

The errors in the derivation of Lorentz Transform part2

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1. Introduction

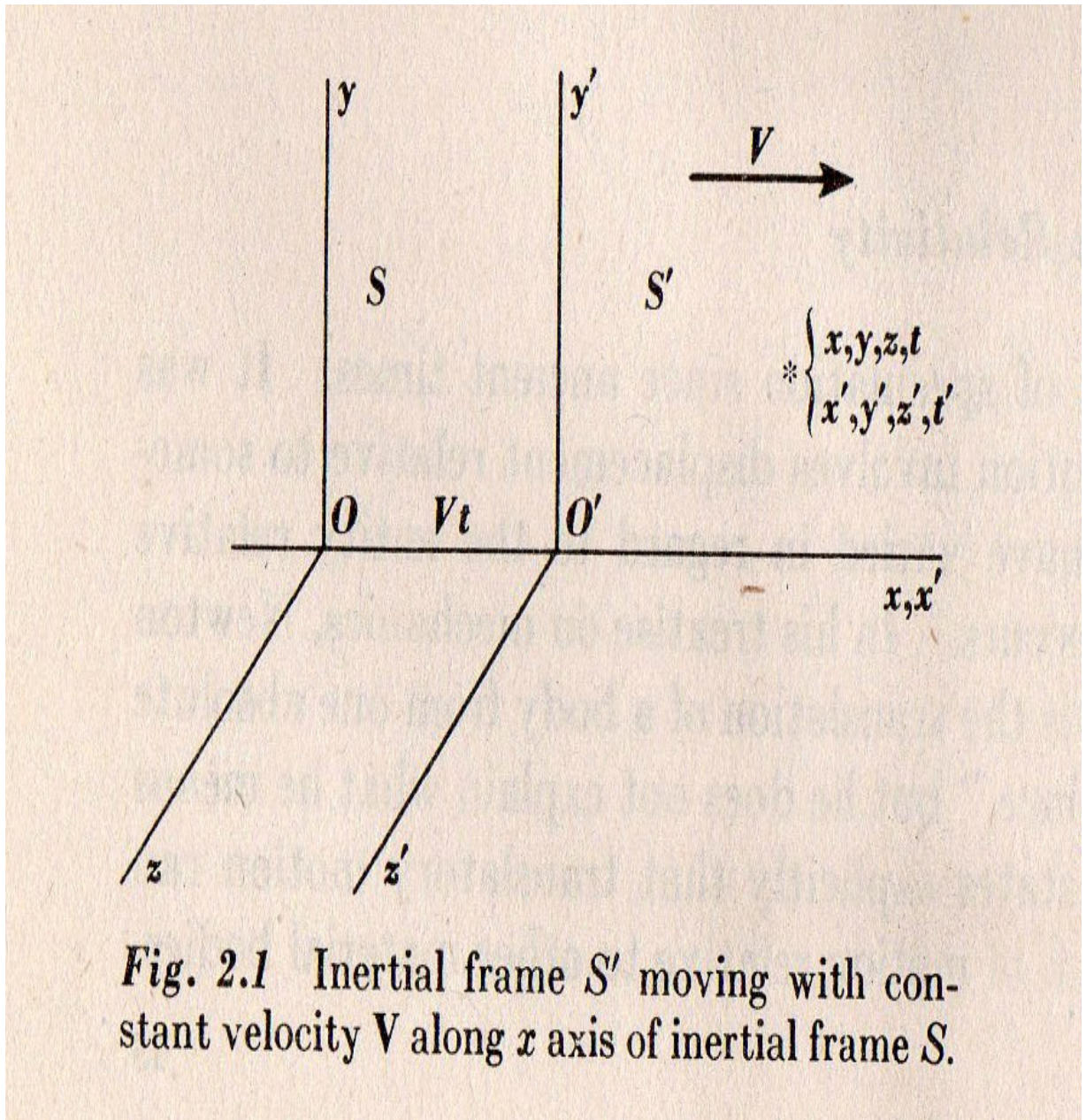
Further to my article “The errors in the derivation of Lorentz transform). I find that the article I was using for reference- Richtmyer-Kennard-Cooper’s had another subtle error in its derivation of the Lorentz transform hidden away.

There are numerous errors in the maths of standard texts on Special relativity (SR) and it is very difficult to spot them all. One error might seek to compensate for a subsequent error, and so forth. While going back to Einstein’s original work on SR just presents one with mountains of mistakes by Einstein. Subsequent works on SR since Einstein have sought to correct the mistakes of Einstein and have added more mistakes. So it is extremely difficult to know where to begin with SR’s maths mistakes. Ideally Einstein’s work being so full of nonsense should have been binned, but unfortunately it formed the basis of modern physics, so we are stuck with a process of trying to fix those mistakes.

2 The subtle mistake

I shall now deal with Richtmyer-Kennard-Cooper’s derivation [1] of the Lorentz transform. In part 1, I used their derivation of Lorentz transformation, only pointing out a few things, this time I am looking at their derivation in more detail to show where it goes wrong in a subtle way:

They ask us to consider the case of inertial frames S and S' .



The two frames are in uniform translatory motion relative to each other.

** (n.b. they use capital V but sometimes I will use lower case v for the same velocity.)

They say:

“Let us call the two frames S and S’ and let the velocity of S’ relative to S be V. Let coordinates and times of any event obtained when the frame S is used be denoted by x,y,z,t and those obtained for the same event when S’ is used by x’, y’,z’, t’. To make the relation between these variables as simple as possible, let us choose our axes so that the x and x’ axes are parallel to V and thus slide along each other; let the y’ and z’ axes be parallel to y and z respectively (fig2.1). Let us also count time from the instant at which the origins of the coordinates O and O’ coincide. The coordinates of O’ are measured in frame S are $x= Vt, y=0, z=0$.”

First they proceed from this for Galilean transformation, then later they pick this up for the Lorentz transformation, and say:

“Let the two origins coincide at $t=0, t’ = 0$, at which instant a light pulse is emitted from the common origin. Imagine that observers in S and S’ have arranged apparatus which enable them to follow the pulse as it moves outward from the source. By Einstein’s second postulate, observer O in frame S and observer O’ in S’ find the locus of the wavefront to be given respectively by”:

$$x^2 + y^2 + z^2 - c^2 t^2 = 0 \quad (2.2a)$$

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = 0 \quad (2.2b)$$

me: I have dealt with an article querying why “c” is the same in both equations. I would write on that occasion:

$$x^2 + y^2 + z^2 - c_1^2 t^2 = 0$$

$$x'^2 + y'^2 + z'^2 - c_2^2 t'^2 = 0$$

And query why should c_1 be equal to c_2 . Although Einstein’s second postulate requires that the speed c of light in vacuo be the same in all directions and for all observers in all inertial frames, it should be established if these c_1 and c_2 are that c. But we shall accept for now as per these authors that they use $c_1 = c_2 = c$.

Thus the authors continue:

“Thus each observer finds the wavefront to be a sphere centered at his own origin, even though the origins of the two systems no longer coincide !”

They point out Galilean transformation does not work. I don't want to go into detail why they are wrong on this, and they get onto their derivation of the Lorentz transformation:

“The equations $y = y'$ and $z = z'$, we accept them without proof; powerful arguments based on the isotropy of space can be advanced for their validity.”

me: I have pointed out the issue on that. But we shall accept it for now, so we can proceed with the rest of their derivation.

They continue:

“Therefore we seek compatible relations between x' , x , t' , and t . As the origins pass, $x = x'$, and we choose $t = 0 = t'$. Because of the homogeneity of space and of the uniformity of natural laws in time we assume the relationships are linear and try”:

$$x' = \alpha x + \eta t \quad (2.3a)$$

$$t' = \epsilon x + \gamma t \quad (2.3b)$$

“where α , η , ϵ , and γ are constants to be determined.”

me : I have pointed out the issue here, but let us continue.

They say:

“At the origin of S' , $x' = 0$ and $x = Vt$, so by (2.3a)”

$$0 = \alpha Vt + \eta t$$

me: This is where the subtle error occurs. If we ignore all the problems they create up to here and accept what they have been saying, they create now a subtle error, namely they say for $x' = 0$ that $x = Vt$.

They have stated it earlier when they were considering Galilean transformation. But now they introduce it here when considering Lorentz transformation, and I want to protest it.

I shall come back to that protest after I allow them the rest of their derivation:

“Thus $\eta = -\alpha V$, and so

$$x' = \alpha (x - Vt)$$

Inserting this value of x' and equation (3b) into equation (2b) yields

$$\alpha^2 x^2 - 2\alpha^2 Vxt + \alpha^2 V^2 t^2 + y^2 + z^2 - c^2 \eta^2 x^2 - 2c^2 \eta \alpha xt - c^2 \alpha^2 t^2 = 0$$

This result is compatible with equation (2a) only if

$$\begin{aligned} \alpha^2 - c^2 \eta^2 &= 0 \\ \alpha^2 Vxt - c^2 \eta \alpha xt &= 0 \\ c^2 \alpha^2 t^2 - \alpha^2 V^2 t^2 &= 0 \end{aligned}$$

These three equations can be solved for the three unknowns α , η , and α in terms of V and c and give

$$\alpha = \eta = 1/\sqrt{1-V^2/c^2}$$

and

$$\eta = -\alpha V/c^2 = -(V/c^2)/\sqrt{1-V^2/c^2}$$

All constants are now determined, and we have:

$$\begin{aligned} x' &= (x - Vt)/\sqrt{1-V^2/c^2} \\ y' &= y \\ z' &= z \\ t' &= (t - (Vx/c^2))/\sqrt{1-V^2/c^2} \end{aligned}$$

That is where they finish their derivation, so back now to the protest.

They have:

“At the origin of S' , $x' = 0$ and $x = Vt$ ”

But they give us at the origin “ $x = x'$, and we choose $t = 0 = t'$.”

So by $x = Vt$ they mean at $t=0$, so they are saying $0=0$. It's back to nonsense about talking about $0=0$.

They give us the general equation of x' as:

$$x' = \alpha x + \eta t \quad (2.3a)$$

so $x = Vt$ is not even the general equation.

If we equate $x = Vt$ to (2.3a) we can do it several ways as-

$$x' = \alpha x + \eta t = Vt \quad (a)$$

or we could accept what they say about x being zero then write :

$$\eta t = Vt \quad (b)$$

The difference between (a) and (b) is important!

From (a) we would deduce that $\alpha = 0$, from (b) we would not deduce that.

So the authors are hiding a mess with saying $x = Vt$ at the origin with their $t=0$, so $x = 0$. They are introducing an error with their $0 = 0$.

The general equation they gave for x was:

$$x' = \alpha x + \eta t$$

When they choose the condition " $x = x'$, and we choose $t = 0 = t'$." and decide $x' = Vt$ they are messing that up. They are just mathematically incompetent. Now I refer you to:

$$x^2 + y^2 + z^2 - c^2 t^2 = 0 \quad (2.2a)$$

The authors are not considering what happens to y and z , so we are dealing with:

$$x^2 - c^2 t^2 = 0$$

from which we can get as positive solution $x = ct$. But the authors have totally messed it up with their $0=0$ nonsense because they have $x = Vt$ from using $0=0$ illegally.

So, they are mixing equations up.

One minute it suits them to have $x = ct$, the next it suits them to have $x = Vt$. They use the mess with $0=0$ to do whatever they like. Effectively $1=2$ when it suits them and $1=1$ when it suits them. Their maths is just a mess of contradictions.

3. Attempt at correction

So picking up this derivation above and provisionally accepting its derivation up to where I protested. (If we pick up at other points before the protest then other corrections occur which have been dealt with in part1, and the maths starts to look different.)

For:

$$x^2 + y^2 + z^2 - c^2 t^2 = 0 \quad (2.2a)$$

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = 0 \quad (2.2b)$$

We are not paying attention to y, y', z and z' so treating them as zero.

** I think it is supposed to be that maybe light travels along x, x', y, y', z and z' in which case $x^2 + y^2 + z^2 = c^2 t^2$ but I want to consider the easier case of light just along x, x' not along y, y', z, z' ; thus I want to deal with the easier case of $y = y' = z = z' = 0$ and $x = ct$, "they" have already messed up by treating x as Vt when it isn't, so it presents problems as to how exactly we correct them.

We have these two equations reduced to:

$$x^2 - c^2 t^2 = 0 \quad (2.2a1)$$

$$x'^2 - c^2 t'^2 = 0 \quad (2.2b1)$$

And we still want to solve:

$$x' = \alpha x + \eta t \quad (2.3a)$$

$$t' = \epsilon x + \gamma t \quad (2.3b)$$

From (2.2a1) we have $x = ct$ (positive solution) place into (2.3a) and (2.3b):

$$x' = \alpha ct + \eta t \quad (2.3a1)$$

$$t' = \epsilon ct + \gamma t \quad (2.3b1)$$

We are given --As the origins pass, $x = x'$, and we choose $t = 0 = t'$.

When that happens (2.3a1) and (2.3b1) become:

$$x' = \alpha c \cdot 0 + \eta \cdot 0 = 0$$

$$t' = \epsilon c \cdot 0 + \gamma \cdot 0 = 0$$

So not much information contained there. Standard SR texts would seek erroneously to get something from it, but they are manipulating $0 = 0$.

The simplest case for x' is the Galilean transformation: $x' = x - Vt$, for $x = ct$, $x' = (c-v)t$, so we suspect x' might be some multiple of $(c-v)t$ let us call it A , so have:

$$x' = A(c - v)t \quad (2.3a2)$$

$$t' = (\epsilon c + \gamma) t \quad (2.3b2)$$

subst into (2.2b1) :

$$(A(c-v)t)^2 - c^2 ((\epsilon c + \gamma)t)^2 = 0$$

divide through by t^2

$$(A(c-v))^2 - c^2 ((\epsilon c + \gamma))^2 = 0$$

Expand out :

$$A^2 (c^2 + v^2 - 2vc) - c^2 (\epsilon^2 c^2 + \gamma^2 + 2 \epsilon \gamma c) = 0$$

trying to collect things that look similar:

$$A^2 c^2 - c^2 \epsilon^2 c^2 = 0$$

$$A^2 v^2 - c^2 \gamma^2 = 0$$

$$-A^2 (2vc) - 2 c^2 \epsilon \gamma c = 0$$

gives us:

$$A^2 - c^2 \epsilon^2 = 0$$

$$A^2 v^2 - c^2 \gamma^2 = 0$$

$$A^2 v + c^2 \epsilon \gamma = 0$$

first equation gives us $A = c \epsilon$ (positive solution)

third equation gives us $Av = -c \gamma$

So (2.3a2 – s.3b2) become:

$$x' = A(c-v)t \quad (2.3a2)$$

$$t' = ((A/c) c - (A/c)v) t \quad (2.3b2)$$

we may as well say $A = 1$ then we have :

$$x' = (c-v)t \quad (2.3a2)$$

$$t' = (1 - v/c) t \quad (2.3b2)$$

recognising $x = ct$ we can write it back :

$$x' = x - vt \quad (a1)$$

$$t' = t - xv/c^2 \quad (a2)$$

It looks very much like how Galilean transformations. The first is how Galilean transformation is defined for x and x' . The second has a difference to how Galilean transformation equation usually relates t and t' .

If we try these equations (a1 – a2) in (2.2b1) we have:

$$(x-vt)^2 - c^2 (t - xv/c^2)^2 =$$

$$x^2 + v^2 t^2 - 2xvt - c^2 (t^2 + x^2 v^2 / c^4 - 2xvt/c^2)$$

subst for $x = ct$

$$c^2 t^2 + v^2 t^2 - 2cvt^2 - c^2 t^2 - v^2 t^2 + 2cvt^2$$

equals zero as required.

i.e. the transformation equations are :

$$x' = x - vt \quad (a1)$$

$$t' = t - xv/c^2 \quad (a2)$$

instead of what is normally given as :

$$x' = (x - Vt)/\sqrt{1 - V^2/c^2} \quad (b1)$$

$$t' = (t - (Vx/c^2))/\sqrt{1 - V^2/c^2} \quad (b2)$$

Indeed if we subst the above two equations into (2.2b1) the relativistic factors $1/\sqrt{1 - V^2/c^2}$ are superfluous and only serve to make the transformation look more complicated than it really is.

Conclusion

We have the corrected Galilean transformation of (a1-a2) being the same as the Lorentz transformation (b1-b2) once we ditch the unneeded relativistic factor.

Galilean transform can be put in many forms so as to work in various different mathematical setups. I have looked at issue of c_2 not equal to c , and other issues regarding defining Galilean transform in other ways in other articles.

Reference

[1] Introduction to Modern Physics, F K Richtmyer, E H Kennard, John N Cooper, Tata McGraw- Hill Publishing company ltd., sixth edition, 1982, New Delhi, India p57- 59

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