

## Gravitational Redshift and Age of the Stars

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José Francisco García Juliá  
C/ Dr. Marco Merenciano, 65, 5.  
46025 Valencia (Spain)  
E-mail: jose.garcia@dival.es

The gravitational redshift, in conjunction with the age of the stars, might explain any case of light redshift.

*Key words:* gravitational redshift, age of the stars.

### 1. Introduction

Generally, it is considered that the universe was originated in the Big Bang, and since then it is expanding. In that theory, the redshift of the light emitted from distant galaxies (the so-called cosmological redshift [1]) is interpreted as a Doppler effect and then considered as an indication of the expansion of the universe, following the law of Hubble. This empirical law is stated as

$$v_r = Hd \quad (1.1)$$

where  $v_r$  is the velocity of recession, namely the speed at which a light source moves away from the observer, due to the expansion of the space between them;  $H$  is the constant of Hubble, and  $d$  is the distance between the observer and the light source. The light redshift parameter is defined as

$$z = \frac{\nu_e - \nu_o}{\nu_o} \quad (1.2)$$

being  $\nu_e$  and  $\nu_o$  the light frequencies emitted and observed, respectively. For low redshift ( $z \ll 1$ ) [1]

$$z = \frac{v_r}{c} = \frac{Hd}{c} \quad (1.3)$$

being  $c$  the light speed in vacuum, therefore the redshift of the galaxies is proportional to their distances to the observer. To greater distance, greater redshift.

However, we are going to consider, using only very elemental arguments, that the redshift in the light coming from the stars might be produced by the gravitational potential.

## 2. The gravitational redshift and the age of the stars

The stars are the most numerous objects in the universe. They represent the 98% of its mass [2] (p. 385). The majority of the stars are in the so-called main sequence phase (MSP), that corresponds to the stage of the hydrogen nuclear fusion (HNF) and that occupies the 90% of their lives. Also, the majority of the stars in the MSP verify the following relations [2] (pp. 401-402):

$$L_{bol} \propto R^{5.2} \quad (2.1)$$

$$L_{bol} \propto M^{3.9} \quad (2.2)$$

being  $L_{bol}$ ,  $R$  and  $M$  the bolometric luminosity, the radius and the mass of the star, respectively. From (2.1) and (2.2),  $M^{3.9} \propto R^{5.2} = R^{3.9} R^{1.3}$ ,  $(M/R)^{3.9} \propto R^{1.3}$ ,  $M/R \propto R^{1.3/3.9}$  and [2] (p. 403)

$$\frac{M}{R} \propto R^{\frac{1}{3}} \quad (2.3)$$

For the gravitational redshift we have that [1]

$$\frac{z}{z+1} = -\frac{\nu_o - \nu_e}{\nu_e} = -\frac{\varphi_e - \varphi_o}{c^2 + \varphi_o} \quad (2.4)$$

being  $\varphi_e$  and  $\varphi_o$  the gravitational potentials at the points of emission and observation, respectively. For  $|\varphi_o| \ll c^2$

$$\frac{z}{z+1} = -\frac{\nu_o - \nu_e}{\nu_e} = -\frac{\varphi_e - \varphi_o}{c^2} \quad (2.5)$$

Note that the equation

$$\frac{\nu_o - \nu_e}{\nu_e} = \frac{\varphi_e - \varphi_o}{c^2} \quad (2.6)$$

is derived for a weak gravitational field in the framework of the General Theory of Relativity (GTR) in [3] (p. 349).

In a homogeneous and isotropic universe (cosmological principle), the gravitational potential does not change on a large scale, then theoretically  $\varphi_e = \varphi_o$ , and, from (2.4),  $z = 0$ . However, this might not be like that when we consider the age of the stars. The radii of the stars in the MSP decrease with the time. In a young star its radius would be greater because the force of contraction produced by the inner gravitational force is counteracted by the force of expansion produced by the energy of the HNF, but this last force decreases when decreases the hydrogen. The gravitational potential varies with the inverse of the distance and always is  $\varphi < 0$ , only  $\varphi(\infty) = 0$ . In the mechanics of Newton, for a star

$$\varphi_e = -\frac{GM}{R} \quad (2.7)$$

being  $G$  the Newton's gravitational constant, and  $M$  and  $R$  the mass and the radius of the star, respectively. For the stars in the MSP we can substitute (2.3) into (2.7). Note also that the distance from a star to us, for a light signal, is

$$d = ct \quad (2.8)$$

being  $t$  the time, and this time has to be computed to calculate ages. With all these premises we can consider, without violating the cosmological principle, that  $\varphi_e$  and  $\varphi_o$  may be different, and, from (2.4),  $z$  may be different of zero. Thus, for  $|\varphi_o| \ll c^2$ ,  $|\varphi_e| \ll |\varphi_o|$  and  $z \ll 1$ , from (2.4) and (2.7), we would have that

$$z \approx \frac{GM}{Rc^2} \quad (2.9)$$

And for the majority of the stars in the MSP, substituting (2.3)

$$z \approx \frac{G}{c^2} R^{\frac{1}{3}} \quad (2.10)$$

and  $z$  increases with  $R$  as  $R^{1/3}$ , or in other words, the younger the star, the larger the radius, so the greater the redshift.

For high redshift we can use the following relation (see appendix)

$$z \approx \left(1 - \frac{2GM}{Rc^2}\right)^{-1/2} - 1 \quad (2.11)$$

And for the majority of the stars in the MSP, substituting (2.3)

$$z \approx \left(1 - \frac{2G}{c^2} R^{\frac{1}{3}}\right)^{-1/2} - 1 \quad (2.12)$$

and  $z$  increases when  $R$  increases, then, as before, the younger the star, the larger the radius, so the greater the redshift.

Note also that the mass density of a star is

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} \quad (2.13)$$

and for a star in the MSP, substituting (2.3), we have that

$$\rho \propto \frac{1}{\frac{4}{3}\pi R^{\frac{5}{3}}} \quad (2.14)$$

and  $\rho$  decreases with  $R$  as  $R^{-5/3}$ , or in other words, the younger the star, the larger the radius, so the lesser the density.

Due to (2.8), the more distant the star, the younger was when emitted its light, and then, (2.10) and (1.3) seem to give similar values. But, from (2.10), considering (2.8), we have that the stars at the same distance to the observer with different redshifts have different ages. However, from (1.3), those stars would have the same redshift. Besides, (1.3) serves only for the redshift, whereas (2.4) serves for the redshift ( $|\varphi_o| < |\varphi_e|$ ,  $\nu_o < \nu_e$ ), non-shift ( $|\varphi_o| = |\varphi_e|$ ,  $\nu_o = \nu_e$ ) or blueshift ( $|\varphi_o| > |\varphi_e|$ ,  $\nu_o > \nu_e$ ).

In 2004, it was reported [4] the discovery of a quasar with high redshift,  $z = 2.114$ , in front of a galaxy NGC 7319 with low redshift,  $z = 0.0225$ . In the enlarged photograph of the event, it can be seen that exists a “V” shaped jet of matter between the quasar and the galaxy that might confirm that the quasar was ejected by the nucleus of the galaxy. Therefore, the quasar would be much younger than the galaxy. Hence, the stars of the quasar would be much younger than the stars of the galaxy, so the redshift of the quasar would be much greater than the redshift of the galaxy. Although (2.12) is valid only for individual stars, it is compatible with the assumption that the quasar would be much younger than the galaxy. However, (1.3) is incompatible with the discovery, because the quasar seems to be even closer to us than the galaxy.

### 3. Conclusion

We conclude that the gravitational redshift, in conjunction with the age of the stars, might explain any case of light redshift.

## Appendix

From the GTR, the Schwarzschild's radius (or gravitational radius)  $r_g$  and interval  $s$  are [3] (p. 398)

$$r_g = \frac{2km}{c^2}$$

$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - r^2 (\sin^2 \theta d\phi^2 + d\theta^2) - \frac{dr^2}{1 - \frac{r_g}{r}}$$

being  $k$  the gravitational constant,  $m$  the total mass of the body that produces the gravitational field and  $r$ ,  $\theta$  and  $\phi$  the spherical coordinates. The metric tensor component  $g_{00}$  is

$$g_{00} = 1 - \frac{r_g}{r}$$

The differential proper time is

$$d\tau = \sqrt{g_{00}} dt$$

The light angular frequency is

$$\omega = 2\pi\nu = \frac{d\alpha}{d\tau}$$

being  $\alpha$  the angle. Therefore

$$\frac{\nu_e}{\nu_o} = \frac{2\pi\nu_e}{2\pi\nu_o} = \frac{\omega_e}{\omega_o} = \frac{d\alpha/d\tau_e}{d\alpha/d\tau_o} = \frac{d\tau_o}{d\tau_e} = \frac{\sqrt{g_{00o}} dt}{\sqrt{g_{00e}} dt} = \frac{\sqrt{g_{00o}}}{\sqrt{g_{00e}}}$$

$$z = \frac{\nu_e - \nu_o}{\nu_o} = \frac{\nu_e}{\nu_o} - 1 = \frac{\sqrt{g_{00o}}}{\sqrt{g_{00e}}} - 1$$

$$z = \frac{\left(1 - \frac{2km_o}{r_o c^2}\right)^{\frac{1}{2}}}{\left(1 - \frac{2km_e}{r_e c^2}\right)^{\frac{1}{2}}} - 1$$

$$z = \frac{\left(1 - \frac{2GM_o}{R_o c^2}\right)^{\frac{1}{2}}}{\left(1 - \frac{2GM_e}{R_e c^2}\right)^{\frac{1}{2}}} - 1$$

and substituting (2.3)

$$z \propto \frac{\left(1 - \frac{2GM_o}{R_o c^2}\right)^{\frac{1}{2}}}{\left(1 - \frac{2G}{c^2} R_e^{\frac{1}{3}}\right)^{\frac{1}{2}}} - 1$$

For  $\frac{2GM_o}{R_o c^2} \ll 1$

$$z \approx \left(1 - \frac{2GM_e}{R_e c^2}\right)^{-1/2} - 1$$

$$z \approx \left(1 - \frac{2G}{c^2} R_e^{\frac{1}{3}}\right)^{-1/2} - 1$$

And when  $\frac{2GM_o}{R_o c^2} \ll \frac{2GM_e}{R_e c^2} \ll 1$ , as for  $\xi \ll 1$ ,  $(1 \pm \xi)^n \approx 1 \pm n\xi$ , then

$$z \approx \frac{GM_e}{R_e c^2}$$

$$z \approx \frac{G}{c^2} R_e^{\frac{1}{3}}$$

Note that when, respectively,  $M_o/R_o$  is lesser, equal or greater than  $M_e/R_e$ ,  $\nu_o$  is lesser, equal or greater than  $\nu_e$ , that corresponds to red, none or blue shift.

## References

- [1] José Francisco García Juliá, Simple Considerations on the Cosmological Redshift, *Apeiron*, Vol. 15, No. 3, July 2008, 325-337.
- [2] P. Bakouline, E. Kononovitch and V. Moroz, *Astronomie Générale*, in French, second edition, Éditions Mir, Moscou (1975). Original edition by Éditions Mir, Moscow (1974).
- [3] L. D. Landau and E. M. Lifshitz, *Teoría clásica de los campos*, in Spanish, second edition, Editorial Reverté, S. A., Barcelona (1973). Original edition by Nauka, Moscow.
- [4] Pasquale Galianni, E. M. Burbidge, H. Arp, V. Junkkarinen, G. Burbidge and Stefano Zibetti, The Discovery of a High Redshift X-Ray Emitting QSO Very Close to the Nucleus of NGC 7319, arXiv: astro-ph/0409215v1 (2004).