}{r^2 c^2}$$

Since two masses attract each other, then they behave like two charges of opposite signs, just like opposite electric currents of opposite senses, we will invert the signe of the velocity v , in order to be in conformity with the electric current convention law.

Then dF_x and dF_y are given by:

$$dF_x = + \frac{k.dm_1.dm_2 \cos(\theta)}{r^2} - \frac{\gamma k dm_1.dm_2 v \sin(\theta)V_y}{r^2 c^2}$$

$$dF_y = + \frac{\gamma k.dm_1.dm_2 \sin(\theta)}{r^2} + \frac{\gamma k dm_1.dm_2 v \sin(\theta)V_x}{r^2 c^2}$$

We notice that a vector product appears between the velocity V and a field the we shall call B_g , supposing a field B_g exists such that:

$$dF_2 = \begin{pmatrix} dF_{x2} \\ dF_{y2} \\ 0 \end{pmatrix} = mV \times B_g = m \begin{pmatrix} V_x \\ V_y \\ 0 \end{pmatrix} \times \begin{pmatrix} dB_{gx} \\ dB_{gy} \\ dB_{gz} \end{pmatrix}$$

$$dF_{x2} = - \frac{\gamma dm_1 \mu_g I_2 dx \cos(\alpha) V_y}{4\pi r^2}$$

$$dF_{y2} = + \frac{\gamma dm_1 \mu_g I_2 dx \cos(\alpha) V_x}{4\pi r^2}$$

After resolution and knowing that V_x can not be expressed as a function of V_y since the associated vectors are orthogonal, we find:

$$B_{gx} = 0$$

$$B_{gy} = 0$$

$$dB_{gz} = - \frac{\mu_g I_2 dx \cos(\alpha)}{4\pi r^2}$$

Written in another form:

$$dF_x = \frac{dm_1 dm_2 \sin(\alpha)}{4\pi \epsilon_g r^2} + \gamma dm_1 V_y dB_{gz}$$

$$dF_y = \frac{\gamma dm_1 dm_2 \cos(\alpha)}{4\pi \epsilon_g r^2} - \gamma dm_1 V_x dB_{gz}$$

If $v \ll c$, $\gamma \approx 1$

The force exerted on m_1 due to the gravitomagnetic field B_g build by the masse current I_2 is given by:

$$F = m_1 g_2 + m_1 V \times B_g$$

We have now shown the existence of the gravitomagnetic field B_g by applying the Lorenz transformation to the gravitational field.

By using the Biot and Savart law to determine B_g and using the same convention, we get:

$$dB_{gx} = 0$$

$$dB_{gy} = 0$$

$$dB_{gz} = - \frac{\mu_g I_2 dx \cos(\alpha)}{4\pi r^2}$$

μ_g being the permeability of gravitomagnetic field.

In that way, by changing the reference frame, the gravitational field components orthogonal to the movement were slightly modified. The gravitomagnetic field is just a relativity effect due to movement and reference frame change.

We deduce that if B_g is the gravitomagnetic field and g_2 the gravitational field build by a masse m_2 moving at a velocity of v along the Ox axis, the force to which a masse particle m_1 moving at a velocity of V is exerted is given by:

$$F = m_1 g_2 + m_1 V \times B_g$$

with

The components of that force are :

$$F_x = q(V_y B_{gz} - V_z B_{gy})$$

$$F_y = q(V_z B_{gx} - V_x B_{gz})$$

$$F_z = q(V_x B_{gy} - V_y B_{gx})$$

Fundamental equations

By using Maxwell-Gauss equation for the gravitational field g , we get:
 $\text{div } g = -m/\epsilon_g$

By using flux conservation equation for gravitomagnetic field B_g , we get:
 $\text{rot } B_g = 0$

Let:

$$dF_x = + \frac{dm_1 dm_2 \sin(\alpha)}{4\pi\epsilon_g r^2} - \frac{\gamma dm_1 \mu_g I_2 dx \cos(\alpha) V_y}{4\pi r^2}$$

$$dF_y = + \frac{\gamma dm_1 dm_2 \cos(\alpha)}{4\pi\epsilon_g r^2} + \frac{\gamma dm_1 \mu_g I_2 dx \cos(\alpha) V_x}{4\pi r^2}$$

We will replace dx and r^2 in the equations before to have α as the only variable, ρ_2 being the masse linear density, we get:

$$x = a \cdot \tan(\alpha)$$

$$dm_2 = \rho_2 dx$$

$$dx = \frac{a \cdot d\alpha}{\cos^2(\alpha)}$$

$$r^2 = \frac{a^2}{\cos^2(\alpha)}$$

$$dF_x = + \frac{dm_1 \rho_2 \sin(\alpha) d\alpha}{4\pi\epsilon_g a} - \frac{\gamma dm_1 \mu_g I_2 \cos(\alpha) V_y d\alpha}{4\pi a}$$

$$dF_y = + \frac{\gamma dm_1 \rho_2 \cos(\alpha) d\alpha}{4\pi\epsilon_g a} + \frac{\gamma dm_1 \mu_g I_2 \cos(\alpha) V_x d\alpha}{4\pi a}$$

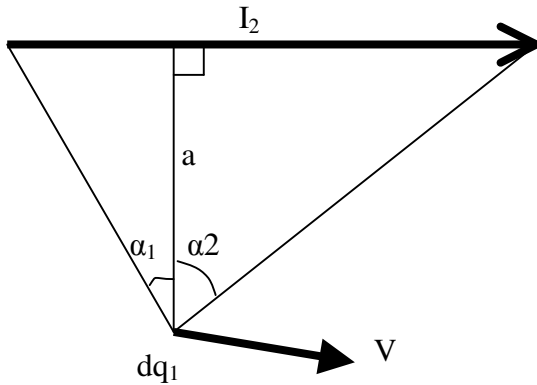
$4\pi\epsilon_g a$ $4\pi a$

By integration from α_1 to α_2 where α varies from $-\pi/2$ to $\pi/2$, we find;

$$dF_x = - \frac{dm_1 \rho_2}{4\pi\epsilon_g a} [\cos(\alpha_2) - \cos(\alpha_1)] - \frac{\gamma dm_1 \mu_g I_2 V_y}{4\pi a} [\sin(\alpha_2) - \sin(\alpha_1)]$$

$$dF_y = + \frac{\gamma dm_1 \rho_2}{4\pi\epsilon_g a} [\sin(\alpha_2) - \sin(\alpha_1)] + \frac{\gamma dm_1 \mu_g I_2 V_x}{4\pi a} [\sin(\alpha_2) - \sin(\alpha_1)]$$

Schematically we get:



1) Masse conductor of infinite length

Implies that, $\alpha_1 = -\pi/2$ and $\alpha_2 = \pi/2$, then F_x and F_y are given by:

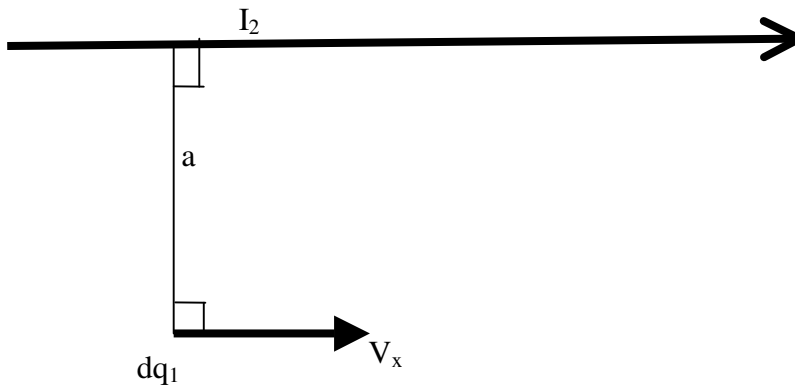
$$F_x = + 0 - \frac{\gamma \cdot \mu_g dm_1 I_2 \cdot V_y}{2\pi a}$$

$$F_y = + \frac{\gamma \cdot dm_1 \cdot \rho_2}{2\pi\epsilon_g a} + \frac{\gamma \cdot \mu_g dm_1 I_2 \cdot V_x}{2\pi a}$$

If the masse dm_1 is moving along the Ox axis, then $V_y = 0$, as shown in the following diagram:

$$F_x = 0$$

$$F_y = + \frac{\gamma \cdot dm_1 \cdot \rho_2}{2\pi\epsilon_g a} + \frac{\gamma \cdot \mu_g dm_1 I_2 \cdot V_x}{2\pi a}$$



If I_1 is the masse current build by a particle having a masse dm_1 and ρ_1 is the masse linear density, we get:

$$dm_1 \cdot Vx = dm_1 \frac{dx}{dt} = \frac{dm_1}{dt} dx = I_1 dx, \text{ where } I_1 = \frac{dm_1}{dt} = \text{masse current}$$

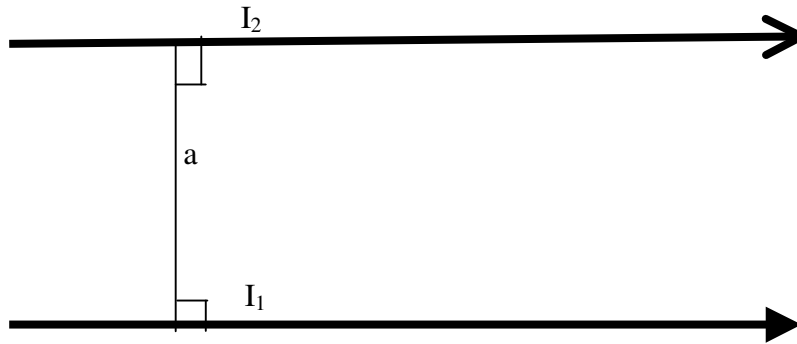
$$dm_1 = \rho_1 dx$$

We get:

$$F_y = + \frac{\gamma \cdot \rho_1 \cdot \rho_2 \cdot dx}{2\pi \epsilon_g a} + \frac{\gamma \cdot \mu_g I_1 I_2 \cdot dx}{2\pi a}$$

$$\frac{F_y}{dx} = + \frac{\gamma \cdot \rho_1 \cdot \rho_2}{2\pi \epsilon_g a} + \frac{\gamma \cdot \mu_g I_1 I_2}{2\pi a}$$

Thus, the force par unit length between two parallel masse pipes passed by two masse currents I_1 and I_2 in the same direction.



Attractive gravitomagnetic force

$$\frac{F_y}{dx} = + \frac{\gamma \cdot \rho_1 \cdot \rho_2}{2\pi \epsilon_g a} + \frac{\gamma \cdot \mu_g I_1 I_2}{2\pi a}$$

Observation; when two parallel masse pipes without masse are passed by two masse currents I_1 and I_2 in the same direction, the force of attraction tends to infinity when v_1 and v_2 tend to c .

Repulsive gravitomagnetic force

Observation; when two parallel masse pipes without masse are passed by two masse currents I_1 and I_2 in opposite directions, a repulsive force appears due to the gravitomagnetic field, as the $I_1 \cdot I_2$ product increases, the force between the two masse currents decreases, at a certain time the force goes down to zero.

Thus:

$$\frac{F_y}{dx} = 0 \Rightarrow + \frac{\gamma \cdot \rho_1 \cdot \rho_2}{2\pi \epsilon_g a} - \frac{\gamma \cdot \mu_g I_1 I_2}{2\pi a} = 0$$

$$\frac{\rho_1 \cdot \rho_2}{\epsilon_g \mu_g} = I_1 I_2$$

$$\rho_1 = \frac{dm_1}{dx_1}, \rho_2 = \frac{dm_2}{dx_2}, I_1 = \frac{dm_1}{dt} \text{ and } I_2 = \frac{dm_2}{dt}$$

$$\rightarrow \frac{1}{\epsilon_g \mu_g} = v_1 \cdot v_2$$

Since our hypothesis was $\epsilon_g \mu_g = 1/c^2$, it's necessary and sufficient that $v_1 = v_2 = c$, the speed of light, the remarkable speed.

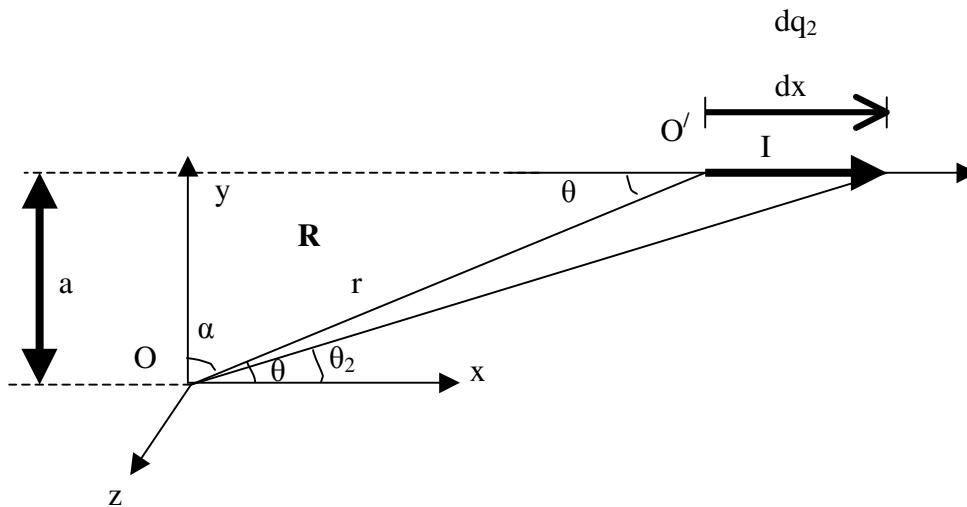
Remark 1: Two masses moving in the same direction along two parallel lines attract each other and two masses moving in the opposite directions along two parallel lines repulse each other due to the gravitomagnetic field.

Second conclusion: In accordance with the principle of reference frame to reference frame force transformation, by applying the Lorentz transformation to the gravitational field, we did show the existence of gravitomagnetic field.

Relationship between magnetic and gravitomagnetic field

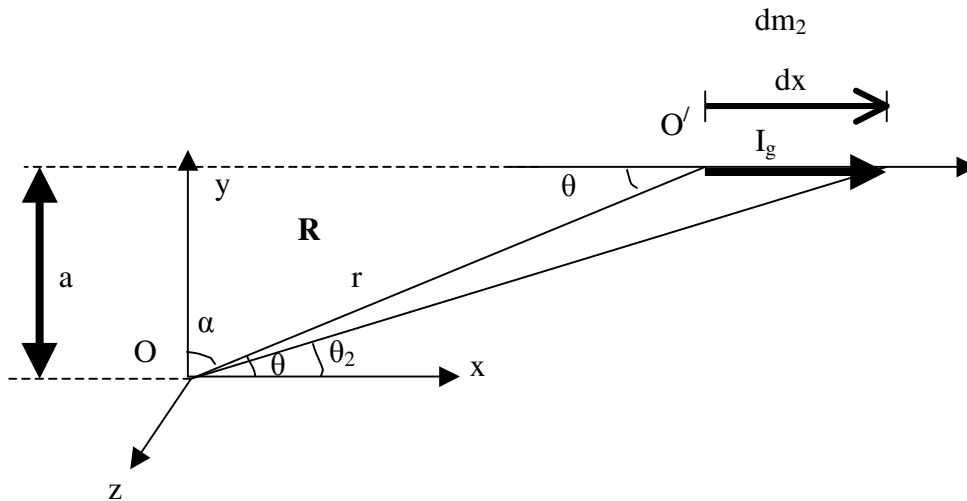
We have already shown that the magnetic field B is given by;

$$dB = - \frac{\mu_0 I dx \cos(\alpha)}{4\pi r^2}$$



We have already shown that the gravitomagnetic field B_g is given by;

$$dB_g = - \frac{\mu_g I_g dx \cos(\alpha)}{4\pi r^2}$$



We are going to express the masse current I_g as function of the electric current I , the electric current is build up of mobile electrons of masse m_e et charge q_e , in that case an electric current builds up a masse current due to the drift of electrons having masse m_e . The coordinates of the masse of an electron are the same as that of the charge. Let:

- q_e be the charge of the electron
- m_e be the masse of the electron
- n be the number of the electrons
- I be the electric current
- I_g be the masse current
- Q be the total electric charge
- t denotes the time

$$I = dQ/dt$$

$$I_g = dm/dt$$

The number of electrons n is given by:

$$n = Q/q_e$$

The total masse m of electrons on drift is given by:

$$m = n \cdot m_e$$

$$I_g = \frac{dm}{dt} = m_e \frac{dn}{dt} = \frac{m_e}{q_e} \frac{dQ}{dt} = \frac{m_e}{q_e} \cdot I$$

The charge of an electron is a negative, scalar quantity; in that case I_g and I are opposite directions.

But

$$dB_g = - \frac{\mu_g I_g dx \cos(\alpha)}{4\pi r^2}$$

By substituting I_g in the equation before, we get:

$$dB_g = - \frac{m_e \cdot \mu_g I dx \cos(\alpha)}{q_e 4\pi r^2}$$

The magnetic force B is given by:

$$dB = - \frac{\mu_0 I dx \cos(\alpha)}{4\pi r^2}$$

By dividing dB_z by dB_{gz} , we get:

$$dB = \frac{q_e \cdot \mu_0}{m_e \mu_g} \cdot dB_g$$

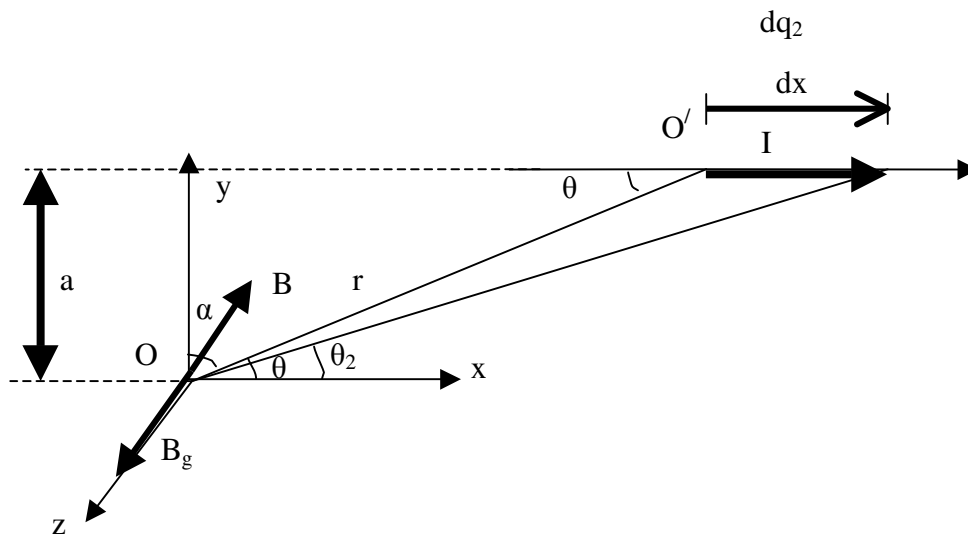
By integration, we find:

$$B = \frac{q_e \cdot \mu_0}{m_e \mu_g} \cdot B_g + (\text{constant} = 0)$$

$$B_g = \frac{m_e \mu_g}{q_e \mu_0} \cdot B$$

The constant of integration is equal to zero, because zero electric current implies zero masse current and vice versa.

The diagram below shows the magnetic field B and the gravitomagnetic field B_g in an electric circuit where the electrons are carriers of the charge and the masse, if protons were the charge and masse carriers then the gravitomagnetic field would be 1600 times stronger because the proton is roughly 1600 times heavier than the electron.



We notice that an electric current builds up a masse current due to the drifting of the masse of the electrons. This is very important because we will later see that the light is not only made up of electromagnetic waves but it is also made up gravitomagnetic waves. We will call this wave, **the electrogravitomagnetic wave.**

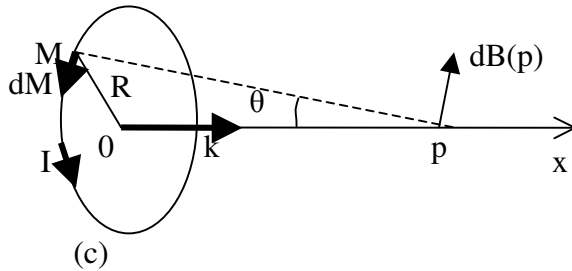
Remark 2: Since the Lorenz transformation to the gravitational field gives exactly the same results as the law of Biot and Savart applied to a constant rectilinear masse current, and if we consider a constant masse current given by any graph as a succession

of infinitely small constant masse currents, by integration we can calculate on a given point the gravimagnetics field by using the law of Biot and Savart as a mathematical tool.

Consequences:

2) Circular masse ring in rotation

Consider a circular ring C having a centre 0 and a radius R, passed by a masse current I (in rotation). The Ox axis is parallel to the unit vector k as shown in the diagram below. Let us determine the gravitomagnetic field using the law of Biot and Savart as a mathematical tool.



M is a point on the ring, in accordance with the law of Biot and Savart, the elementary gravitomagnetic field $dB(p)$ build on the point P on x axis by an elementary length of the ring, and in the same direction as the current, is given by:

$$dB(p) = \frac{\mu_g I dM \times u}{4\pi \|MP\|^2}$$

Where u is a unit vector of MP, $u = MP / \|MP\|$.

Two elementary symmetrical currents with respect to 0, build at point P, on the plain yOz , two gravitomagnetic fields of opposite signs but equal in magnitude, these fields null each other. We will thereby just have to determine the projection of $dB(p)$ on the Ox axis.

θ denotes the angle OpM . The angle between $dB(p)$ and k is then $\pi/2 - \theta$ and the projection of $dB(p)$ on the Ox axis is given by:

$$dB(p)_x = \frac{\mu_g I dM \sin(\theta)}{4\pi \|MP\|^2}$$

By integration on the whole length of the ring, we get:

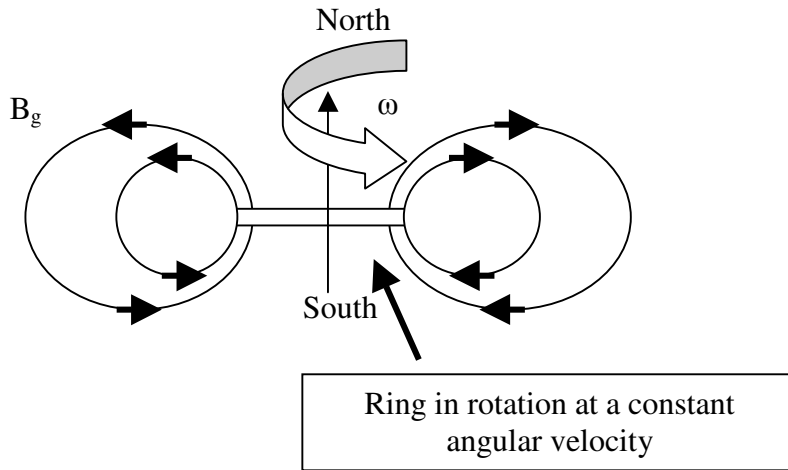
$$B(p)_x = \frac{\mu_g I 2\pi R \sin(\theta)}{4\pi \|MP\|^2}$$

$$\|MP\| = R/\sin(\theta)$$

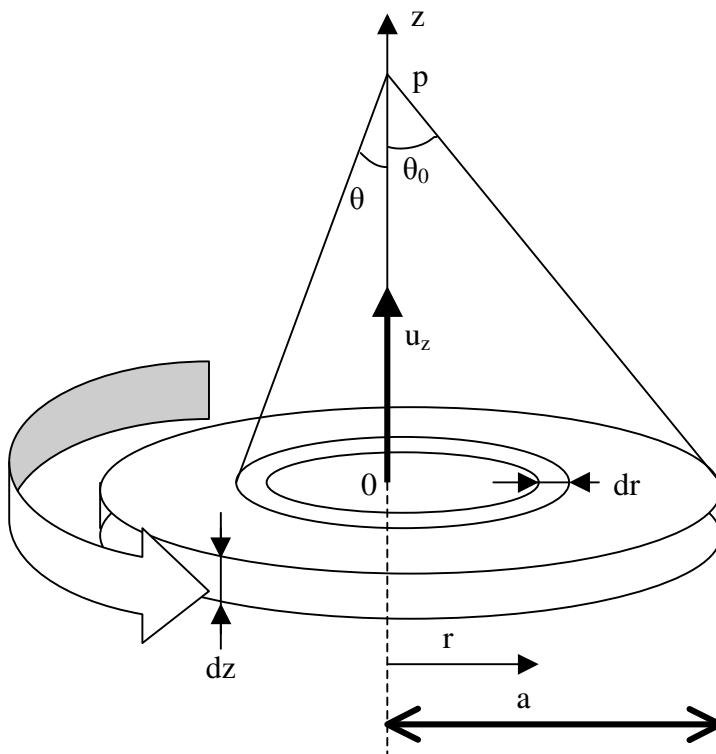
⇒ The gravitomagnetic field is given by the following expression:

$$B(p)_x = \frac{\mu_g I \sin^3(\theta)}{2R}$$

The following diagram shows the gravitomagnetic field around the ring;



3) Masse disc in rotation



Let σ be the volume density.

A disc in rotation around the $0z$ axis is equivalent to a succession of circular rings centred at 0 , having a radius of r , a width of dr , a depth of dz and passed by masse current of I . Each ring builds at the point p , along the $0z$ axis, an elementary gravitomagnetic field $dB(p,dz)$, $\omega = 2\pi f$, f being the frequency of rotation:

$$dB(p,dz)_z = \mu_g \frac{I \sin^3(\theta)}{2r}$$

$I = dm/dt$ is the quantity of masse that traverses (crosses) the surface $dr.dz$ per unit of time.
 $I = dm/dt = \sigma.drdz.speed = \sigma.drdz.r\omega$

$$dB(p,dz)_z = \frac{\mu_g \cdot \sigma.drdz.r\omega \sin^3(\theta)}{2r}$$

$$dB(p,dz)_z = \frac{\mu_g \cdot \sigma.\omega drdz. \sin^3(\theta)}{2}$$

By differentiating the equation $\tan(\theta) = r/z$, we find $dr = z d\theta / \cos^2(\theta)$, We thereby get the expression of the elementary gravitomagnetic field as a function of θ :

$$dB(p,dz)_z = \frac{\mu_g \cdot \sigma.\omega z dz. \sin^3(\theta) d\theta}{2 \cos^2(\theta)}$$

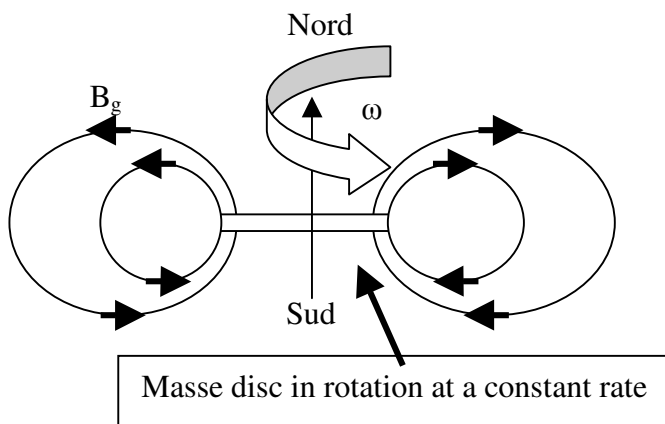
The total gravitomagnetic field build by a rotating disc becomes, taking the extreme angle θ_0 such that $\tan(\theta_0) = a/z$ and $\cos(\theta_0) = z/\sqrt{a^2 + z^2}$:

$$\frac{\sin^3(\theta)}{\cos^2(\theta)} = \frac{\sin^2(\theta) \sin(\theta)}{\cos^2(\theta)} = \frac{(1 - \cos^2(\theta)) \sin(\theta)}{\cos^2(\theta)} = \frac{\sin(\theta)}{\cos^2(\theta)} - \sin(\theta)$$

By integration from 0 to θ_0 ;

$$dB(p,dz)_z = \frac{\mu_g \cdot \sigma.\omega z dz. (\frac{1}{\cos(\theta_0)} + \cos(\theta_0) - 2)}{2}$$

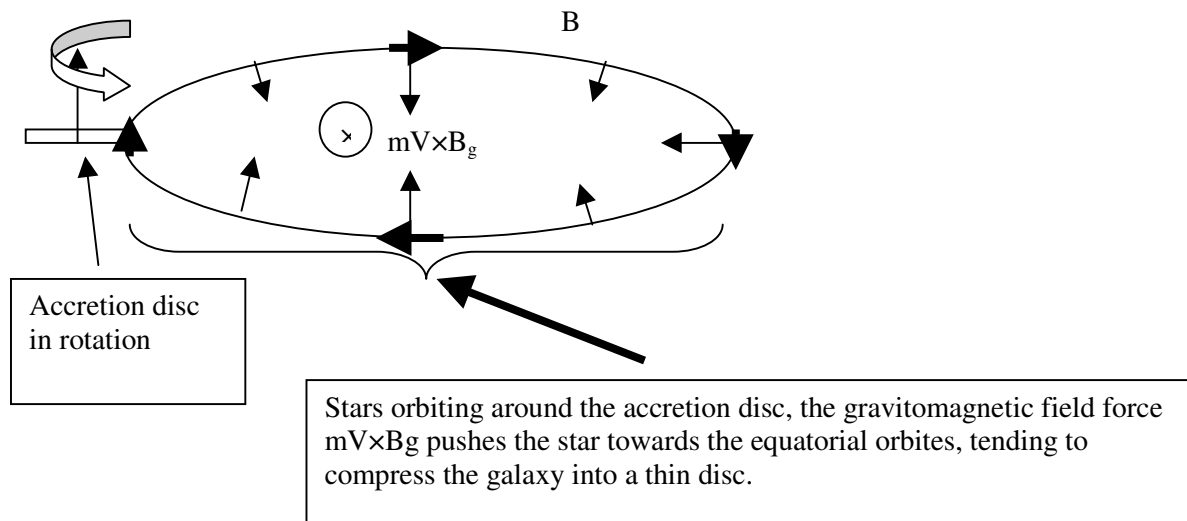
The following diagram shows the schematic form of the gravitomagnetic field of a disc in rotation at a constant angular velocity:



Remark 3: consequence of remark 1, two rings/discs in rotation at a constant angular velocity in the same axis and in the same angular direction, attract each other. Two rings/discs in rotation at a constant angular velocity in the same axis and in opposite angular directions, repulse each other. Masses in rotation can thereby be considered as having polarities, north and south just like electromagnets.

Gravitomagnétique effect on the shape of galaxies

When we observe the disc galaxies, they are incredibly flat. The rotation of the centre of galaxy builds a gravitomagnetic field (B_g) as shown in the diagram below:



The shape of the galaxies is not a surprise, the forced exerted on stars orbiting around the centre of the galaxy is given by:

$$F = (mg + m\mathbf{V} \times \mathbf{B}_g + mr\omega^2) \mathbf{u}_r$$

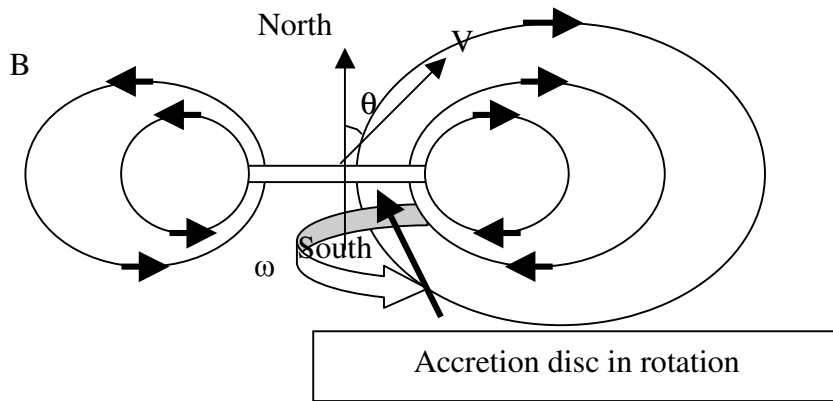
Where \mathbf{u}_r is the unit radial vector, g is the gravitational field build by the centre of the galaxy and the masses in the inner part of the radius r , m is the masse of the studied object, $mr\omega^2$ is the centrifugal force, r is the radial distance, $\omega = 2\pi f$, f the frequency of rotation.

As we can see, the equatorial orbits are the only stable orbits, due to the gravitomagnetic field, the nearby orbits are pushed down if the are above the equator and are pushed up if the are below the equator. The retrograde orbits are prohibited, with time, the orbits become prograde, and move towards the equator of the rotating centre of galaxy. With time, even a spherical rotating galaxy becomes ellipsoidal and finally becomes a disc galaxy. If the force of gravity becomes too big, a spiral galaxy starts to be formed.

The flat form of Saturn rings and the fact that all planets, except that of Pluto, are almost in the same plain is due to the gravitomagnetic field. In general, revolution and rotation axis are prograde in the solar system.

Bipolar jets of matter

If the matter, in the accretion disc, attain the escape velocity, it will be ejected at a certain direction with respect to axis of rotation, let θ be the angle between the axis of rotation and the initial escape velocity V , Let us consider the north pole of the accretion disc as shown in the diagram below.



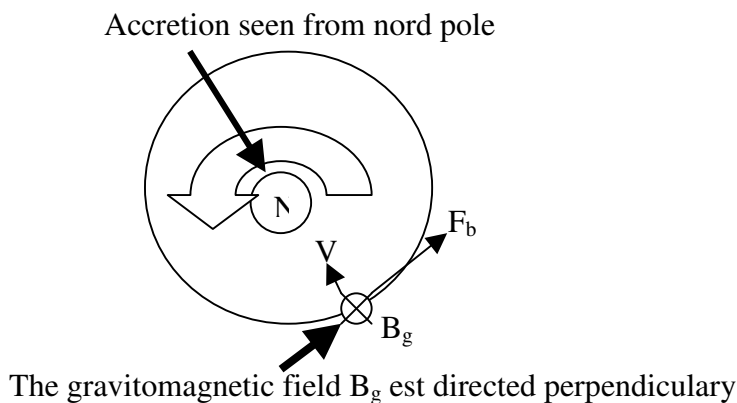
The smaller the angle the less the matter cuts the gravitomagnetic field lines, thereby the less the matter is deviated. The greater the angle, the greater the matter cuts the gravitomagnetics field lines, thereby the greater the matter is deviated. In that way, the matter follows a helicoidally trajectory with increasing radius as the radial distance increases. There are three possibilities;

- The matter falls back in the accretion disc.
- The matter goes in orbit or enters in collision with other bodies.
- The matter escapes in space.

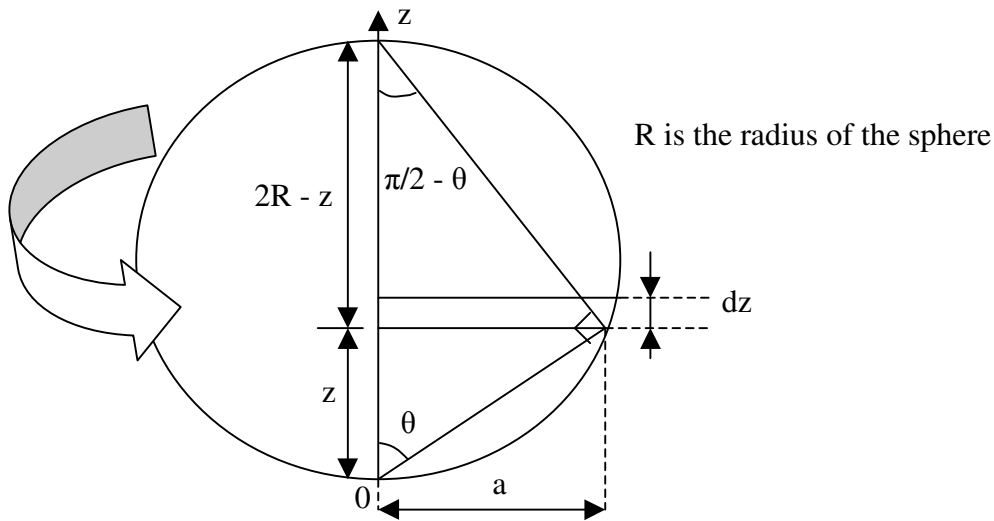
The latter case is the most interesting since the matter escapes in space following a helicoidally trajectory, the set of trajectories has almost a form of a cone in rotation.

Spiral aspiration of matter

When the matter approaches the accretion disc at a velocity of V , a force F_b perpendicular to the velocity is exerted to the matter by the gravitomagnetic field B_b , that way the matter is sucked in a spiral form by the accretion disc as shown by the following diagram.



4) Full sphere in rotation



A sphere in rotation is equivalent to a succession of discs having Oz axis as the centre, of a radius a, of thickness dz and of density σ . Each disc builds on the point O along the Oz axis the elementary gravitomagnetic field, $\omega = 2\pi f$, f being the frequency of rotation. **Take care**; the permeability μ_g of gravitomagnetic field in matter is not perhaps equal to that one in vacuum, the same case regarding the gravitational interaction speed.

$$dB(0)_z = \frac{\mu_g \sigma \omega z dz}{2} \left(\frac{1}{\cos(\theta)} + \cos(\theta) - 2 \right)$$

We will determine z dz:

$$\frac{a}{z} = \tan(\theta) \quad \text{et} \quad \frac{a}{(2R - z)} = \tan(\pi/2 - \theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\rightarrow z = 2R \cos^2(\theta)$$

$$dz = -4R \sin(\theta) \cos(\theta) d\theta$$

$$z dz = -8R^2 \sin(\theta) \cos^3(\theta) d\theta$$

By replacing z dz in function $dB(0)_z$, we get:

$$dB(0)_z = -4\mu_g \sigma \omega R^2 [\sin(\theta) \cos^2(\theta) + \sin(\theta) \cos^4(\theta) - 2\sin(\theta) \cos^3(\theta)] d\theta$$

By integration from $\pi/2$ to 0:

$$B(0)_z = -4\mu_g \sigma \omega R^2 \left[-\frac{\cos^3(\theta)}{3} - \frac{\cos^5(\theta)}{5} + \frac{\cos^4(\theta)}{2} \right]_{\pi/2}^0$$

$$B(0)_z = -4\mu_g \sigma \omega R^2 \left(-\frac{1}{3} - \frac{1}{5} + \frac{1}{2} \right)$$

$$B(0)_z = -4\mu_g \sigma \omega R^2 \left(-\frac{1}{30} \right)$$

$$B(0)_z = + \frac{2\mu_g \sigma \cdot \omega R^2}{15} = \frac{2\mu_g \sigma 2\pi f R^2}{15} = \frac{\mu_g}{5TR} \cdot \frac{\sigma 4\pi R^3}{3}$$

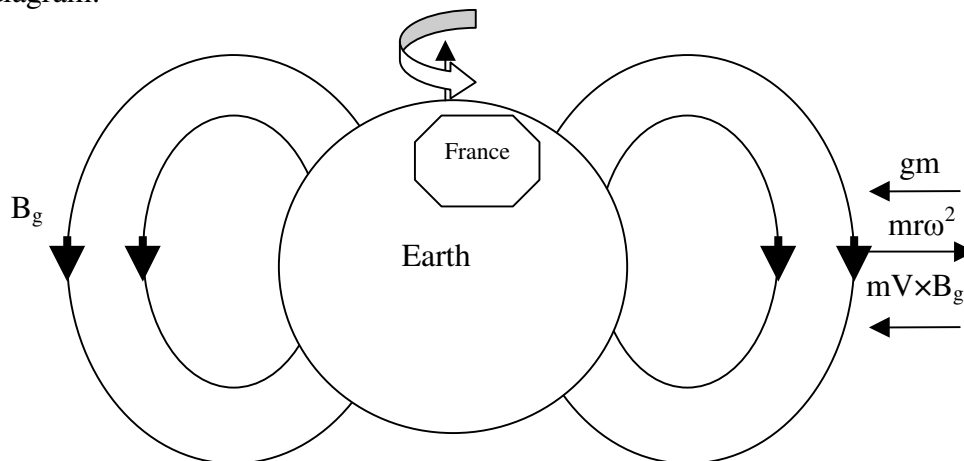
But $\frac{\sigma 4\pi R^3}{3}$ = volume density \times sphere volume = M masse of the sphere.

$$B(0)_z = \frac{\mu_g M}{5TR}$$

T being the period of rotation.

Gravitomagnetic field in Earths poles

The rotation of the Earth builds a gravitomagnetic field as shown in the following diagram:



The gravitomagnetic field is given by:

$$B(0)_z = \frac{\mu_g M}{5TR}$$

This field $B(0)_z$ is vectored towards the geographic north.

G is the constant of gravity.

T is the period of rotation = 60 \times 60 \times 24 seconds.

$$G = 1/4\pi\epsilon_g$$

$$\mu_g \epsilon_g c^2 = 1$$

c is the speed of light (hypothesis: The speed of gravitational interaction is equal to the speed of light)

ϵ_g is the permittivity of gravitational field, μ_g is the permeability of gravitomagnetic field in vacuum.

$$\mu_g = 9.4 \times 10^{-27} \text{ m/kg}$$

$$M = 5.973 \times 10^{24} \text{ kg}$$

$$R = 1.275 \times 10^7 \text{ m}$$

$$T = 86400 \text{ s}$$

$$B(0)_z = 1 \times 10^{-14} \text{ s}^{-1}$$

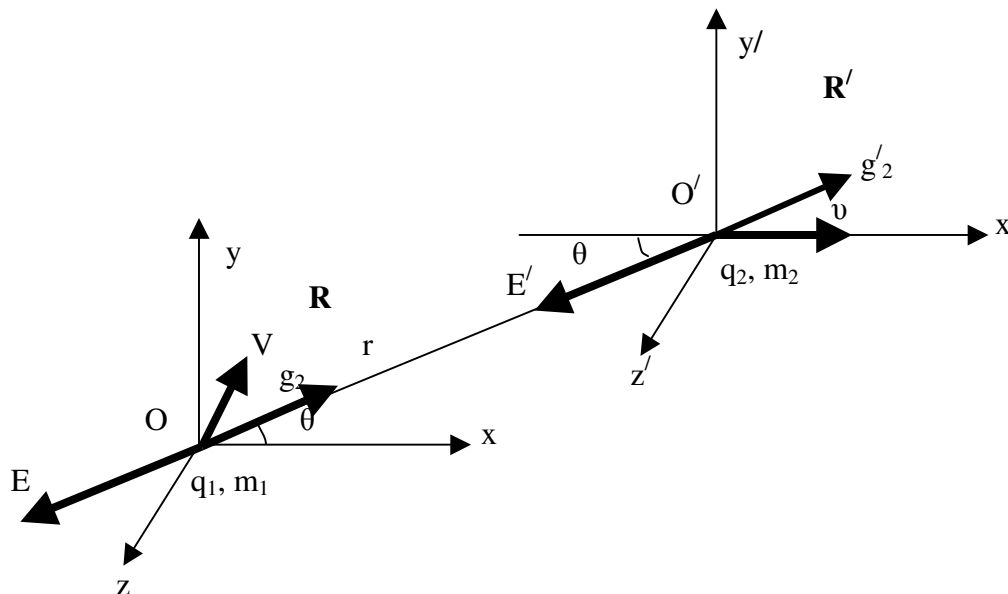
(units in 1/seconds, just like the frequency)

Masse and charged particle

We deduce that if B is the magnetic field, E_2 the electric field, B_g the gravitaomagnetic field and g_2 the gravitational field all build by a particle of charge q_2 and of masse m_2 moving at a velocity of v along the Ox axis, the force to which a particle of charge q_1 and of masse m_1 moving at a velocity V is exerted is given by:

$$F = q_1 E_2 + m_1 g_2 + V \times (q_1 B + m_1 B_g)$$

Fundamental electrogravitomagnetic equation



Third Conclusion

In accordance with the principle of reference frame to reference frame force transformation, by applying the Lorenz transformation to the gravitational field, we have shown the existence of gravitomagnetic field.

Propagation of gravitomagnetic waves

To determine the gravitomagnetic wave, we will be obliged to distinguish three areas of radiation:

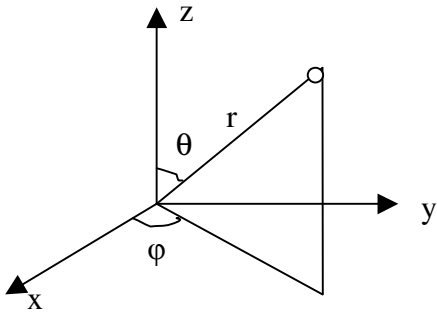
- the close radiation area, quasi stationary field ($r \ll \lambda$) ;
- the intermediate radiation area ($r \approx \lambda$) ;
- the far radiation area ($r \gg \lambda$) ;

We will confine ourselves to the last area of radiation.

Electromagnetism and gravitomagnetism

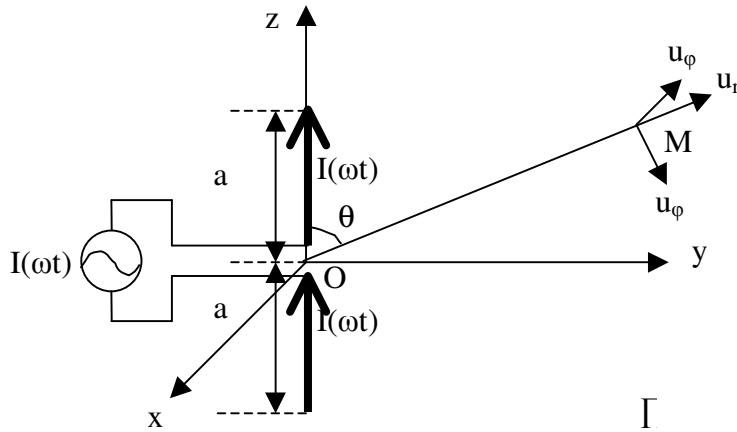
Electromagnetic wave, far radiation area:

We will use the spherical coordinates as shown in the diagram below:



Let us consider a Hertz antenna, we assume the electric current is uniform (amplitude and phase independent). Let the antenna have a length of $2a$. The retarded vector potential $A(M,t)$ build by an elementary electric current at a point M, that is very far away, as shown in the diagram below, is given by :

With $i = I_0 \cos(\omega t)$



Symetrical Hertz antenna

$$A(M,t) = \mu_0 \int_0^{2a} \frac{dl}{4\pi r} i(t-r/c)$$

$$i(t-r/c) = I_0 e^{j\omega(t-r/c)} = e^{j\omega t} e^{-ikr} u_z$$

With $k = \frac{\omega}{c} u_r$ et $j = \sqrt{-1}$

We get:

$$A(M, t) = \frac{\mu_0 2a I_0}{4\pi r} e^{j\omega t} e^{jkr} u_z$$

Let the real part of the vector be:

$$\underline{A}(\underline{M}, t) = \frac{\underline{\mu}_0 \underline{a} \underline{I}_0}{4\pi r} \cos(\omega t - kr) \underline{u}_z$$

If \underline{B} is magnetic field, using the approximation of a plain wave, we get:

$$\begin{aligned} \underline{B} &= \text{rot}(\underline{A}) = -j\mathbf{k} \times \underline{A} \\ &= -j \|\mathbf{k}\| \underline{u}_r \times \|\underline{A}\| \underline{u}_z = j \|\mathbf{k}\| \|\underline{A}\| \sin(\theta) \underline{u}_\varphi \end{aligned}$$

Let:

$$\underline{B} = \frac{j k \mu_0 \underline{a} \underline{I}_0 \sin(\theta)}{4\pi r} e^{j\omega t} e^{jkr} \underline{u}_\varphi$$

But $\underline{B} = \frac{\mathbf{k} \times \underline{E}}{\omega}$, we deduce:

$$\underline{E} = \frac{j k \mu_0 \omega \underline{a} \underline{I}_0 \sin(\theta)}{4\pi r} e^{j\omega t} e^{jkr} \underline{u}_\theta$$

The real parts of the fields are:

$$\underline{B} = -\frac{\mu_0 \underline{a} \underline{I}_0 \sin(\theta)}{2\pi r} \sin(\omega t - kr) \underline{u}_\varphi$$

$$\underline{E} = -\frac{\mu_0 \omega \underline{a} \underline{I}_0 \sin(\theta)}{2\pi r} \sin(\omega t - kr) \underline{u}_\theta$$

The average Poynting vector is written as:

$$\underline{S} = \frac{1}{2\mu_0} \underline{R} [\underline{E} \times \underline{B}^*] = \frac{\mu_0 \underline{a}^2 (\underline{I}_0)^2 \omega^2 \sin^2(\theta)}{8\pi^2 r^2 c} \underline{u}_r$$

\underline{B}^* = the conjugate of \underline{B} $(a + jb)^* = (a - jb)$ with $(a, b) \in \mathfrak{R}^2$ and $\mathbf{j} = \sqrt{-1}$.

If \underline{B}_g is the gravitomagnetic field and \underline{B} the magnetic field, we have already shown that:

$$\underline{B}_g = \frac{\underline{m}_e \underline{\mu}_g}{q_e \mu_0} \cdot \underline{B} \cdot \underline{u}_b, \underline{u}_b \text{ being a unit vector.}$$

In accordance with Einstein general relativity $m_e = m_{e0} / \sqrt{1 - v^2/c^2}$, m_{e0} being the masse of the electron at rest. $v = V_{\max} \cdot \cos(\omega t)$, where V_{\max} is the maximum speed of the electrons and v is the velocity at time t .

Let $V_{\max} \ll c$, $\rightarrow m_e \approx m_{e0}$

After substituting B in the equation relating B and B_g , B_g is given by:

$$B_g = - \mu_0 \frac{k a \underline{m}_e \underline{\mu}_g I_0 \sin(\theta) \sin(\omega t - kr)}{2\pi r \underline{q}_e \mu_0} \underline{u}_\phi$$

The set of Maxwell and Faraday equations constitute a linear application. In mathematics, a **linear application** (also know as linear operator) is an application between two vector spaces which respect addition of vectors and the scalar multiplication defined in those spaces, or, in other terms, that « preserve linear combination ».

Definition

Let

$f : E \rightarrow F$ from E towards F

An application where E and F are two \mathbb{K} vector spaces.

f is a **linear application** if:

- $\forall x \in E, \forall y \in E, f(x + y) = f(x) + f(y)$
- $\forall \lambda \in \mathbb{K}, \forall x \in E, f(\lambda \cdot x) = \lambda \cdot f(x)$

An application that posses the first property is said to be additive, and, for the second, **homogenous**.

Let

$$\text{Scalar (S)} = \frac{\underline{m}_e \underline{\mu}_g}{\underline{q}_e \mu_0}$$

S is a multiplying scalar corresponding to λ in the linear application, not to mistake with the wave length.

*Since the set of Maxwell and Faraday equations constitute a linear application that is additive and **homogenous**. Since the gravitomagnetic field is proportional to the magnetic field and since this application (set of Maxwell and Faraday equations) is invariant with respect to proportionality (multiplying scalar,).*

Knowing that in electromagnetism that the magnetic field is derived from:

$$B = \frac{k \times E}{\omega}, \text{ Where } E \text{ is the electric field build by the charge}$$

We can affirm that there exists a field g from which the the gravitomagnetic is derived, such that:

$$B_g = \frac{k \times g}{\omega} \quad , \quad \text{Where } g \text{ is a field build by the masse:}$$

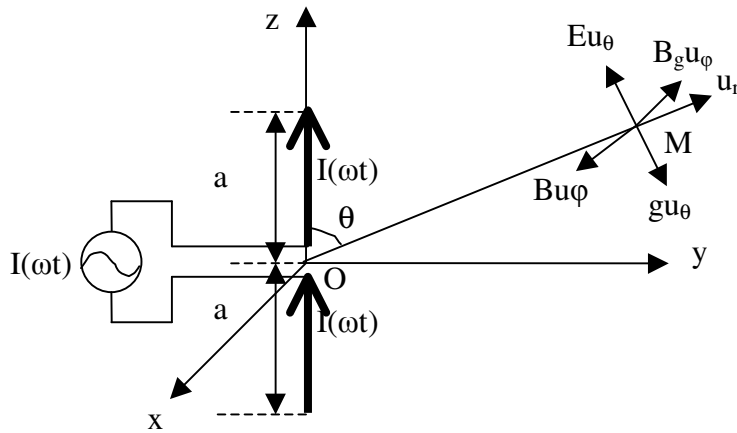
$$g = - \frac{\mu_0 \omega a \underline{m_e} \underline{\mu_g} I_0 \sin(\theta) \sin(\omega t - kr)}{2\pi r q_e \mu_0} u_\theta$$

Le field g is the gravitational field, thereby:

$$B_g = - \frac{\mu_0 k a \underline{m_e} \underline{\mu_g} I_0 \sin(\theta) \sin(\omega t - kr)}{2\pi r q_e \mu_0} u_\phi$$

$$g = - \frac{\mu_0 \omega a \underline{m_e} \underline{\mu_g} I_0 \sin(\theta) \sin(\omega t - kr)}{2\pi r q_e \mu_0} u_\theta$$

The charge q_e is a negative quantity; we have to take care about the orientation of these fields. The following diagram shows the electromagnetic and the gravitomagnetic fields.



Symetrical Hertz antenna

We can compare the magnitude of the electromagnetic and the magnitude of the gravitomagnetic waves, by dividing B_g by B and g by E , from the following equations;

Electromagnetic wave is given by;

$$B = - \frac{\mu_0 k a I_0 \sin(\theta) \sin(\omega t - kr)}{2\pi r} u_\phi$$

$$E = - \frac{\mu_0 \omega a I_0 \sin(\theta) \sin(\omega t - kr)}{2\pi r} u_\theta$$

The gravitomagnetic wave is given by;

$$\mathbf{B}_g = - \mu_0 \frac{ka \mathbf{m}_e \underline{\mu}_g}{2\pi r \mathbf{q}_e \mu_0} I_0 \sin(\theta) \sin(\omega t - kr) \mathbf{u}_\varphi$$

$$\mathbf{g} = - \mu_0 \frac{\omega a \mathbf{m}_e \underline{\mu}_g}{2\pi r \mathbf{q}_e \mu_0} I_0 \sin(\theta) \sin(\omega t - kr) \mathbf{u}_\theta$$

After division, we find:

$$\mathbf{B}_g = \begin{bmatrix} \mathbf{m}_e & \cdot \underline{\mu}_g \\ \mathbf{q}_e & \mu_0 \end{bmatrix} \cdot \mathbf{B}$$

$$\mathbf{g} = \begin{bmatrix} \mathbf{m}_e & \cdot \underline{\mu}_g \\ \mathbf{q}_e & \mu_0 \end{bmatrix} \cdot \mathbf{E}$$

*Remainder: Since the set of Maxwell and Faraday equations constitute a linear application that is **additive and homogenous**. Since the gravitomagnetic wave is proportional to the magnetic wave and since this application (set of Maxwell and Faraday equations) is invariant with respect to proportionality (multiplying scalar,). We can affirm that the gravitomagnetic waves are also governed by the set of Maxwell and Faraday equations in vacuum, such that:*

$$\text{div } \mathbf{g} = 0$$

$$\text{div } \mathbf{B}_g = 0$$

$$\text{rot } \mathbf{g} = - \frac{\partial \mathbf{B}_g}{\partial t}$$

$$\text{rot } \mathbf{B}_g = \mu_g \epsilon_g \frac{\partial \mathbf{g}}{\partial t}$$

With

$$\Delta \mathbf{g} - \epsilon_g \mu_g \frac{\partial^2 \mathbf{g}}{\partial t^2} = 0$$

$$\Delta \mathbf{B}_g - \epsilon_g \mu_g \frac{\partial^2 \mathbf{B}_g}{\partial t^2} = 0$$

The gravitomagnetic waves are propagated at a speed of c in vacuum such that;

$$c^2 \epsilon_g \mu_g = 1$$

The gravitomagnetic waves transport energy. The energy local density U is given by:

$$U = \epsilon_g \frac{\|\mathbf{g}\|^2}{2} + \frac{\|\mathbf{B}_g\|^2}{2\mu_g} \quad , \quad (\text{kg/m}^1 \text{s}^2)$$

The energy current is given by the vector of Poynting Π ;

$$\Pi = \frac{\mathbf{g} \times \mathbf{B}_g}{\mu_g} \quad (\text{kg/s}^4)$$

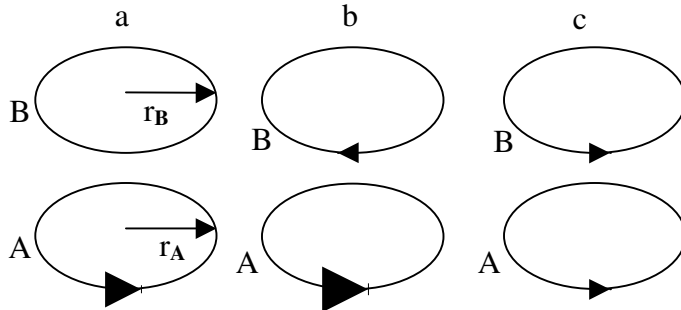
Gravitomagnetic induction

The gravitomagnetic induction leads to the production of a potential difference between ends of a masse conductor which is exposed to a variable gravitomagnetic field. We will call this potential difference, the gravitomotive force.

- The law of Maxwell-Faraday gives the relationship between the induced g.m.f (gravitomotive force) e_g and the gravitomagnetic field B_g flux Φ that traverses a masse circuit, it is given by;

$$e_g = - \frac{d\Phi}{dt} \quad \text{où} \quad \Phi = \iint_S (\mathbf{B}_g \cdot d\mathbf{S})$$

The following diagram shows the principle of induction



The phenomena of induction between two masse currents (c, b and c): when the ring A is passed by a stationary current, no current is induced in the ring B (a). If, in the contrary, the current in the ring A increases, it induces a current in the opposite sense in the ring B(b), and if it decreases (c) it induces a current in the same sense B(c). This is in accordance with the model published by Maxwell in 1891, in his third edition of Treatise on Electricity and Magnetism.

Let:

- m_A is the masse of the ring A
- m_B is the masse of the ring B
- v_A is the rotating speed of the ring A
- v_B is the rotating speed of the ring B
- I_A is the moment of inertia of the ring A
- I_B is the moment of inertia of the ring B
- $\omega = 2\pi \times \text{frequency of rotation}$
- P le power input.

In accordance with the principle of conservation of energy;

$$P = m_A \cdot v_A \frac{d v_A}{dt} + m_A \cdot v_A \frac{d v_A}{dt} + \text{waste (radiation)}$$

Or

$$P = I_A \cdot \omega_A \frac{d \omega_A}{dt} + I_A \cdot \omega_A \frac{d \omega_A}{dt} + \text{waste (radiation)}$$

Remark 4: An accelerating masse induces masse currents in space by exciting the nearby masses. In that case we have to take into account the masses near the accelerating masse in order to determine its acceleration. This is because the nearby masses tend to break the accelerating masse by absorbing its kinetic energy by induction effect. When considering any element, we have to take into account the contents of its environment.

Remark 5: The electromagnetic waves generation implies the generation of gravitomagnetic waves because the charges (electrons or protons) that generate these waves have a masse but on the contrary the gavitomagnetic waves can be generated by neutral vibrating masses without the generation of electromagnetic waves.

Electrogravitomagnetic wave energy

Accord the remark 5, the light is not only composed of electromagnetic waves, it is also composed of gravitomagnetic waves; we shall call this wave, electrogravitomagnetic wave.

The electrogravitomagnetic wave transports energy. The energy local density U of the electrogravitomagnetic wave is given by:

$$U = \frac{\epsilon_0 \|\mathbf{E}\|^2}{2} + \frac{\|\mathbf{B}\|^2}{2\mu_0} + \frac{\epsilon_g \|\mathbf{g}\|^2}{2} + \frac{\|\mathbf{B}_g\|^2}{2\mu_g} \quad , (\text{ kg/m}^1\text{s}^2)$$

With;

$$\mathbf{B}_g = \begin{bmatrix} \underline{\mathbf{m}}_e & \cdot \underline{\boldsymbol{\mu}}_g \\ \mathbf{q}_e & \boldsymbol{\mu}_0 \end{bmatrix} \cdot \mathbf{B}$$

$$\mathbf{g} = \begin{bmatrix} \underline{\mathbf{m}}_e & \cdot \underline{\boldsymbol{\mu}}_g \\ \mathbf{q}_e & \boldsymbol{\mu}_0 \end{bmatrix} \cdot \mathbf{E}$$

After substitution, we get:

Electrogravitomagnetic wave equation of energy

$$U = \frac{\left[\epsilon_0 + \epsilon_g \frac{m_e u_g}{q_e u_0} \right]^2}{2} \|E\|^2 + \frac{\left[1 + \frac{1}{\mu_0 \mu_g} \frac{m_e u_g}{q_e u_0} \right]^2}{2} \|B\|^2$$

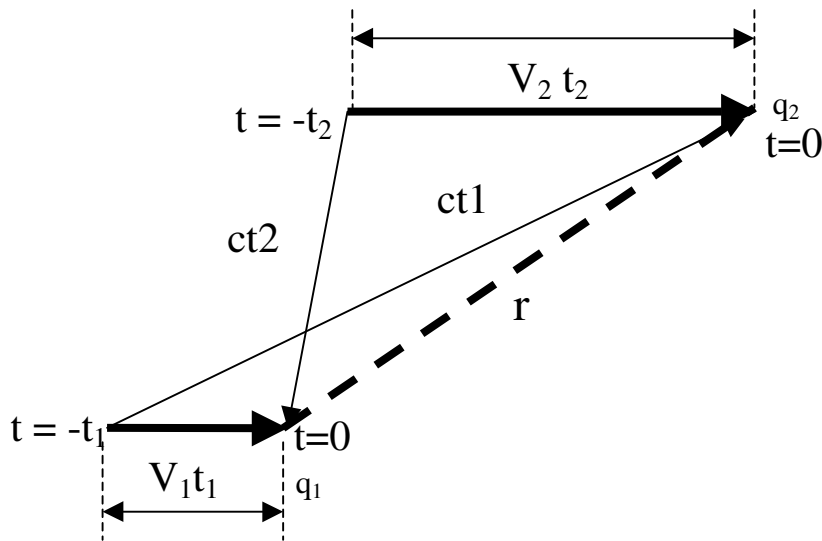
Physical constants

Name	Symbol	Value	Origin
Constant of gravity	G	$\approx 6.674 \times 10^{-11}$ $m^3/kg.s^2$	measured
Speed of gravitational interaction in vacuum	c	$\approx 2.99 \times 10^8$ m/s	hypothesis
Permittivity of gravitational field	ϵ_g	$\approx 1.18 \times 10^9$ $kg.s^2/m^3$	$\epsilon_g = 1/4\pi G$
Permeability of gravitomagnetic field	μ_g	$\approx 9.4 \times 10^{-27}$ m/kg	$c^2 \epsilon_g \mu_g = 1$
Characteristic impedance of gravitomagnetic waves in vacuum	Z_g	$\approx 2.82 \times 10^{-18}$ $m^2/kg.s$	$\mu_g c$
Speed of interaction of electric field	c	$\approx 2.99 \times 10^8$ m/s	measured
Permittivity of electric field	ϵ_0	$4\pi \times 10^{-7} Kg/A^2.s^2$	definition
Permeability of magnetic field	μ_0	8.85×10^{-12} $A^2.s^2/kg.m$	$c^2 \epsilon_0 \mu_0 = 1$
Characteristic impedance of electromagnetic waves in vacuum	Z_0	376.7 Ω	$\mu_0 c$
Elementary charge	q_e	1.6×10^{-19} A.s	measured
Masse of the electron	m_e	$9.1.9 \times 10^{-31}$ kg	measured
Masse of the proton	m_p	1.672×10^{-27} kg	measured
Masse of the neutron	m_n	1.674×10^{-27} kg	measured

Physical quantities

Nom	symbol	units
Gravitational field	g	m/s^2
Gravitomagnetic field	B_g	s^{-1}
Electric field	E	
Magnetic field	B	

Time and space delay



Consider two particles q_1 and q_2 at time $t = 0$ moving in two parallel lines respectively with a velocity of v_1 and v_2 , separated by a distance r . Due to time and space delay, the force exerted on q_1 by q_2 is the force that was built by q_2 at time $t = -t_2$ when this particle occupied its ancient position and the force exerted on q_2 by q_1 is the force that was built by q_1 at time $t = -t_1$ when this particle occupied its ancient position. In the classical mathematical model, we consider r , at time $t = 0$, as the distance that separates the two particles but this does not correspond to a physical reality. The Lorenz transformation corrects the time and space delay by introducing the magnetic and gravitomagnetic field.

We never watch the TV in live since the present time has not yet arrived.

31st October 2005 Toulouse **FRANCE**
Joseph Wathuta Nduriri.