DEFINITION OF FUNDAMENTAL QUANTITIES WITH RESPECT TO SPACE AND TIME

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Abstract

In the SI system, the dimensions of physical quantities are derived from seven base dimensions corresponding to the seven fundamental quantities. This document shows how it is possible to derive the dimensions of all quantities from space and time only.

Introduction

In the SI system, seven base elements constitute the fundamental quantities and their dimension is not derived from another one. This document shows that it is however possible to derive the dimension of all quantities from space and time only. For this, we will present an innovative method using Planck quantities' values in order to discover their dimensions.

The dimensional system

The SI system defines seven fundamental quantities whose dimensions are the base from which are derived the dimensions of all other physical quantities. These basic dimensions are:

Name	**Unit**	**Symbol**
Distance	Meter	L
Mass	Kilogram	M
Time	Second	T
Current	Ampere	I
Light	Candela	J
Heat	Kelvin	K
Concentration	Mole	N

Table 1 - The fundamental dimensions

Any physical quantity Q can then be dimensionally expressed as:

$$[Q] = L^a M^b T^c I^d J^e K^f N^g \qquad (1)$$

With exponents, “a … g”, representing the influence of each constituent on the final quantity and the sign of the exponent indicating direct or inverse (1/x) proportionality. The notation [Q] means “the dimension of Q”.

This system allows us to perform dimensional analysis based on the requirement that each side of an equation must be of the same dimension. This is why it became the mandatory tool to validate a formula.

Considering that the unit of concentration (Mole) is dimensionless and that heat and light can be derived from energy, it means that it is enough to define mass and electric current.

The idea to derive mass and electric current dimensions from space and time only is not new and J.C. Maxwell already touched the subject in his treatise on electromagnetism[1] when he stated " *If, as in the astronomical system, the unit of mass is defined with respect to its attractive power, the dimensions of [M] are* $[L^3T^{-2}]$.". But we can easily demonstrate from Newton's law of gravitation that it is in fact G*M that dimensionally corresponds to L^3T^{-2} because:

$$F = M * a = G * M^2 / r^2 \qquad (2)$$

Then:

$$G * M = a * r^2 \qquad (3)$$

Which leads to:

$$[GM] = LT^{-2} * L^2 = L^3T^{-2} \qquad (4)$$

Maxwell did choose to assume, without explanation, that the gravitational constant G is dimensionless and that [M] corresponds to L^3T^{-2}. But the truth is that nothing really supports this assumption.

Concerning the electric charge, its definition is usually based on the mass's definition, but another difficulty arises because apparently two incompatible possibilities exist. The electrostatic version where the Coulomb's constant K is assumed dimensionless and $[Q] = M^{1/2}L^{3/2}T^{-1}$, and the electromagnetic version where it is $[Q] = M^{1/2}L^{1/2}$ because μ, the permeability, is then considered dimensionless.

Therefore, although previous similar studies exist, unfortunately, up to now, no convincing evidence has been offered to support any of the proposed solutions without an initial arbitrary choice. In addition, nothing in current knowledge allow us to make a sound choice describing [M] and [G] separately.

Basic principles

There is however a way to easily find the correlation between the dimensions of quantities and space-time. We base this demonstration on a matrix presenting the "Planck values" of physical quantities, with the help of modern computer power.

Consider a matrix whose horizontal axis corresponds to space, while the vertical axis represents time, as illustrated:

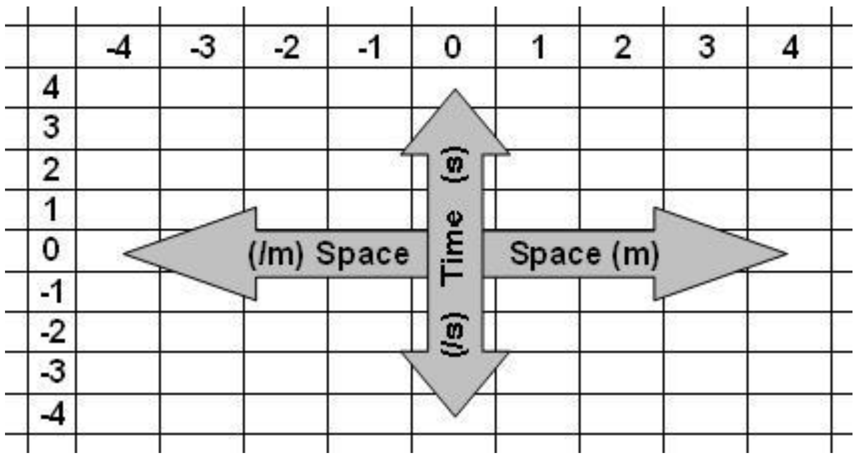

Figure 1 - The space and time matrix

The indices of the matrix directly correspond to the exponents of time and space dimensions. Based on these principles, it is easy to locate the basic elements such as:

	-4	-3	-2	-1	0	1	2	3	4
4									
3									
2									
1					Time				
0					1	Length	Surface	Volume	
-1					Freq.				
-2									
-3									
-4									

Figure 2 - The basic elements

If we replace meters and seconds by the Planck length and time respectively, we can then directly locate the "Planck quantities":

	-4	-3	-2	-1	0	1	2	3	4
4									
3									
2									
1					5,3912E-44				
0					**1**	1,6163E-35	2,6123E-70	4,2221E-105	
-1					1,8549E+43				
-2									
-3									
-4									

Figure 3 - Planck quantities

We then fill the cells of the matrix with a formula that reflects the relation between a quantity and space-time such as:

$$Qp = Lp^{x} * Tp^{y} \qquad (5)$$

With Qp = Planck quantity, Lp = Planck length, Tp = Palck time, x and y = exponents of the dimensions of Lp and Tp (or indices on the Matrix).

	-4	-3	-2	-1	0	1	2	3	4
4	1,2380E-34	2,0009E-69	3,2340E-104	5,2269E-139	8,4480E-174	1,3654E-208	2,2068E-243	3,5668E-278	0
3	2,2963E+09	3,7114E-26	5,9986E-61	9,6952E-96	1,5670E-130	2,5326E-165	4,0934E-200	6,6160E-235	1,0693E-269
2	4,2593E+52	6,8841E+17	1,1127E-17	1,7983E-52	2,9065E-87	4,6977E-122	7,5927E-157	1,2272E-191	1,9834E-226
1	7,9004E+95	1,2769E+61	2,0638E+26	3,3356E-09	**5,3912E-44**	8,7136E-79	1,4083E-113	2,2762E-148	3,6790E-183
0	1,4654E+139	2,3685E+104	3,8281E+69	6,1872E+34	**1**	**1,6163E-35**	**2,6123E-70**	**4,2221E-105**	6,8240E-140
-1	2,7182E+182	4,3932E+147	7,1006E+112	1,1476E+78	1,8549E+43	**2,9979E+08**	4,8454E-27	7,8314E-62	1,2658E-96
-2	5,0418E+225	8,1488E+190	1,3171E+156	2,1287E+121	3,4405E+86	5,5607E+51	8,9876E+16	1,4526E-18	2,3478E-53
-3	9,3519E+268	1,5115E+234	2,4430E+199	3,9484E+164	6,3817E+129	1,0314E+95	1,6671E+60	2,6944E+25	4,3548E-10
-4	#NOMBRE!	2,8036E+277	4,5314E+242	7,3238E+207	1,1837E+173	1,9132E+138	3,0922E+103	4,9977E+68	8,0776E+33

Figure 4 - Filling the matrix

We immediately recognize the speed of light which corresponds to $Lp * Tp^{-1}$ and is consequently located at position [1, -1], which validates our base formula. It is obvious that all combinations based on Planck space and time will appear on this matrix, within the limits of the software tool as can be seen in cells [-4, -4] and [4, 4]. This is because this limit exist that calculations for the thermic domain had to be done with a calculator.

Therefore, if our hypothesis is correct concerning the fact that dimensions of physical quantities can be derived from space and time only, their "Planck value" will appear on this matrix at the location corresponding to their space and time dimensions.

This study however highlighted the presence of simple multiplicative factors, specific to each domain, whose meaning is still unknown. Hopefully, these factors appear as 1eX, which is neutral in a multiplication and doesn't modify the number itself, but only the power exponent. Therefore, and although it could be a third basic dimension in addition to space and time, they have no influence on the two dimension matrix visual output. They however affect the numerical value, but do not hide them, by changing the power exponent. Once these factors acknowledged, we will show that it is possible to find all Planck quantities on the matrix, proving that their dimensions are derivable from space and time only.

Note that for a better reading, future figures will only display the values relevant to the discussion.

The dimension of mass

If we suppose that G and M are both on the matrix, then G*M must also be there. Let's take then the dimension of G*M, that we know for sure to be L^3T^{-2}. The numerical value of G*M is:

$$6.6742E\text{-}11 * 2.1764E\text{-}8 = 1.4526E\text{-}18 \qquad (6)$$

From the dimensional point of view, if we replace L and T by their Planck equivalents, we obtain:

$$Lp^3Tp^{-2} = (1.6162E\text{-}35)^3 * (5.3912E\text{-}44)^{-2} = 1.4526E\text{-}18 \qquad (7)$$

And we discover on the matrix that the numerical value of G*M appears at the expected location:

	-4	-3	-2	-1	0	1	2	3	4
4									
3					1,5670E-130				
2					2,9065E-87				
1					5,3912E-44				
0		2,3685E+104	3,8281E+69	6,1872E+34	1	1,6163E-35	2,6123E-70	4,2221E-105	
-1					1,8549E+43	2,9979E+08			
-2					3,4405E+86			1,4526E-18	
-3					6,3817E+129				
-4									

Figure 5 - The position of G*M

As previously mentioned, there is an infinity of possibilities for G and M and nothing supports the assertion that [M] = L3T-2. Our basic hypothesis states that the numerical values of G and M must be on the matrix and our researches show that the numerical value of the Planck mass appears at the location [7, -7], which means:

$$\mathbf{[M] = L^7 T^{-7}} \quad (8)$$

The gravitic domain's multiplicative factors being 1e67 and 1e-67, the constitutive formula of the Planck mass is:

$$Mp = Lp^7 * Tp^{-7} / 1e67 \quad (9)$$

This numerically gives:

$$2.1764E\text{-}08 = (1.6162E\text{-}35)^7 * (5.3912E\text{-}44)^{-7} / 1e67 \quad (10)$$

We also have a visual confirmation on the matrix, with respect to the multiplicative factor:

	-2	-1	0	1	2	3	4	5	6	7
7										
6										
5										
4										
3			1,5670E-130							
2			2,9065E-87							
1			5,3912E-44							
0		6,1872E+34	1	1,6163E-35	2,6123E-70	4,2221E-105	6,8240E-140	1,1029E-174	1,7826E-209	2,8811E-244
-1			1,8549E+43	2,9979E+08						
-2			3,4405E+86			1,4526E-18				
-3			6,3817E+129							
-4			1,1837E+173							
-5			2,1956E+216							
-6			4,0726E+259							
-7			7,5541E+302							2,1764E+59

Figure 6 - The Planck mass position

Although number similarity is striking, this could have been only a coincidence. However, this hypothesis is invalidated by the presence of all the Planck quantities at perfectly compliant and dimensionally coherent locations.

We know that to comply with $[GM] = L^3T^{-2}$, the dimension of G becomes:

$$[G] = M^{-1} L^3T^{-2} = L^{-4}T^5 \quad (11)$$

By using the multiplicative factor, the numerical value of G is then:

$$G = 6.6742E\text{-}11 = (1.6162E\text{-}35)^{-4} * (5.3912E\text{-}44)^5 * 1e\text{-}67 \quad (12)$$

This is directly confirmed on the Matrix:

	-5	-4	-3	-2	-1	0	1	2	3
7									
6									
5		6,6743E-78				4,5545E-217			
4						8,4480E-174			
3						1,5670E-130			
2						2,9065E-87			
1						5,3912E-44			
0		1,4654E+139	2,3685E+104	3,8281E+69	6,1872E+34	1	1,6163E-35	2,6123E-70	4,2221E-105
-1						1,8549E+43	2,9979E+08		
-2						3,4405E+86			1,4526E-18
-3						6,3817E+129			
-4						1,1837E+173			
-5						2,1956E+216			
-6						4,0726E+259			
-7						7,5541E+302			

Figure 7 - The gravitational constant position

The quantities associated with the energetic domain (energy, power, moment, action, etc …) are usually defined from mass, space and time. Therefore, once the position of the mass is found, Newton's second law (F=ma) tells us that the dimension of Force becomes L^8T^{-9}, and accordingly at position [8,-9] on the Matrix we find the expected value of the Planck force. This is also confirmed by the numerical value:

$$1.2103E44 = (1.6162E-35)^8 * (5.3912E-44)^{-9} * 1e-67 \qquad (13)$$

From the dimension of Force, it is trivial to derive the dimensions of the energetic, thermic and pressuric domains.

The dimension of electric current

Electric current, seen as the expression of a flow of electric charges, can be derived directly from electric charge (I = Q / t). However, the value of the Planck electric charge does not appear directly on the matrix. But the presence of the square of this charge (3.5176E-36 C^2) at location [8,7] is a clear indication that the dimension of electric charge is:

$$\mathbf{[Q] = L^4T^{-3.5}} \qquad (14)$$

Considering that for electric and magnetic domains, the multiplicative factors are 1e30, 1e37, 1e7 and 1e-7, the constitutive formula of the Planck charge is:

$$Qp = Lp^4 * Tp^{-3.5} / 1e30 \qquad (15)$$

This gives the following numerical value:

$$1.8755E\text{-}18 = (1.6162E\text{-}35)^4 * (5.3912E\text{-}44)^{-3.5} / 1e30 \qquad (16)$$

Because the time dimension exponent is not an integer, it implies a non-integer dimension. To represent this on the matrix, it is necessary to use the square root of Planck length and time as bases for the axis of the matrix. Then, the exponents of space and time will take the values 0, 0.5, 1, 1.5 … and the electric charge automatically appears

	-1	-0,5	0	0,5	1	1,5	2	2,5	3	3,5	4
4											
3,5											
3											
2,5											
2											
1,5			1,2518E-65								
1			5,3912E-44								
0,5			2,3219E-22								
0	6,1872E+34	2,4874E+17	1	4,0203E-18	1,6163E-35	6,4978E-53	2,6123E-70	1,0502E-87	4,2221E-105	1,6974E-122	6,8240E-140
-0,5			4,3068E+21								
-1			1,8549E+43		2,9979E+08						
-1,5			7,9885E+64								
-2			3,4405E+86								
-2,5			1,4818E+108								
-3			6,3817E+129								
-3,5			2,7485E+151								1,8755E+12
-4											

Figure 8 - The electric charge position

This dimension of the electric charge must satisfy either the electrostatic or the electromagnetic versions. Because of the L^2T^{-2} relation between μ and K, the two options are exclusive and cannot be both true. More precisely:

$$K = 1 / (4 * pi * \varepsilon) \qquad (17)$$

And

$$\varepsilon = 1 / (c^2 * \mu) \qquad (18)$$

It follows that:

$$K = (c^2 * \mu) / (4 * pi) \qquad (19)$$

So:

$$[K] = [\mu] * L^2T^{-2} \qquad (20)$$

We can say that if [K] = 1 then $[\mu] = L^{-2}T^2$ and if [μ] = 1 then $[K] = L^2T^{-2}$.

The electrostatic version is based on the unsupported assumption that the Coulomb's constant is dimensionless. In fact, the electromagnetic expression is correct because the permeability μ is equal to 4 * pi * 1e-7, where the 4 * pi term corresponds to a spherical integration in space, and the 1e-7 portion is the multiplicative factor of the quantity. Moreover, with their 1e-7 factor, the impedance and Coulomb's constant numerical values appear directly at the [1,-1] and [2,-2] positions, which confirms the parallel between impedance and velocity and the correspondence between the Coulomb's constant and the square of the speed of light. Also, by using Plancks values for the mass and electric charge, we see that Qp = $(MpLp)^{1/2}$ is true when introducing the multiplicative factors such as:

$$1.8755E\text{-}18 * 1e30 = ((2.1764E\text{-}08 * 1e67) * (1.6162E\text{-}35 * 1))^{1/2} \quad (21)$$

Finally, we see that if $[Q] = L^4T^{-3.5}$ and $[M] = L^7T^{-7}$ then again $[Q^2] = ML$ is dimensionally verified. When the real dimension of the Coulomb's constant (L^2T^{-2}) is introduced into the electrostatic version, it then reduces to:

$$Q^2 = F * r^2 / K \quad (22)$$

So

$$[Q^2] = ML^3T^{-2} / [K] \quad (23)$$

Then

$$[Q^2] = ML^3T^{-2} * L^{-2}T^2 = ML \quad (24)$$

Therefore, the apparent incompatibility vanishes, the electric charge has only one definition and no inconsistency really exist.

Synthesis

Based on the presented method, dimensions of all physical quantities were derived from space and time only in a perfectly numerically and dimensionally coherent system. The main physical quantities, their dimensional properties as well as their Planck values are presented in the following table:

Dom	Quantity	Symbol	Dimension SI/MKS	Dimension LT	Factor	Value	Unit
Dynamic	Length	l	L	L	1	1,6163E-35	m
	Surface	s	L2	L2	1	2,6123E-70	m2
	Volume	v	L3	L3	1	4,2221E-105	m3
	Time	t	T	T	1	5,3912E-44	s
	Frequency	f	T-1	T-1	1	1,8549E+43	/s
	velocity	c	LT-1	LT-1	1	2,9979E+08	m/s
	acceleration	a	LT-2	LT-2	1	5,5607E+51	m/s2
Energetic	Energy	N	ML2T-2	L9T-9	1E+67	1,9561E+09	J
	Force	F	MLT-2	L8T-9	1E+67	1,2102E+44	N
	Power	P	MLT-3	L9T-10	1E+67	3,6282E+52	W
	Power distribution (Poyting)	S	ML-1T-3	L7T-10	1E+67	1,3889E+122	W/m2
	Momentum	M	MLT-1	L8T-8	1E+67	6,5247E+00	Js/m
	Action	A	ML2T-1	L9T-8	1E+67	1,0546E-34	Js
	Energy density	U	ML-1T-2	L6T-9	1E+67	4,6329E+113	J/m3
	Energy flux	Φ	ML3T-2	L10T-9	1E+67	3,1615E-26	Jm
Gravitic	Mass	M	M	L7T-7	1E+67	2,1764E-08	Kg
	Mass density	ρ	ML-3	L4T-7	1E+67	5,1548E+96	Kg/m3
	Gravitational constant	G	M-1L3T-2	L-4T5	1E-67	6,6743E-11	Nm2/Kg2
	Field	g	LT-2	LT-2	1	5,56073E+51	m/s2
	Potential	ψ	L2T-2	L2T-2	1	8,9875E+16	m2/s2
	Mass flow	m	MT-1	L7T-8	1E+67	4,0370E+35	Kg/s
	Mass flux	Φ	ML-2T-1	L5T-8	1E+67	1,5454E+105	Kg/m2s
Pressuric	Volume	v	L3	L3	1	4,2221E-105	m3
	Flow	Q	L3T-1	L3T-1	1	7,8314E-62	m3/s
	Pressure	P	ML-1T-2	L6T-9	1E+67	4,6329E+113	Pa
Thermic	Heat	Q	ML2T-2	L5T1,5	1E+216	1,3807E-23	K
	Boltzmann's constant	K	ML2T-2	L5T1,5	1E+216	1,3807E-23	J/K
	Heat flow	Φ	ML2T-3	L5T0.5	1E+216	2,5610E+20	K/s
	Heat flux	Φq	MT-3	L3T0.5	1E+216	9,8039E+89	K/m2s
	Temperature	T	K	L4T-10,5	1E+283	1,4167E+32	K
	Capacity	C	ML2T-2K-1	LT11.5	1E-67	4,1969E-466	J/K
	conductivity	c	MLT-3K-1	T10,5	1E-67	4,8164E-388	W/mK
	Conductance	G	ML2T-3K-1	LT10,5	1E-67	7,7846E-423	W/K
	Resistance	R	M-1L-2T3K	L-1T-10.5	1E-67	1,2845E422	K/W
Electric	Charge	Q	TA	L4T-3,5	1E+30	1,8755E-18	C
	Charge distribution	D	L-2TA	L2T-3,5	1E+30	7,1797E+51	C/m2
	Charge density	ρ	L-3TA	LT-3,5	1E+30	1,1604E+17	C/m3
	Current	I	A	L4T-4,5	1E+30	3,4789E+25	A
	Current distribution	J	L-2A	L2T-4,5	1E+30	1,3317E+95	A/m2
	Dipole moment	M	LTA	L5T-3,5	1E+30	3,0313E-53	Cm
	Potential	U	ML2T-3A-1	L5T-5,5	1E+37	1,0429E+27	V
	Field	E	MLT-3A-1	L4T-5,5	1E+37	6,4528E+61	V/m
	Flux	Φ	ML3T-3A-1	L6T-5,5	1E+37	1,6856E-08	Vm
	Resistance	R	ML2T-3A-2	LT-1	1E+07	2,9979E+01	Ω
	Resistivity	r	MLT-3A-2	L2T-1	1E+07	1,8549E+36	Ωm
	Coulomb's constant	K	MLT-4A-2	L2T-2	1E+07	8,9875E+09	Ωm/s
	Permeability	μ	MLT-2A-2	1	1E+07	1,0000E-07	H/m
	Inductance	L	ML2T-2A-2	L	1E+07	1,6163E-42	H
	conductance	G	M-1L-2T3A2	L-1T	1E-07	3,3356E-02	S
	Conductivity	σ	M-1L-3T3A2	L-2T	1E-07	2,0638E+33	S/m
	Permittivity	ε	M-1L-3T4A2	L-2T2	1E-07	1,1127E-10	F/m
	Capacity	C	M-1L-2T4A2	L-1T2	1E-07	1,7983E-45	F
Magnetic	Potential	A	A	L4T-4,5	1E+37	3,4789E+18	Tm
	Field	B	L-1A	L3T-4,5	1E+37	2,1524E+53	T
	Auxiliary field	H	L-1A	L3T-4,5	1E+30	2,1524E+60	A/m
	Flux	Φ	LA	L5T-4,5	1E+37	5,6227E-17	Am
	Dipole moment	M	L2A	L6T-4,5	1E+30	9,0877E-45	Am2

Table 2 - Physical quantities and their dimensions

Conclusion

It was shown that by organizing Planck quantities according to the proposed configuration, it is possible to dimensionally define all physical quantities with respect to space and time only. This perspective offers numerous possibilities related to the understanding and the determination of physical quantities.

Acknowledgment

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References

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[1] J. C. Maxwell : “A Treatise on Electricity and Magnetism”, chapter 5