

The errors in the derivation of Lorentz Transform

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This article deals with correcting the mistakes made with the Lorentz transformation.

Einstein's Special Relativity (SR) amounts to its basis being the equations:

$x^2 + y^2 + z^2 - c^2 t^2 = 0$ for reference frame of observer O with coordinates x,y,z,t observing light.

$x'^2 + y'^2 + z'^2 - c^2 t'^2 = 0$ for reference frame of observer O' with coordinates x',y',z',t' observing the same light.

I will now look at Richtmyer-Kennard-Cooper derivation of the Lorentz transforms [1] from these equations:

Einstein's second postulate requires that the speed c of light in vacuo be the same in all directions and for all observers in all inertial frames; thus c becomes a universal constant.

Consider the inertial frames S and S'.

Let the two origins coincide at $t = 0, t' = 0$, at which instant a light pulse is emitted from the common origin. Imagine that observers in S and S' have arranged apparatus which enable them to follow the pulse as it moves outward from the source. By Einstein's second postulate [constancy of lightspeed], observer O in frame S and observer O' in S' find the locus of the wavefront to be given respectively by:

$$x^2 + y^2 + z^2 - c^2 t^2 = 0 \quad (2a)$$

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = 0 \quad (2b)$$

Thus each observer finds the wavefront to be a sphere centered at his own origin, even though the origins of the two systems no longer coincide !

The equations $y = y'$ and $z = z'$, we accept them without proof; powerful arguments based on the isotropy of space can be advanced for their validity.

me: Saying “powerful arguments” is a bluff. As pointed out in article “Feynman’s relativistic mistake”, [2] there is no real justification for believing $y=y'$ and $z=z'$. So, the article is really asking for $y=y'$ and $z=z'$ to be assumed. And the authors Richtmyer-Kennard-Cooper don’t really know why it is assumed, so have to bluff.

Therefore we seek compatible relations between x' , x , t' , and t . As the origins pass, $x = x'$, and we choose $t = 0 = t'$. Because of the homogeneity of space and of the uniformity of natural laws in time we assume the relationships are linear and try:

$$x' = \alpha x + \eta t \quad (3a)$$

$$t' = \epsilon x + \gamma t \quad (3b)$$

where α , η , ϵ , and γ are constants to be determined.

me : Notice that the authors Richtmyer-Kennard-Cooper has to make yet another assumption to get these equations. Einstein’s Special relativity is supposed to be based on two assumptions relativity and constancy of lightspeed. But to really derive any equations, arbitrary extra assumptions have to be added!!

At the origin of S' , $x' = 0$ and $x = Vt$, so by (3a)

$$0 = \alpha Vt + \eta t$$

Thus $\eta = -\alpha V$, and so

$$x' = \alpha (x - Vt)$$

Inserting this value of x' and equation (3b) into equation (2b) yields

$$\alpha^2 x^2 - 2\alpha^2 Vxt + \alpha^2 V^2 t^2 + y^2 + z^2 - c^2 \eta^2 x^2 - 2c^2 \eta \gamma xt - c^2 \gamma^2 t^2 = 0$$

This result is compatible with equation (2a) only if

$$\begin{aligned}\alpha^2 - c^2 \eta^2 &= 0 \\ \alpha^2 V \eta - c^2 \eta \gamma &= 0 \\ c^2 \gamma^2 - \alpha^2 V^2 &= 0\end{aligned}$$

These three equations can be solved for the three unknowns α , γ , and η in terms of V and c and give

$$\alpha = \gamma = 1/\sqrt{1 - V^2/c^2}$$

and

$$\eta = -\gamma V/c^2 = -(V/c^2)/\sqrt{1 - V^2/c^2}$$

All constants are now determined, and we have:

$$\begin{aligned}x' &= (x - Vt)/\sqrt{1 - V^2/c^2} \\ y' &= y \\ z' &= z \\ t' &= (t - (Vx/c^2))/\sqrt{1 - V^2/c^2}\end{aligned}$$

These equations, named the Lorentz transformation by Poincare, were discovered by Voigt in 1887 and independently derived by Lorentz in 1904 in the course of his study of matter moving in an electromagnetic field. Lorentz assumed one frame of reference to be at rest in the ether and attached an immediate physical meaning only to measurements made with this frame. The new principle of relativity implies that all inertial frames are to be treated on an equal footing.

me: i.e. the theory before Einstein's SR was Lorentz's theory using these equations. And Einstein added the principle of relativity to the idea of using these Lorentz equations, for all inertial frames, not just the one frame of reference that Lorentz was assuming.

me: So Einstein's basic equations (2a- 2b) – the foundation for his SR have two theories connected with it. The Lorentz theory comes from (2a-2b) assuming one reference frame at rest. And SR comes from (2a-2b) by adding the principle of relativity as the way to interpret these equations. From SR we could re-interpret Lorentz's one frame of reference as a preferred frame.

me: Now some the mistakes in this derivation of the Lorentz equations, I have already pointed out, namely the “bluff” and the adding of extra assumptions. Adding extra assumptions in a mathematical model I think is okay, but the main error with it is that most relativity texts like to present the false image that SR is derived only from two assumptions, when actually there are more than two.

me: If we look at the Lorentz equations we can try substituting them into the left hand side of (2b) and see if they equal zero as required. And if we do that then we find it works – the Lorentz equations are consistent with the starting equation of SR. Usually that’s where many texts leave the derivation. But really there should be after that derivation, an investigation as to whether any other transformation equations than Lorentz’s equations satisfy (2a-2b). And as noted the bluff with $y=y'$, $z=z'$ hides one possibility, dealt with in article “Feynman’s relativistic mistake”. [2]

me: Also as part of checking other possible transformation equations, it should be checked that Galilean transformation equations don’t satisfy (2a-2b). Because after all if those equations (2a-2b) are satisfied by Galilean transformation equations then there was no need to step over from Newtonian physics to what is claimed to be Einsteinian physics. i.e. there would be no need for the Einstein Revolution overturning the existing Newtonian paradigm. The authors Richtmyer-Kennard-Cooper do actually cover that possibility. They check the Galilean transformation equations in (2a-2b) declare they don’t work, hence need for Einstein’s Revolution. My claim is that they do that wrong. So let’s now look at that derivation:

The Galilean transformation equations

$$x' = x - Vt$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

Direct substitution of the Galilean transformation into (2b) yields

$$x^2 - 2xVt + V^2 t^2 + y^2 + z^2 - c^2 t^2 = 0$$

which is clearly incompatible with (2a)

me: What is that the substitution of those Galilean transformation equations into (2b) does not make it look like (2a) as required, and hence the Galilean transformation equations they give is not a solution of (2a-2b).

me: I agree the Galilean transformation equations they give is not a solution of (2a-2b). But I disagree about the Galilean transformation equations they use as being correct. So let's look at things in more detail. The important part of (2a-2b) if we take $y=y'$ and $z=z'$ is:

$$x^2 - c^2 t^2 = 0 \quad (2a(i))$$

$$x'^2 - c^2 t'^2 = 0 \quad (2b(i))$$

We are given:

$$x' = x - Vt$$

In (2a(i)) a distance x is being travelled by light in time interval t given by

$$x^2 = c^2 t^2$$

$$x = ct$$

i.e. distance x is traversed by light with speed c in time interval t .

For the given:

$$x' = x - Vt$$

We have a different distance than x , we know $x = ct$, so the equation becomes:

$$x' = (c - V)t$$

i.e. distance x' is traversed by a speed $(c-V)$ in time interval t .

But light travels speed c , so light would travel that distance $(c - V) t$ in less time than interval t , namely:

$$ct' = (c-V)t$$

light covers distance $(c-V)t$ in time interval t' .

Then t and t' connected by

$$t' = (1-V/c)t$$

Recap:

For $x' = x - Vt$, we have $t' = (1-V/c)t$.

Whoever gave for the Galilean transformation that $t=t'$ was grossly in error when $x' = x - Vt$.

It is true that in Newtonian (Galilean) physics that people all observe the same time durations, which might lead one to say $t=t'$. But that would be an error. The observer in O frame is seeing light travel a different distance than the observer in the O' frame. This is highlighted by having the O frame observer observe the distance x and the observer in the O' frame have distance $x-Vt$. It would be a nonsense to say that the light covers both distances x and $x-Vt$ in the same time interval (barring case of $V=0$). But that is precisely the nonsense that has been allowed to be setup for what is claimed to be the Galilean transformation equations- it claims the nonsense that the distances x and $x-Vt$ are traversed by light in the same times! It is no wonder the Galilean transformation does not work; nonsense was substituted for what the Galilean transformation really should be. [3]

The correct Galilean transformation equations are:

$$\begin{aligned} x' &= x - Vt, \\ t' &= (1-V/c)t. \end{aligned}$$

With x a different distance to x' and t a different time interval to t' .

Now to check, we substitute into (2(b(i)))

$$\begin{aligned} &x^2 - 2xVt + V^2 t^2 - c^2 (1 - 2V/c + V^2/c^2)t^2 \\ &= x^2 - 2cVt^2 + V^2 t^2 - c^2 t^2 + 2Vc t^2 + V^2 t^2 \end{aligned}$$

$$= x^2 - c^2 t^2$$

= equation (2(a(i))) as required.

So the correct Galilean transformation equations are consistent with the same basic equations that SR starts from. There was no need to transfer from Newtonian Physics to Einsteinian revolution. The existing physics was quite adequate, provided no maths mistakes were made.

If we want to, we can write the corrected Galilean transformation equations (using $x = ct$) as:

$$x' = x - Vt,$$

$$t' = (t - Vx/c^2)$$

Then compare them to the Lorentz transformation equations:

$$x' = (x - Vt)/\sqrt{1 - V^2/c^2}$$

$$t' = (t - (Vx/c^2))/\sqrt{1 - V^2/c^2}$$

And we see that they are the same equations if we dispose of the relativistic factor $\sqrt{1 - V^2/c^2}$.

n.b. the situation is a bit different when we consider the scenario of y not equal to y' and z not equal to z' . In this article's derivation we have assumed $y=y'$, $z=z'$ as I have pointed out.

Really I think the case under consideration is that y, z, y', z' has no effect on it, so that case is $y=y'=z=z'=0$. i.e. light not travelling along those directions; instead only along x, x' direction. If considering general along distance r where $r^2 = x^2 + y^2 + z^2$ (i.e light travelling along r) then case different. **

Conclusion

The whole basis of the Einsteinian Revolution was mathematical mistakes. And once that basis of mistakes had been built, the impetus was to carry on making more maths mistakes, and adding them to the existing collection.

Special Relativity and Galilean Relativity are the same theory as I have claimed. But what hides that connection that they are the same theory is that there are numerous mathematical mistakes making both theories look different. And the maths that is associated with existing Special Relativity is so bad that it is contradictory i.e. not forming a coherent consistent set of maths. This is because standard SR maths is a collection of different people's mathematical mistakes.

References

[1] Introduction to Modern Physics, F K Richtmyer, E H Kennard, John N Cooper, Tata McGraw- Hill Publishing company Ltd., sixth edition, 1982, New Delhi, India p57- 59

[2] Feynman's relativistic Mistake
<http://www.wbabin.net/science/anderton20.pdf>

[3] add on: What self-respecting Newtonian-Galilean person would say that two different distances would be traversed in the same time interval by a constant velocity object; because that's what the incorrect Galilean transformation equations in relativity texts were insisting they would do.

c. RJAnderton2009

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