Max Born's Relativistic Mistake

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Max Born was a friend of Einstein and a main promoter of Einstein's theory relativity, so now let's look at his relativistic maths mistake and see what mess he makes of Special Relativity (SR).

1. Max Born's derivation

I shall present his derivation of the Lorentz Transformations, and then point out the error. He starts as follows [1]:

We once more repeat the hypothesis of Einstein's kinematics.

1. The Principle of Relativity .—There are an infinite number of systems of reference (inertial systems) moving uniformly and rectilinearly with respect to each other, in which all physical laws assume their simplest form (originally derived for absolute space or the stationary ether).

2. The Principle of the Constancy of the Velocity of Light.— In all inertial systems the velocity of light has the same value when measured with length-measures and clocks of the same kind.

Our problem is to derive the relations between lengths and times in the various inertial systems. In doing so we shall again restrict ourselves to motions that occur parallel to a definite direction in space, the x-direction.

Let us consider two inertial systems S and S' which have the relative velocity v. The origin of the system S' thus has the co-ordinate $x = vt$ relative to the system S at the time t. Its (world-line) is characterized in the system S' by the condition $x' = 0$. The two equations must denote the same and hence $x - vt$ must be proportional to x' . We set

$$\alpha x' = x - vt.$$

According to the principle of relativity, however, both systems are fully equivalent. Thus we may equally well apply the same argument to the motion of the origin of S relative to S', except that now the relative velocity v has the reverse sign. Therefore $x' + vt'$ must be proportional to x, and, on account of the equivalence of both systems, the factor of proportionality a will be the same in each case :

$$\alpha x = x' + vt'$$

From this and the preceding equation t' may be expressed in terms of x and t. We get

$$vt' = \alpha x - x' = \alpha x - (x - vt)/\alpha = (1/\alpha) \{(\alpha^2 - 1)x + vt)$$

thus

$$\alpha t' = (\alpha^2 - 1)x/v + t$$

This equation, combined with the first, allows x' and t' to be calculated when x and t are known. The factor of proportionality alpha is as yet indeterminate, but it must be chosen so that the principle of the constancy of the velocity of light remains preserved.

The velocity of a uniform motion is represented in the system S by $U = x/t$ and in the system S' by $U' = x'/t'$. If we divide the two equations which allow x' and t' to be expressed by x and t into each other, the factor a cancels, and we find

$$U' = x'/t' = (x - vt)/((\alpha^2 - 1)x/v + t)$$

If we divide the numerator and the denominator on the right by t and introduce $U = x/t$ we get

$$U' = (U - v)/((\alpha^2 - 1)U/v + 1) \quad \ldots(70)$$

If, in particular, we are concerned with the uniform motion of a ray of light along the x-axis then, by the principle of the constancy of the velocity of light, we must have $v = U'$, and the value of each is that of the velocity of light c. If, then, we set $U = c$ and at the same time $U' = c$, we must have

$$c = (c - v)/((\alpha^2 - 1)c/v + 1)$$

or $(\alpha^2 - 1)c^2/v + c = c - v$

Hence it follows that

$$\alpha^2 - 1 = -v^2/c^2 = -\beta^2$$

or $\alpha^2 + \beta^2 = 1$

This gives us the factor of proportionality alpha, namely

$$\alpha = \sqrt{1 - \beta^2} \quad . . . (71)$$

The transformation formulae now become

$$\alpha x' = x - vt$$

$$\alpha t' = -\beta^2 x/v + t.$$

We shall write them down once again in full, adding the co-ordinates y and z that are perpendicular to the direction of motion and that do not change:

$$x' = (x - vt)/\sqrt{1 - v^2/c^2}$$

$$y' = y$$

$$z' = z$$

$$t' = (t - vx/c^2)/\sqrt{1 - v^2/c^2}$$

(72)

These rules, according to which the place and time of a world-point in the system S' may be calculated, are said to constitute a Lorentz transformation.

2. The Critique

If one is seduced by the derivation above then it can appear as a valid line of mathematical reasoning. But of course it is nonsense. And once pointed out is nonsense, there is no way that one can see how to correct the error to what Max Born might really have meant.

The error occurs at what he (Max Born)labels equation (70).

If we accept all the conditions he gives up to the derivation of equation (70) then the equation seems okay. The problem starts after equation (70) so let us look at what he does:

$$U' = (U - v)/((\alpha^2 - 1)U/v + 1) \quad \ldots(70)$$

First of all he says: “If, in particular, we are concerned with the uniform motion of a ray of light along the x-axis then, by the principle of the constancy of the velocity of light, we must have v = U', and the value of each is that of the velocity of light c.”

He is setting U’ = v = c. It is more likely he means U=U’ and so means to set U=U’ = c, because that makes more sense with what he next does:

“If, then, we set U = c and at the same time U' = c, we must have”

$$c = (c - v)/((\alpha^2 - 1)c/v + 1)$$

$$\text{or } (\alpha^2 - 1)c^2/v + c = c - v$$

i.e. he has substituted U=U’=c into (70) and derived the equations above.

He then continues:

Hence it follows that

alpha^2 -1 = -v^2/c^2 = - beta^2

or alpha ^2 +beta^2 = 1

This gives us the factor of proportionality alpha, namely

alpha = sqrt (1 - beta ^2) . . . (71)

The transformation formulae now become

alpha x' = x - vt

alpha t' = - beta^2x/v + t.

And from that derives Lorentz transformation.

At first sight this seems okay, but in fact it leads to total ambiguity.

For the condition U=U’= c we have:

alpha^2 - 1 =-v^2/c^2

We substitute this back into (70) giving us:

c = (c-v)/((((-v^2/c^2)c)/v)+1)

c = (c-v)/(-v/c +1) = (c-v)/(1-v/c)

multiply through by (1-v/c)

c(1-v/c) = (c-v)

(c-v) = (c-v)

i.e. the equation is consistent; left hand side equals right hand side.

So

U’ = (U -v)/(1-v/c) is a valid equation for U’=U= c.

Let us label it:

U’ = (U–v)/(1–v/c)…….(a1)

Now let us try another speed into (70) let us say U=U’= 0.5c

for that we have (alpha^2 –1) = –v^2/(0.5c)^2

substitute into (70) gives us for U=U’ = 0.5c:

0.5c = (0.5c – v)/(((–v^2/(0.5c)^2)0.5c)/v) +1)

0.5c = (0.5c – v)/(1 –v/(0.5c))

multiply through by (1 –v/(0.5c))

0.5c(1 –v/(0.5c)) = (0.5c – v)

(0.5c –v) = (0.5c –v)

So once again consistent, so equation (70) works for U=U’= 0.5c.

i.e.

U’ = (U–v)/(1–v/(0.5c))…….(a2)

works for U= U’ = 0.5c

Notice that (a1) and (a2) are different!

If we were to do a general case of U=U’, then the general equation would be:

U’ = (U–v)/(1–v/U)…….(a3)

with(a3) works for U=U’.

Now the question is – where is the c?

the (1–v/U) part keeps changing for different values of U, it does not stay fixed as c.

If we now look at how relativistic velocity addition, it is defined [2]:

$$w' = \frac{w - v}{1 - wv/c^2}. \quad \text{(b)}$$

For the U=U’ = c case this is w’=w =c.

In that equation we have:

c = (c-v)/(1- cv/c^2) = (c-v)/(1-v/c)

This is the same as equation (a1) so we are okay there.

But for w=w’ = U=U’ = 0.5c case into (b) we have:

0.5c = (0.5c – v)/(1-0.5cv/c^2)

0.5c = (0.5c – v)/(1 – 0.5v/c)

This is not the same as equation (a2)

U’ = (U-v)/(1-v/(0.5c))…….(a2)

for U=U’ = 0.5c have (a2) as:

0.5c = (0.5c -v)/(1-v/(0.5c))

We have a difference in the equations. And most significantly equation (a3):

U’ = (U-v)/(1-v/U)…….(a3)

(a3) works for U=U’.

Allows us to add speeds greater than c say U’=U = 2c then (a3) is:

2c = (2c-v)/(1-v/2c)

whereas U=U’ = w=w’ = 2c into (b) gives

2c = (2c-v)/(1-v/c)

multiply through both sides:

2c(1-v/c) = (2c-v)

left hand side equals 2c–2v, right hand side equals 2c–v; so left hand side and right hand are not equal. Which the mainstream interprets as by equation (b) there are no speeds greater than c.

But from (a3) with its:

$2c = (2c-v)/(1-v/2c)$

multiply both sides by (1–v/2c) gives:

$2c\ (1-v/2c) = (2c\ -v)$

$(2c-\ v) = (2c\ -v)$

Left hand side still equals right hand side, so that by equation (a3) speeds greater than c are allowed.

So the issue is what did Max Born really mean by equation (70) did he mean equation (a3) or (b). He left it all vague and ambiguous.

i.e. another inconsistency in SR this time by Max Born.

Because - SR is not properly defined.

Going by other SR texts what seems to be meant is that SR relativistic velocity addition is equation (b). That means equation (70) which does not reduce uniquely to being (b) is not probably what SR means by an equation of SR.

i.e. equation (70) allows more than what standard SR seems to want to admit to allowing.

That invalidates Max Born’s whole derivation of the Lorentz transformation.

Max Born derived the Lorentz transformation but it involves equations that standard SR does not recognise as being SR. (e.g. it allows speeds greater than c, when standard SR is supposed to not allow it.) Whatever maths he has - it is not standard SR theory, its something else.

3.To overcome the difficulty

Max Born’s derivation of the Lorentz transformation was to overcome difficulties that Einstein left with his derivation. But as we can see Max Born’s derivation has its own problems.

Richtmyer-Kennard-Cooper seem to try to overcome the difficulties that Max Bohr faced with his derivation, and they give their derivation of the Lorentz transformation.[3] But they create difficulties – one moment they have x defined as x=ct then the next they have x defined as x = Vt. Ideally V should be any range of values, but the steps they take define it as being only V=c. So their derivation of Lorentz transformation is invalid also, and is only true for the case of V= c, and cannot deal with the general case of the Lorentz transformation where V is supposed to be any value.

The method of the relativists is to find different ways to bodge their maths.

Their derivation of the Lorentz transformation is invalid. All they can do is bodge the maths by different means to get their Lorentz transformation. Mathematically what they do with the maths is invalid.

Quite simply the maths of standard SR just does not work. It is inconsistent, ambiguous and vague and liable to different interpretations. It is not a clearly defined logical mathematical theory. Instead the maths is bodged to try to fit whatever the experimental data is. When we try to trace back why SR is like this, we trace it back to Einstein who formed the theory and made a totally mess of his maths. Those who then follow Einstein then do likewise – namely mess up the maths.

Reference

[1] Einstein’s theory of Relativity, Max Born, translated by Henry L. Brose, third edition, E.P. Dutton and company, UK, 1922, p. 198- 200

[2] Composition of velocities

If the observer in S sees an object moving along the x axis at velocity w, then the observer in the S' system, a frame of reference moving at velocity v in the x direction with respect to S, will see the object moving with velocity w' where

$$w' = \frac{w - v}{1 - wv/c^2}.$$

This equation can be derived from the space and time transformations above.

$$w' = \frac{dx'}{dt'} = \frac{\gamma(dx - v dt)}{\gamma(dt - v dx/c^2)} = \frac{(dx/dt) - v}{1 - (v/c^2)(dx/dt)}$$

Notice that if the object were moving at the speed of light in the S system (i.e. $w = c$), then it would also be moving at the speed of light in the S' system. Also, if both w and v are small with respect to the speed of light, we will recover the intuitive Galilean transformation of velocities:

http://en.wikipedia.org/wiki/Special_relativity#Simultaneity from 31Jan2009

[3] The errors in the derivation of Lorentz Transform part2
http://www.wbabin.net/science/anderton25.pdf