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Ancient, Traditional, and "Common Sense" Notions of Space and Time

Todd Matthews Kelso
c/o Margaret R. Kelso kelsomeg@sbcglobal.net
http://home.att.net/~zei/TMKelso/index.htm

Atoms Moving Through the Void

At least since the time of Thales we have seemed to inhabit a world that consists of moving objects and to be moving objects ourselves. In order to describe motion at all, one needs two metric spaces. That way, there is a set of times that are at various temporal distances from each other and there is a set of places that are at various spatial distances from each other. A physical object can then be said to be moving if it is in different places at different times, or if its parts are. We could say that much even without the metrics. With the metrics, however, we can divide the spatial distance by the temporal distance and assign a speed to the motion. We can also define the other concepts that we need, such as continuous paths, so that we may distinguish motion from popping out of existence at one place and reappearing at some other place. In addition, defining continuous paths can explain why the universe consists of things that, by and large, retain their identities while they move from place to place and are, so far as we can tell, in intermediate places at intermediate times.

There are a large number of different metric spaces, more than merely infinitely many of them. Some of them have no such property as dimension. Some of them are n-dimensional for various numbers n. The propositions that are consonant with our perceptions are that time is one-dimensional and space is three-dimensional, at least during our history and as far out as we can see. Of the one-dimensional and three-dimensional metric spaces, some are infinite in length or volume and some are finite. All the infinite ones have finite subspaces of all sizes.

For the space and time of the actual universe, assuming that such concepts are parts of the correct description of the actual universe, (assuming, indeed, that there is an actual universe and that it has a correct description) we can access only a finite region of either. There are a few thousand years of recorded history and a few billion years before that that can plausibly be inferred by indirect reasoning processes using data such as the amounts of radioactive materials and their decay products found in rocks. Collectively, we experience the major portion of the surface of the earth directly but not much else until we went to the moon. We experience a few billion light-years exterior to our living space indirectly, more only so indirectly that very few people understand the indirect evidence of more to be that.

Euclidean Metrics

A theory of physics, assuming that it is to contain a subset that is a theory of kinematics, is constrained on page one to state axioms that specify space and time to be certain metric spaces. These are preferably axioms 1 and 2. The concepts of kinematics are so basic that any other axioms about the nature of physical objects are likely to establish relationships either of the concepts of space and time to each other or between the concepts of space and time and something else. Axiom 3, for example, might posit the existence of objects that have positions in space that vary from instant to instant. The decision about whether space and time are finite or infinite could conceivably be held in abeyance, but specificity requires that the decision be made at the time that the axioms of kinematics are stated.

These postulates, whichever postulates they are, are not going to be testable. Nobody can prove that the world was not created six minutes ago containing what appears to be internal evidence that it is older. Nobody can prove that, given any period of duration one hour, there was another period of duration one hour that preceded it. Nobody can prove that there exist points or instants with irrational numbers as their co-ordinates, or that there aren't. Nobody can prove that the geometry of the real world is Euclidean rather than either hyperbolic or elliptic, provided that its curvature, if not zero, is small enough to be undetectable by human beings.

Still, the simplest possible proposition consonant with what is known is that space and time have Euclidean metrics and consist of the entirety of, respectively, a one-dimensional Euclidean time and a three-dimensional Euclidean space.

The decision to prefer or even to consider a non-Euclidean metric would be very strange. No such metric is simpler in structure than the Euclidean metric and there is a total lack of any empirical data or philosophical reasonings whatever that even suggest that a Euclidean metric is in any way unsuitable.

The Continuous Structure of Time

The physical universe, the real world, exists in an open-ended linear continuum of states that replace one another in a certain order and at a certain rate. The notion of time abstracts from the real world the continuous structure, the metric properties and the ordering relations of this continuous change while ignoring entirely the structure of that which changes.

One thing that never changes is the structure of time. That the structure of time never changes is not an arbitrary presumption. If anything at all is to change, there must be some standard that does not change so that the rates of change of those things that do change can be measured by comparison to that standard. To suppose that the standard with reference to which rates of change are measured is itself one of the things that changes is incoherent because it makes complete nonsense out of the concept 'rate of change'.

Time as a Set

Time is not a physical object and instants are not physical objects. They are abstract entities, sets, to be exact. Time is the set of all instants. The instants are each examples of the special kind of set known as individuals. These are sets each of which is its own only element. The instants and sets of two or more instants, up to and including the set of all instants, play essential roles in the correct description of physical objects and of how physical objects are related to each other. These roles are as essential as the roles of numbers and sets of numbers, which are also abstract entities and sets.

Time is a set. The elements of time are instants. 'Instants', in this context, is just the generic name for the elements of time. Thus time is the set of all instants.

The instants are some distance each from each, each such distance being r seconds for some non-negative real number r.

Given any instants I and J, the distance t(I,J) of the instant J from the instant I is 0 seconds if and only if I and J are the same instant. The distance t(I,J) is r seconds for some positive real number r if I and J are distinct instants. The distance 0 seconds is the zero distance in time. Any other distance in time is a positive distance.

It is to be understood that the product r(s seconds) of a non-negative real number r and a distance s seconds is the distance rs seconds, the ratio (r seconds)/s of a distance r seconds to a positive number s is the distance r/s seconds and the ratio (r seconds)/(s seconds) of a distance r seconds to a positive distance s seconds is the non-negative real number r/s.

The sum r seconds + s seconds of two distances r seconds and s seconds is defined to be the distance r + s seconds. A distance r seconds is defined to be less than, at least as large as, equal to, no greater than or greater than a distance s seconds. Likewise, the non-negative real number r is respectively less than, at least as large as, equal to, no greater than or greater than the non-negative real number s. The difference r seconds - s seconds obtained by subtracting from the distance r seconds the distance s seconds is the distance r - s seconds, provided that the non-negative real number s is not greater than the non-negative real number r. Independently of whether r is greater than s, one can define a symmetric difference between a distance of r seconds and a distance of s seconds to be a distance of |r - s| seconds, where |r - s| = ((r - s)2)1/2.

Since the product of a positive real number and a non-negative real number is a non-negative real number, instants r seconds apart can also be said to have a separation that is the product of a non-negative real number r/s and a unit of distance equal to s seconds, s being any positive real number. The use of the second as a unit of distance in time is thus arbitrary; a unit equal to the multiple of the second by any positive real number would serve as well.

Said another way, the separation of any two distinct instants may serve as a unit of distance in time. The separation of any other pair of instants is then some multiple of the given unit by a non-negative real number.

Since the product of a positive real number and the number 0 is the number 0, the zero distance in time is not only 0 seconds but 0 also of any other unit of distance in time. A positive distance, on the other hand, is always the product of a positive real number and a unit of distance, but which positive real number represents which distance depends on the size of the unit in which distances are expressed.

The instants have also this property: if I, J and K are any instants then the distance of I from J is never greater than the sum of the distance of J from K and the distance of K from I. There exist triples I, J and K of instants for which t(I,J) = t(J,K) + t(K,I) and other triples I, J and K of instants for which t(I,J) < t(J,K) + t(K,I).

In addition, for any two distinct instants I and J, the intervals (I,J), [I,J], [I,J) and (I,J] contain the same number of instants as does all of time, no matter how far apart or close together the instants I and J are. The length of each of these intervals is t(I,J), which is r seconds for some or another positive real number r. For any instant I, the interval (I,I) is the empty set, the cardinal of which is 0, and the interval [I,I] is the singleton set {I} the only element of which is the instant I, the cardinal of which is 1. The length of both (I,I) and [I,I] is t(I,I), which is 0 seconds. The set {I} the only element of which is the instant I may conveniently be taken to be the same entity as the instant I.

The Metric Property of Time

A metric space is an ordered pair (A,a) where A is a non-empty set and a is a function from the set A x A of all ordered pairs (x,y) of elements x and y of A into the set R of all real numbers, such that

1) for any elements x and y of A, a(x,y) = 0 if and only if x = y,

and

2) for any elements x, y and z of A, a(x,y) + a(y,z) | a(z,x).

If A and a are, respectively, any such set and function, then the set A is said to be the underlying set of the metric space (A,a), the function a is said to be a metric on the set A and, for any elements x and y of A, the real number a(x,y) is said to be the a-distance from x to y or the a-distance of y from x.

Let (A,a) be a metric space. Let x and y be any elements of A. Then

a(x,y) = 0 + a(x,y) = a(x,x) + a(x,y) | a(y,x)

and, similarly,

a(y,x) = 0 + a(y,x) = a(y,y) + a(y,x) | a(x,y),

so that, in fact,

a(y,x) = a(x,y),

or y is the same a-distance from x and x is from y, for which reason this distance is also called the a-distance between x and y.

Also, then,

$$a(x,y) = (a(x,y) + a(y,x))/2 \mid a(x,x)/2 = 0.$$

Thus the a-distance of y from x is never a negative number and a is, in fact, a function from A x A into the set R0 of non-negative real numbers. We see also that the a-distance between x and y is the real number 0 only if x = y and that, otherwise, the a-distance between x and y is a positive real number.

Ordering Relations of Time

If we take any one of the possible units of distance in time and define a function d from the set of all ordered pairs (I,J) of instants into the set R of real numbers by d(I,J) = r if and only if t(I,J) = r of those units, then d is a metric on time and the ordered pair (T,d) where T is time, the set of all instants, is a metric space the underlying set of which is time.

An instant K is said to occur between instants I and J if and only if the instant K is neither the instant I nor the instant J and the distance of the instant J from the instant I is equal to the sum of the distance of the instant J from the instant K and the distance of the instant K from the instant I.

The open interval of time (I,J) or (J,I) determined by instants I and J is the set of instants that occur between the instant I and the instant J. The closed interval of time [I,J] or [J,I] determined by the instants I and J is the set the elements of which are the instants I and J and the instants that occur between the instants I and J. If I and J are not the same instant, there are also the half-open, half-closed intervals determined by I and J, which contain the instants that occur between the instants I and J and either the instant I or the instant J but not both. The interval [I,J) or (J,I] contains I but not J. The interval (I,J] or [J,I) contains J but not I.

The separation of the instant I from the instant J is also called the length or the duration of any of the intervals determined by the instants I and J. Units of temporal separation are thus also called units of length of intervals of time or units of duration.

The instants are so numerous that there exist one-to-one functions from time onto the set R of real numbers. The cardinal of time is thus the cardinal of the set R of real numbers or the power of the continuum, !1 on the continuum hypothesis.

The instants of time are so arrayed, with respect to their distances from each other, that there exists a one-to-one function O from time onto the set R of real numbers such that there exists a unit of temporal distance such that, for any instants I and J and any real numbers t1 and t2, if O(I) = t1 and O(J) = t2 then the distance of the instant J from the instant I is the product of the given unit of temporal distance by the non-negative real number $|t1 - t2|$. Any such function from time onto R, and nothing else, is a Cartesian ordinate system for time.

There are as many Cartesian ordinate systems for time as there are real numbers. If O1 and O2 are any two of them, then, for any instant I, $O2(I) = t1 + (t2 - t1)O1(I)$ where t1 and t2 are the distinct real numbers $t1 = O2(O1\text{-}1(0))$ and $t2 = O2(O1\text{-}1(1))$.

Conversely, if O1 is a Cartesian ordinate system for time and t1 and t2 are any distinct real numbers and a function O2 from time onto R is defined by $O2(I) = t1 + (t2 - t1)O1(I)$ for each instant I, then O2 is also a Cartesian ordinate system for time.

The instants of time occur in a certain order. They have the same ordering as do the real numbers. Just as, of any two distinct real numbers a and b, one of them is less than the other and the other is greater than the one, so too, of any two distinct instants I and J, one is earlier than the other and the other is later than the one.

An instant K occurs between instants I and J if and only if either I is earlier than K and K is earlier than J or else J is earlier than K and K is earlier than I.

One customarily writes an interval of time with the first component being the earlier instant.

Cartesian ordinate systems for time fall into one or the other of two classes. Regressive systems assign larger numbers to earlier instants. Progressive systems assign larger numbers to later instants. Except in unusual circumstances, such as a countdown to an anticipated event, it is customary to employ only progressive ordinate systems for time.

The assignment of any two distinct real numbers to any two distinct instants determines a unique Cartesian ordinate system for time. The two assignments and the criterion that the duration of any interval of time be calculable by subtracting the ordinate of one endpoint of the interval from the ordinate of the other endpoint of the interval determine the ordinates of all the other instants.

One may therefore think of Cartesian ordinate systems for time as differing essentially only in the matter of to which instants the numbers 0 and 1 are assigned. Any instant whatever may be assigned the number 0. Any second instant, distinct from the first, may be assigned the number 1.

Each Cartesian ordinate system for time has associated with it a unique one of the possible units of temporal distance, these being the various temporal separations of distinct instants, or the multiples of the second by positive real numbers. The unit associated with a given ordinate system is the separation of the instants to which the numbers 0 and 1 have been assigned, which is also the separation of any other two instants the ordinates of which differ by 1. Thus one may also think of ordinate systems as differing from each other in the matter of assignment of the number 0 to different instants, in the matter of employing different units of duration and in being either progressive or regressive.

For each positive real number r and each instant I, there are two Cartesian ordinate systems for time, one progressive and one regressive, that assign the number 0 to the instant I and use a unit of duration of magnitude r seconds.

Note that there is a complete structural symmetry between the set of all instants later than any given instant and the set of all instants earlier than that instant. These are isometric metric spaces and they are also isomorphic as ordered sets, the ordering relation 'later than' of either set corresponding to the ordering relation 'earlier than' in the other set. This isometry and order isomorphism of past and future applies only to the abstract concept 'time' and does not extend to the physical universe from the nature and behavior of which the concept 'time' is abstracted. This is because physical objects are not related to time in a symmetric way.

Space as a Set

Space is more complicated than time because it is multi-dimensional. In general, the same tools can be used to understand the nature of space as essentially the same concept that has been understood in the past by classical mechanics. Unfortunately, in the twenty-first century science no longer considers any absolute spatial coordinate system. The following background material is used if one were to set up an absolute space.

Space is a set. The elements of space are points. 'Point', in this context, is just the generic name for the elements of space. Thus space is the set of all points.

The points are some distance each from each, each such distance being r meters for some non-negative real number r.

Given any points P and Q, the distance s(P,Q) of the point Q from the point P is 0 meters if and only if P and Q are the same point. The distance s(P,Q) is r meters for some positive real number r if P and Q are distinct points. The distance 0 meters is the zero distance in space. Any other distance in space is a positive distance.

It is to be understood that the product of a non-negative real number r and a distance s meters is the distance rs meters. The ratio of a distance r meters to a positive number s is the distance r/s meters and the ratio of a distance r meters to a positive distance s meters is the real number r/s.

The real number r is respectively less than, at least as large as, equal to, no greater than or greater than the real number s. Therefore, the sum of two distances r meters and s meters is defined to be the distance r + s meters. A distance r meters is defined to be less than, at least as large as, equal to, no greater than or greater than a distance s meters. The difference obtained by subtracting from the distance r meters the distance s meters is the distance r - s meters. This is true provided that the non-negative real number s is not greater than the non-negative real number r. The difference between a distance of r meters and a distance of s meters is a distance of $|r - s|$ meters. This is true without regard to which of r and s is the greater real number.

Since the product of a positive real number and a non-negative real number is a non-negative real number, points r meters apart can also be said to have a separation that is the product of a non-negative real number r/s and a unit of distance equal to s meters, s being any positive real number. The use of the meter as a unit of distance in space is thus arbitrary; a unit equal to the multiple of the meter by any positive real number would serve as well.

Said another way, the separation of any two distinct points may be used as a unit of distance in space. The separation of any other pair of points is then some multiple of the given unit by a non-negative real number.

Since the product of a positive real number and the number 0 is the number 0, the zero distance is not only 0 meters but 0 also of any other unit of distance in space. A positive distance, on the other hand, is always the product of a positive real number and a unit of distance, but which positive real number represents which distance depends on the size of the unit in which distances are expressed.

The points have also this property: if P, Q and R are any points then the distance of P from Q is never greater than the sum of the distance of Q from R and the distance of R from P. There exist triples P, Q and R of points for which s(P,Q) = s(Q,R) + s(R,P) and other triples P, Q and R of points for which s(P,Q) < s(Q,R) + s(R,P).

If we select a unit of distance in space and define a function ╪ from the set of all ordered pairs (P,Q) of points into the set R of real numbers by ╪ (P,Q) = r if and only if s(P,Q) = r of those units, then ╪ is a metric on space and the ordered pair (S,╪) where S is space, the set of all points, is a metric space the underlying set of which is space.

A point R is said to lie between points P and Q if and only if the point r is neither the point p nor the point Q and the distance of the point Q from the point P is the sum of the distance of the point P from the point R and the distance of the point R from the point Q.

The open interval of space determined by points P and Q is the set (P,Q) or (Q,P) of points that lie between the point P and the point Q. The closed interval of space determined by the points P and Q is the set [P,Q] or [Q,P] the elements of which are the points P and Q and the points that lie between the points P and Q. If P and Q are not the same point, there are also the half-open, half-closed intervals determined by P and Q, which contain the points that lie between the points P and Q and either the point P or the point Q but not both. [P,Q) or (Q,P] contains P but not Q and (P,Q] or [Q,P) contains Q but not P. In any interval of points, the choice between '[' or ']' and '(' or ')' determines whether the adjacent point does or does not belong to the interval, but the order in which the endpoints are written is arbitrary. '[' or '(' are used before the name of the left endpoint of the interval and ']' or ')' are used after the name of the right endpoint, 'left' and 'right' here referring to the relations of the symbols to each other, not of the points to each other.

The separation of the point P from the point Q is also called the length of any of the intervals determined by the points P and Q. Units of distance in space are thus also called units of length of intervals of space or, simply, units of length.

Subsets of Space

Certain subsets of space other than the intervals are important in the subsequent discussion.

If P and Q are distinct points then the set of all points R such that P is between R and Q or R is P or R is between P and Q or R is Q or Q is between P and R is the line determined by P and Q or the line PQ.

If P, Q and R are three points such that there is no line that contains all three of them, then the set of all points S such that there exist points T and U such that T is on the line PQ and U is on the line PR and not both T and U are the point P and S is on the line TU is the plane determined by the points P, Q and R, or by the lines PQ and PR.

If P is a point and r is a positive real number, then the set of all points Q such that s(P,Q) = r meters is the sphere with center P and radius r meters.

The intersection of a plane and a sphere is the empty set or a singleton set or a set the cardinal of which is the power of the continuum. In the last case, the set of all points on both the sphere and the plane is a circle. For each point P and each positive real number r, the intersection of a sphere centered at P and of radius r meters with a plane containing the point P is a circle centered at P and of radius r meters.

The structure of space makes it more appropriate to map space onto the set R3 of ordered triples of real numbers than to map space onto the set R of real numbers, but these sets all have the same cardinal, the power of the continuum, !1 on the continuum hypothesis.

The points of space are so arrayed, with respect to their distances from each other, that there exists a one-to-one function C from space onto the set R3 of ordered triples of real numbers such that there exists a unit of spatial distance such that, for any points P and Q and any real numbers x1, y1, z1, x2, y2 and z2, if C(P) = (x1,y1,z1) and C(Q) = (x2,y2,z2) then the distance of the point P from the point Q is the product of the given unit of spatial distance by the non-negative real number ((x1 - x2)2 + (y1 - y2)2 + (z1 - z2)2)1/2. Any such function from space onto R3, and nothing else, is a Cartesian co-ordinate system for space.

Vector Algebraic Interpretations of Space

It is convenient to invent at this time an algebraic system composed of ordered triples of real numbers that, in this context, are called 'vectors'. (When other kinds of vectors need to be discussed, the ones with which we are here concerned must be identified as 3-vectors over the field of real numbers.)

We understand two vectors u = (a,b,c) and v = (d,e,f) to be the same vector and write u = v if and only if a = d and b = e and c = f.

The product of a real number m and a vector u = (a,b,c) is

mu = m(a,b,c) = (ma, mb, mc)

The ratio of a vector u = (a,b,c) to a non-zero real number m is

u/m = (a,b,c)/m = (a/m, b/m, c/m)

The negative of a vector u = (a,b,c) is

-u = -(a,b,c) = (-a, -b, -c)

The sum of two vectors u = (a,b,c) and v = (d,e,f) is

u + v = (a,b,c) + (d,e,f) = (a + d, b + e, c + f)

The difference obtained by subtracting from the vector u = (a,b,c) the vector v = (d,e,f) is

u - v = (a,b,c) - (d,e,f) = (a - d, b - e, c - f)

The dot product of two vectors u = (a,b,c) and v = (d,e,f) is

u • v = (a,b,c) • (d,e,f) = ad + be + cf

The cross product of two vectors u = (a,b,c) and v = (d,e,f) is

u x v = (a,b,c) x (d,e,f) = (bf - ce, cd - af, ae - bd)

The square of a vector u = (a,b,c) is the dot product

u2 = u • u = a2 + b2 + c2
of that vector with itself.

The magnitude |u| of a vector u = (a,b,c) is the square root

|u| = (u2)1/2 = (a2 + b2 + c2)1/2

of the square of that vector. The zero vector, which is the vector (0,0,0) has magnitude 0. Any other vector, or any non-zero vector, has a magnitude that is a positive real number.

The direction <u> of a non-zero vector u = (a,b,c) is the ratio

<u> = u/|u| =

(a,b,c)/(a2 + b2 + c2)1/2 =

(a/(a2 + b2 + c2)1/2, b/(a2 + b2 + c2)1/2, c/(a2 + b2 + c2)1/2))

of the vector u to its length |u|. The zero vector has no direction.

A vector is a unit vector if and only if the magnitude of that vector is the number 1. The direction of a non-zero vector is a unit vector.

Two non-zero vectors are perpendicular each to the other if and only their dot product is the number 0. The zero vector is not perpendicular to any vector and no vector is perpendicular to the zero vector.

Two non-zero vectors are parallel each to the other if and only if their directions are the same vector. Two non-zero vectors are antiparallel each to the other if and only if the direction of one is the negative of the direction of the other, in which case the direction of each is the negative of the direction of the other. The zero-vector is neither parallel nor antiparallel to any vector and no vector is either parallel or antiparallel to the zero vector.

Co-Ordinate Systems for Space

There are as many Cartesian co-ordinate systems for space as there are real numbers. If C1 and C2 are any two of them, then there exist a vector u, a positive real number m and mutually perpendicular unit vectors i and j such that, for any point P and any real numbers x, y and z, C1(P) = (x,y,z) if and only if C2(P) = u + m(xi + yj + zk) where either k = i x j or k = j x i.

Conversely, if C1 is a Cartesian co-ordinate system for space and u is a vector and i and j are mutually perpendicular unit vectors and k is either i x j or else j x i and m is a positive real number and a function C2 from space into R3 is defined by C2(P) = u + m(xi + yj + zk) if any only if C1(P) = (x,y,z) for each point P and any real numbers x, y and z, then C2 is also a Cartesian co-ordinate system for space.

Each Cartesian co-ordinate system for space has associated with it a unique one of the possible units of distance, these being the various separations of distinct points, or the multiples of the meter by positive real numbers. The unit associated with a given co-ordinate system is the separation of the points to which the triples (0,0,0) and (1,0,0) have been assigned, which is also the separation of any other two points to which are assigned co-ordinates (x1,y1,z1) and (x2,y2,z2) such that ((x1 - x2)2 + (y1 - y2)2 + (z1 - z2)2)1/2 = 1.

Cartesian co-ordinate systems for space differ essentially only in the matter of to which points the triples (0,0,0), (1,0,0), (0,1,0) and (0,0,1) are assigned. These four assignments and the criterion that the length of any interval of points be calculable by subtracting the co-ordinates of one endpoint of the interval from the co-ordinates of the other endpoint of the interval and taking the square root of the sum of the squares of the three differences so obtained determine the co-ordinates of all the other points.

The formula for calculating the distance between points from the co-ordinates of the points also puts some constraints on the sets of four points to which the triples (0,0,0), (1,0,0), (0,1,0) and (0,0,1) may be assigned. Any point whatever may be assigned the triple (0,0,0). Any second point, distinct from the first, may be assigned the triple (1,0,0). The unit of spatial distance employed for the Cartesian co-ordinate systems for space that contain these two assignments is thus established as the distance of these two points from each other.

Alternatively, one may first select a point to be assigned the triple (0,0,0) and a unit of distance. There is then a sphere each point of which is that distance from the point to which the triple (0,0,0) has been assigned. Any one of these points may be assigned the triple (1,0,0).

The assignments of the triples (0,0,0) and (1,0,0) having been made, there is now a circle each point of which is the same distance from the point to which the triple (0,0,0) has been assigned as is the point to which the triple (1,0,0) has been assigned and each of which is 21/2 times as far from the point to which the triple (1,0,0) has been assigned as it is from the point to which the triple (0,0,0) has been assigned. Any one of these points may be assigned the triple (0,1,0).

There are now two points both of which are the same distance from the point to which the triple (0,0,0) has been assigned as are the points to which the triples (1,0,0) and (0,1,0) have been assigned and both of which are 21/2 times as far both from the point to which the triple (1,0,0) has been assigned and from the point to which the triple (0,1,0) has been assigned as either is from the point to which the triple (0,0,0) has been assigned. Either of these points may be assigned the triple (0,0,1).

For any two distinct points P and Q, the intervals (P,Q), [P,Q], [P,Q) and (P,Q] contain the same number of points as does all of space, no matter how far apart or close together the points P and Q are. The length of each of these intervals is s(P,Q), which is r meters for some or another positive real number r. For any point P, the interval (P,P) is the empty set, the cardinal of which is 0, and the interval [P,P] is the singleton set {P} the only element of which is the point P, the cardinal of which is 1. The length of both (P,P) and [P,P] is s(P,P), which is 0 meters. The set {P} the only element of which is the point P may conveniently be taken to be the same entity as the point P.

Possible Notion: Left Handedness and Right Handedness in Space
If one imagines a humanoid figure placed with its head at the point to which the triple (0,0,0) has been assigned, its feet at the point to which the triple (1,0,0) has been assigned and facing the point to which the triple (0,1,0) has been assigned, then the point to which the triple (0,0,1) has been assigned lies either to its left or to its right. Cartesian co-ordinate systems for space thus fall into one of two classes, called left-handed systems

and right-handed systems, respectively. It is customary to employ right-handed co-ordinate systems for space and to avoid the use of left-handed ones.

Matching the symmetry of time with the roles of before and after reversed, there is complete symmetry of space with the roles of right-handed and left-handed interchanged with each other.

This symmetry of space does not of and in itself imply a similar symmetry of the physical universe or of the internal structure or the behavior of the physical objects that inhabit space and move about in space while time passes. Whether there is such a symmetry depends on how physical objects interact with each other, on whether the law that specifies the nature of the interaction of physical objects with each other reflects, or the laws that specify the nature of the interaction of physical objects with each other reflect, whether the objects are left-handed or right-handed with respect to their internal structure or have left-handed or right-handed relations to each other.

This is an unanswered question, since the laws of interaction are not known. If these laws are like Newton's law of gravity, then they make no distinction between right-handed and left-handed. However, either Newton's law of gravity is incorrect or Newton's law of gravity is correct as far as it goes but it is not the only law of interaction between physical objects.

The physical universe apparently has some sort of asymmetry to it that may reflect a right-left distinction. For example, negatively charged electrons are common and positively charged positrons are rare, whereas positively charged protons are common but negatively charged antiprotons are rare. Nobody knows why. Nobody knows, either, whether, and if so then how, positive and negative charges are related to left-handed and right-handed. These are matters still under study and investigation.

Similarly, it appears that all neutrinos are left-handed while all antineutrinos are right-handed. Again, nobody knows why or what, exactly, this may mean.

Possible Notion: Infinity of Time and Space

Granted the Euclidean metric, the decision to speak of a beginning or an end of time or a boundary of space would be completely arbitrary and ad hoc and could be motivated only by some extra-logical and extra-experiential consideration such as an irrational dread of the infinite or a theologically motivated preference for a creation theory.

There are other reasons for assuming an infinite space and time. One is that if they are finite, we must then make a completely arbitrary decision about how big they are, there being no reason to prefer any size over any other, so long, anyway, as we choose sizes sufficiently large to encompass our visible environment and recorded or reconstructable history. Another is that a beginning or an end to time is incompatible with any conservation law. If physical stuff (matter, energy, momentum, whatever) popped into existence from time to time or vanished from time to time, and in particular if there was more of something at some times and less of it at other times, then it would be possible that there have been or will be times when there was none of it. However, the real world seems to be governed by a set a laws that say that this stuff can be here or there but it can never be nowhere, that you can move it about and concentrate it or rarefy it, but you can't make any more of it than there already is and you can't get rid of any of it. You can't, however, have, say, conservation of momentum unless time goes on forever and has always been doing so. A limit to space is also incompatible with conservation laws. An object moving at a some speed in some direction and subject to no external force continues moving at the same speed and in the same direction forever, not just until it arrives at "the edge of space."

Such arguments, however, prove nothing. It is always possible that quantities that appear to be constant are changing at imperceptible rates. In the end we must merely make such postulates as appear reasonable. We can have some hope that they are false and that evidence from the real world will contradict them so that we may learn something. We have to understand that if this does not happen, that fact does not prove that the postulates are true. We have to understand that there is no way whatever to show that they are true. Indeed, we have to understand that there is no way even to show that they are reasonable. 'Reasonable' is a judgment call and doesn't mean much more than 'I am interested in exploring the consequences of these propositions'.

If we decide to postulate a time of infinite duration and a space of infinite volume, we are extrapolating from the known to the unknown. Actually in this case, we are extrapolating from the unknown on the small to the unknown on the large or even better, from the unknown at distances intermediate between the small and the large to the unknown on the large. There is always a danger in extrapolation. Following the same principle, nobody in ancient England would have believed that there were elephants or black people in Africa, or that the sun in Africa was in the northern part of the sky.

Subsidiary Notions of Absolute Time and Space

Once we have a theory of kinematics that posits the existence of a space within which physical objects are to move about and a time that passes as they do so, there are a number of subsidiary notions that are part of this picture that we shall want to retain at least until our theory can't explain something or is actually contradicted by some observation. In a universe that consists of moving objects:

1. There must be a space and a time, since otherwise there is no way to have motion, which consists of having different locations in space at different locations in time and, more specifically, in continuous change of location in space with respect to location in time;

2. Space and time must be metric spaces, since otherwise there are no distances in space and durations in time and hence no way to define motion, as opposed to merely being somewhere at one time and somewhere else at some other time;

3. Metric spaces are point sets, so space must be a set of points and time must be a set the elements of which, called instants, have the same size and shape as points, that is, the

same relationship to distances between instants, or durations of intervals of time, that points have to distances between points, or lengths of intervals of space;

4. The physical universe, the set theoretical union of the set of moving objects, must be a set of things that occupy singleton point sets at the instants of time, for anything larger would have parts and hence would be a subset rather than an element of the universe;

5. These atoms must move all at the same constant speed, for otherwise there would be no way to distinguish a co-ordinate system for space from a moving spatial reference frame nor a uniform clock from a non-uniform clock, and no sense to the proposition that any atom even has a speed, much less to the proposition that any atom has a varying speed or that any two atoms have different speeds;

6. The elements of the physical universe must therefore each be completely surrounded by empty space and isolated from each other, since there is no way for continuous aggregates of atoms to move at variable speeds or with variable directions if their punctiform parts must all have the same constant speed;

7. The atoms must interact with each other according to some such law as Newton's law of gravity, since otherwise they would move only in straight lines and retain constant amounts of mass, energy, momentum and angular momentum, which is not the case, for physical objects are observed to move at variable speeds and in variable directions and to exchange energy, momentum and angular momentum with each other;

8. Since the atoms are separated from each other by positive distances, these interactions must be "action at a distance."

Notions that are Incompatible with Absolute Time and Space

Once we have a theory of kinematics that posits the existence of a space within which physical objects are to move about and a time that passes as they do so, there are a number of subsidiary notions that cannot reasonably extend from it. A theory of kinematics that posits the existence of a space within which physical objects are to move about and a time that passes as they do so, is incompatible with the following suggestions:

- Space is a physical object that can itself move or wiggle or be compressed or rarefied or expand or contract.
- Time moves, speeds up or slows down.
- Physical objects move through time.
- Lengths or durations depend on the relative speeds of the objects whose lengths are being measured or the processes whose durations are being measured with respect to the observer who is doing the measuring. There is no difference between a co-ordinate system for space and a moving spatial reference frame.
- Space and time together form one four-dimensional manifold.
- What is one distance and duration to one observer is another distance and duration to another observer.
- Physical objects interact with space or time.

- Space or time cause has some effect on the behavior of physical objects.
- Physical objects cause changes in the structure of space and time.
- Physical objects are changes in the structure of space and time.
- There exist objects that have momentum, but do not have definite positions.
- There exist objects that have positions but do not have definite speeds or directions of motion.
- A physical object has position and momentum as a single property that looks like either position or momentum depending on what experiment is done on it but does not have both position and momentum at once.