

An Analysis of Einstein's Special Relativity from a Boscovichian Perspective

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1. Introduction

My analysis of how Special Relativity was derived will try to clarify certain issues that have led to great misunderstandings. Classical or Newtonian Physics went through a transition from Newton's perspective to Boscovich's. It is my intention to analyse by a Boscovichian perspective the physics that led to Relativity Theory. In this perspective I show that Relativity Theory is a natural progression of Classical Physics, and only a great deal of misunderstandings has prevented that this should be more clearly known. The mathematics of Einstein's relativity is correct, but there has been subtle mistakes with understanding the maths leading to people incorrectly using it.

2. Newton's Physics

Newton's equations of motion are:

$$\begin{aligned}v &= u + at \\v^2 &= u^2 + 2as \\s &= ut + (1/2)at^2 \\s &= (1/2)(v + u)t\end{aligned}$$

v = final speed; u = initial speed; a = uniform acceleration; t = time; s = distance.

There are other equations of motion we can derive from these, but we can treat these as the basic equations of motion.

Newton's gravity equation is:

$$F = - (G m_1 m_2)/r^2$$

F = force, m_1 = mass of first object; m_2 = mass of second object; r = distance between objects.

There are other equations than these which we can write such as Kinetic Energy and momentum.

Let us now look at the Law of Conservation of Momentum in Newtonian theory, given that momentum is mass multiplied by velocity:

$$m_1 v_1 + m_2 v_2 = m_1 v_3 + m_2 v_4$$

treat v as a vector.

m_1 = mass of first object; v_1 = velocity of first object before collision; m_2 = mass of second object; v_2 = velocity of second object before collision; v_3 = velocity of first object after collision; v_4 = velocity of second object after collision.

The sum of the momentum of two objects colliding, is the same as the sum of the momentum afterwards.

This assumes several things such as there are no losses, no air resistance etc., and it would be good to treat the objects as idealisations of mass-points.

So, what we are talking about here with this scenario is an idealisation.

An actual physical experiment would be different to this idealisation; but we match the physical observation to idealisation as much as possible, and any difference measured we assign to some other effect outside the descriptive of the idealisation.

Now these two mass-points as they collide, by Newton's gravity theory the masses must interact through gravity and there is a force at work. But the idealisation of the Momentum case is not taking into account that acceleration and force i.e. it is only considering velocities. Also it assumes masses are unchanged etc. In other words the idealisation is considering only certain things.

Relativity theory is supposed to give a different version of Momentum conservation law.

When we deal with the idealisation approach, we must keep bearing in mind and realise that only certain things are being considered.

Now let us look at Special Relativity.

3. Special Relativity (SR)

The essential assumption of Special Relativity is that speed of light is the same for all inertial observers.

Inertial observer is someone who has velocity as zero, or moves at constant velocity.

In other words SR is only considering constant velocity scenario. (That is in the original way that SR was first formed, others have tried to extend it.)

For an observer that is inertial, he does not feel the motion. So if travel in a vehicle at constant velocity, and look at the surroundings, then:

case (1) vehicle moves and surroundings stationary

case (2) vehicle stationary and surroundings moving

Both cases have the same form of maths.

We usually assume that first scenario is the case; but from maths can treat the other case as valid as well. And that is the Relativity principle.

Einstein's amazing insight was to add to this the constancy of light speed; so that all inertial observers measure same light speed.

Newton's equations of motion for this case of constant speed becomes:

$$a = 0$$

$$v = u$$

$$v^2 = u^2$$

$$s^2 = ut$$

$$s = vt = ut$$

When we consider mass, really we have acceleration according to Newton's gravity theory. But this SR scenario is not considering that in its idealisation.

The equations of motion and of gravity were hard fought equations to win, and there is nothing so far to say change from them; hence we should stick with using them.

The derivation of the Lorentz equations of SR can be found rather simply by considering light bouncing between two mirrors:

For Observer O

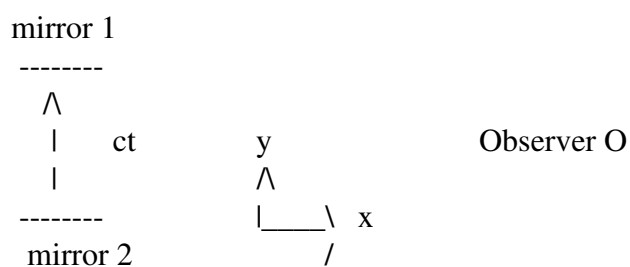


Fig.1

The light with speed c moves the distance between the two mirrors in time t in the y direction, the distance is thus ct .

Observer O' has similar arrangement, and sees exactly the same as observer O when stationary, but assigns distance ct' instead of ct .

For Observer O'

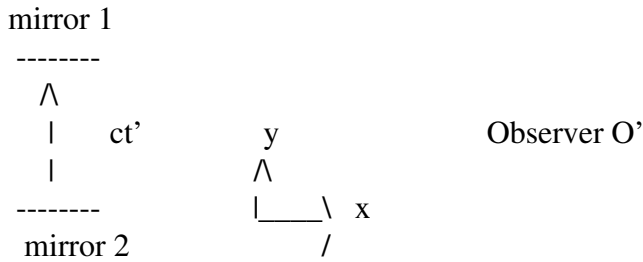


Fig.2

When O' moves with velocity v in the x direction with respect to observer O, then things change.

Observer O observation of O' arrangement is that in time t^* , O' moves distance vt^* in the x direction, and during this time t^* light travels distance ct^* . This forms a triangle with the hypotenuse as ct^* and the base of vt^* . The vertical side of the triangle is still the distance ct' from Fig.2. So, we have the Pythagorean Triangle:

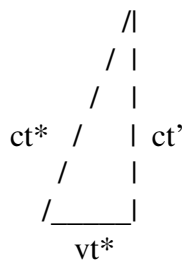


Fig. 3

What O observes is light with speed c travelling along the hypotenuse, he assigns time interval t^* for this distance and in that time interval distance vt^* is travelled along the horizontal. He does not observe light travelling along the vertical between the two mirrors. It is O' that observes light travelling vertically between the two mirrors; O observes light travelling a greater distance along the hypotenuse. It might be expected that O observes light speed c along the vertical and speed v along the horizontal, giving the velocity vector triangle of:

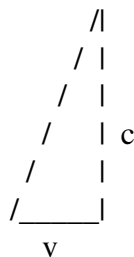


Fig.4

The hypotenuse of this triangle would be:

$$(c^2 + v^2)^{1/2}$$

And be greater than the speed of light c . This however is not the case, by Einstein's assumption the observer measures light speed as c and not as a greater light speed than c .

Proceeding with scenario of Fig.3 and let us drop the use of the star *, so that we consider time interval as t not t^* ; the proviso is that we should remember that this t is not the same time interval represented by t in Fig.1. Fig.3 then becomes:

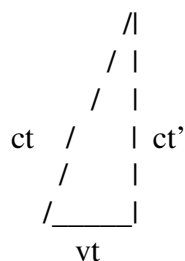


Fig.5

From this triangle we have:

$$c^2 t'^2 = v^2 t^2 + c^2 t^2$$

We can rearrange this to:

$$c^2 t'^2 = (c^2 - v^2) t^2$$

Divide through by c^2 :

$$t'^2 = (1 - v^2/c^2) t^2$$

Take the square root; and the positive solution:

$$t' = (1 - v^2/c^2)^{1/2} t$$

This is the Lorentz equation for time; it connects the time measured by O' with time measured by O .

Given that speed of light is constant, distance covered in time t' by O' is L' and this equals c ; similarly distance covered by O is L I time t and this equals c . So:

$$L'/t' = L/t = c$$

Rearrange this gives:

$$L' = Lt'/t$$

$$\text{Substitute for } t' = (1 - v^2/c^2)^{1/2} t$$

this gives:

$$L' = L (1 - v^2/c^2)^{1/2}$$

So, we now have the Lorentz equation connecting lengths as measured by O and by O'

Special Relativity would then go on to consider things like momentum, and form other equations. And as noted Special Relativity in its idealisation is not considering the gravity effects of mass; so care has to be taken that such equations are not applied to physical situations where the effects that are being ignored are significant.

There is nothing wrong with this. One has to realise that one is dealing with an idealisation and what is being considered is only the Principle of Relativity, constant speed, light speed as constant between the two observers, no acceleration, no gravity, and no mass etc. This then gives a mathematical result that is correct; one then matches it to physical measurements. The trouble is that certain subtle things have been misunderstood; which we now analyse.

4. Re-analysis of Inertial observers

The question of what is the vertical of this triangle has caused great misunderstanding in the subject called Special Relativity.

Observer O' should actually be assigned as observing light moving between mirrors as:

$$c't'$$

where c' is O' observation of light speed, and t' is Observer O' observation of time it takes for light to move between mirrors.

Now by Einstein's Special Relativity it is taken that $c = c'$, thus the vertical equals ct' .

Then from this Triangle, we can derive the equations of Length Contraction and Time Dilation, that was done in section (3).

So, instead of Fig 5 triangle we have:

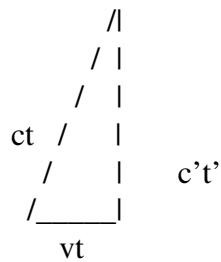


Fig. 6

The only difference between Fig 5 and Fig 6 is that the vertical in Fig 4 is $c't'$ and ct' in Fig 3.

From the Triangle of Fig.6 we have simply by Pythagorean theorem:

$$c't' = (c^2 - v^2)^{1/2} t$$

The possibilities are:

- (a) $c = c'$ and $t' = (c^2 - v^2)^{1/2} t$
- (b) $t' = t$ and $c' = (c^2 - v^2)^{1/2} c$

Special Relativity takes option (a) as shown in section (3).

While Newtonian Physics with the hidden assumption that all observers would take the same value for a given time interval takes option (b).

It was supposed to be a great switch in perception of Physics when there was a switch from option (b) to option (a), however a great many misunderstandings took place.

In option (b) perspective c' is the new speed of light, and c is the old speed of light. Observer O sees c' as different from c .

Now, by option (a) it is thought that light speed is constant, because both O and O' are observing the same value for light speed. This creates a subtle error.

It is correct that when O' makes an observation he believes that light speed is the same value whether he moves with velocity v or is stationary; i.e. when he is in an inertial frame.

It is similarly for observer O that he believes that light speed is the same whether he moves with velocity v or is stationary.

However, when O observes O' he observes the Triangle Fig.7:

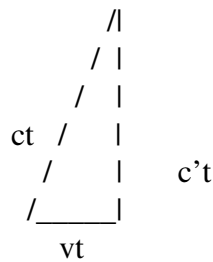


Fig. 7

The Observer O observes light move along the hypotenuse, it travels a distance ct in time interval t , and travels distance vt along the horizontal in time interval t . The Observer O is still measuring with time interval t along the vertical, and since this vertical is a smaller distance than the hypotenuse it must be a slower speed than c , which let us call c' . This c' is the speed of light that O would assign O', i.e. he deduces that light speed is different in frame of O', hence says:

$$c' = (c^2 - v^2)^{1/2} c$$

because he believes that t must be the same as t'

However, O' can tell him that there is no change in speed of light from O' frame. Therefore O then concludes that O' must be observing time and distance measurements different to himself, and has:

$$t' = (c^2 - v^2)^{1/2} t$$

as the transformation equation between the time interval as measured by O' with the time interval as measured by O.

Yet if O insists that all measurements be made from his O frame then he can use option (b), and from this frame the speed of light is variable.

It is just that we need not take measurements from frame O, rather we can take measurements from any frame, and if we do it from any frame we need transformation equations for time and space distances between different frames.

So, recapping if we choose a universal frame of reference to take measurements from then we have speed of light as variable, because from this frame that is what appears to happen.

What has happened is because it is pointed out that O and O' make different space and time measurements, what gets measured from any specific frame has been confused by various people.

What we need to remember is that all measurements in frame of O is measured in terms of t and c . Thus the vertical from this frame is: $c \cdot t$ not ct'

The equations of time dilation and length contraction of Special Relativity are relating how time and length measurements are connected from different frames.

5. Mass

As noted Special Relativity would then proceed to get the equation for Relativistic mass etc. after the derivation of the Lorentz equations, but the mass is not being considered as to its gravitational effect, i.e. not being considered what effect the gravitational force equation of Newton has.

What we are dealing with idealisations, and the Special Relativity scenario that does not deal with the gravitational effect of mass is ignoring that effect. In similar way other effects are being ignored in such a simple scenario, i.e. light is considered without any effect different mediums might have on it; it is assumed it travel in vacuum that has no effect on it.

Newton's theory raised various questions which Newton had not fully addressed, one of these was the Principle of Relativity. Galileo was the first to raise this idea during the Copernican Revolution. Copernicus' idea that the Earth moved was contrary to the existing belief that the Earth was stationary. It was thought that if the Earth moved then that movement should be felt, but it was not felt, and it looked like the Sun, planets etc. moved round the Earth. Galileo in order to overcome this difficulty between what was seemingly perceived (namely that there was no motion of the Earth) and the theory where there was motion of the Earth, introduced the Relativity Principle. According to this Principle whether one is stationary or moving at constant velocity one does not feel the motion. The stationary and moving frames of constant velocity are called inertial frames; and one cannot distinguish between inertial frames by what one feels as to which is moving or not moving. The Principle raises the idea that there might be one frame which is truly stationary, which would then be a suitable frame in which to measure all other motion against. The Principle was not adequately explored by Newton, and was taken up by Boscovich in his theory. It was Boscovich's theory with its relative motion that the Catholic Church accepted. The issue of a suitable frame in which to measure all motion with respect was taken up in the question of whether there was an ether. An absolute frame in the sense of a frame that Nature chooses as the stationary frame was seemingly not discovered. However, there is the question of a Universal frame in which people can decide as a datum a frame in which they want to measure all motion with respect to. i.e.

frame of rest defined from Nature -- called absolute frame

frame of rest defined by man -- called Universal frame

The frame of rest defined by man is of course arbitrary; different people choose different ones; it requires an agreement by all people as to what frame to use as a datum.

The question of a frame of rest defined by Nature is tied to Mach's principle-- whereby the average speed of all moving objects in the universe could be set to zero and chosen as this frame. The problem here being the difficulty in actually measuring this speed.

Thus barring Mach's principle; any inertial frame by the Principle of Relativity can be chosen as a rest frame to measure any other motion with respect to it.

Thus as far as I know Newton did not consider the Relativity issue of deciding how measurements of length and time etc. by different inertial observers was connected, it was an issue taken up by Boscovich. The equations of Newton for motion have been hard fought for, and the equations so far for Special Relativity takes the case where acceleration $a = 0$ for these equations.

There is no need to change from those Newton's motion equations and generalising to acceleration being non-zero we have the Triangle for O observing O' becoming:

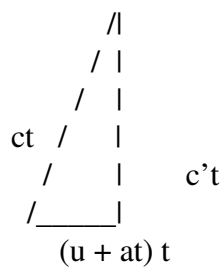


Fig.8

The issue of showing that Newton's equations of motion with acceleration set to zero is still applicable in the Michelson-Morley experiment and hence in Special Relativity was shown in my paper on Ether. [1] It also demonstrated that the motion of water waves was the same maths as motion of light waves, and that Ether could be defined as existing if one detaches the idea of ether from the difficulty of the absolute frame.

All of this means we can just stick with Euclidean geometry.

General Relativity takes the position of generalising the type of geometry to non-Euclidean geometry.

The maths of this Euclidean Geometry based upon its Classical Physics can be transformed to General Relativity with its non-Euclidean Geometry. The two systems of maths: Euclidean and non-Euclidean are both mathematically consistent systems, thus what can be done mathematically in one system can be transformed by suitable transformation equations to be the maths of the other system.

Conclusions

The peculiarity that velocity vector addition is not as per Fig 4 has resulted in different time and space intervals being measured by inertial observers; and the need for transformations between different inertial observers for the space and time intervals has resulted in people getting these different space and time intervals confused.

Once corrections have been made for this, there is a natural progression in Classical Physics from Newton's theory to Boscovich's theory regarding the Relativity issue which maintains a unified description of physics.

Reference

[1] A Re-examination of the concept of Ether in Einstein's Special Theory of Relativity, General Science Journal, Roger J Anderton, www.wbabin.net/science/anderton2.pdf

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