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Direct Measurement of Pulsar Distances

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Introduction -- The formula used for the dispersion of the radio waves in the interstellar vacuum don't allow the calculation of the pulsars distance. In this article we demonstrate that the exact formula allows that calculation.

From the Lorentz's equations:

$$\begin{cases} x = \dfrac{x_0 + vt_0}{\sqrt{1 - v^2/c^2}} \\ t = \dfrac{t_0 + vx_0/c^2}{\sqrt{1 - v^2/c^2}} \end{cases} \Leftrightarrow \begin{cases} v^2(c^2{t_0}^2 + x^2) + 2vc^2x_0t_0 + c^2({x_0}^2 - x^2) = 0 \\ v^2(c^2t^2 + {x_0}^2) + 2vc^2x_0t_0 + c^4({t_0}^2 - t^2) = 0 \end{cases}$$

Equalling the coefficients:

$$\Leftrightarrow \quad c^2t^2 - x^2 = c^2{t_0}^2 - {x_0}^2$$

So, for all and any v

$$\Leftrightarrow \quad c^2{t_n}^2 - {x_n}^2 = k \quad \text{or} \quad {x_n}^2 - c^2{t_n}^2 = k \qquad (k \text{ is a constant})$$

Doing $\quad w_n = x_n / t_n \quad$ and $\quad f_n = 1/t_n \quad \Leftrightarrow$

$$w_n = \sqrt{c^2 - k{f_n}^2} \quad (1) \qquad \text{or} \qquad w_n = \sqrt{c^2 + k{f_n}^2} \quad (2)$$

c is the light speed, f_n is the wave frequency and w_n it's the propagation speed.

Those formulas can explain the phenomena of positive (1) and negative (2) dispersion in optical mediums. As we know, by experience, the interstellar vacuum has negative dispersion.

$$w = x/t \quad ; \quad w_0 = x_0/t_0 \quad ; \quad w = \sqrt{c^2 + kf^2} \quad ; \quad w_0 = \sqrt{c^2 + k{f_0}^2}$$

$$\begin{cases} x = \dfrac{x_0 + vt_0}{\sqrt{1 - v^2 / c^2}} \\ t = \dfrac{t_0 + vx_0 / c^2}{\sqrt{1 - v^2 / c^2}} \end{cases} \quad \Leftrightarrow \quad \begin{cases} w = c^2 \dfrac{w_0 + v}{c^2 + vw_0} \\ f = \dfrac{cf_0 \sqrt{c^2 - v^2}}{c^2 + vw_0} \end{cases}$$

Derivation of the exact formula of the dispersion in the interstellar medium

According to several studies we have done, v is a constant.

$$w = c^2 \frac{\sqrt{c^2 + kf_0{}^2} + v}{c^2 + v\sqrt{c^2 + kf_0{}^2}} \quad \Leftrightarrow \quad \frac{dw}{df_0} = c^2 kf_0 \frac{c^2 - v^2}{\left(c^2 + v\sqrt{c^2 + kf_0{}^2}\right)^2 \sqrt{c^2 + kf_0{}^2}}$$

$$f = \frac{cf_0 \sqrt{c^2 - v^2}}{c^2 + v\sqrt{c^2 + kf_0{}^2}} \quad \Leftrightarrow \quad c^2 - v^2 = \frac{f^2\left(c^2 + v\sqrt{c^2 + kf_0{}^2}\right)^2}{c^2 f_0{}^2}$$

$$\frac{dw}{df_0} = \frac{kf^2}{f_0\sqrt{c^2 + kf_0{}^2}} \quad \text{and} \quad kf^2 = w^2 - c^2 \quad \Leftrightarrow$$

$$\Leftrightarrow \quad \frac{dw}{w^2 - c^2} = \frac{df_0}{f_0\sqrt{c^2 + kf_0{}^2}} \quad \Leftrightarrow$$

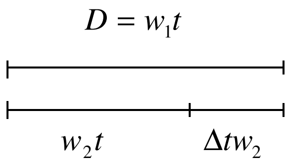
$$\Leftrightarrow \quad \log\left|\frac{w+c}{w-c}\right| + 2\log\left|\frac{kf_0}{c + \sqrt{c^2 + kf_0{}^2}}\right| = \log R \quad \text{e} \quad \sqrt{c^2 + kf_0{}^2} \approx c$$

$$\Leftrightarrow \quad \frac{w+c}{w-c} = R\frac{4c^2}{k^2 f_0{}^2} \quad \text{and} \quad \frac{4c^2 R}{k^2} = F^2 \quad \Leftrightarrow$$

$$\Leftrightarrow \quad w = c\frac{F^2 + f_0{}^2}{F^2 - f_0{}^2}$$

This is the exact formula of the light speed in the interstellar medium. Notice that the speed w and the frequency f_0 are measured in different frames.

Time shift between two frequencies



$$\Delta t = D\frac{w_1 - w_2}{w_1 w_2}$$

$$w_1 = c\frac{F^2 + f_1^2}{F^2 - f_1^2} \quad ; \quad w_2 = c\frac{F^2 + f_2^2}{F^2 - f_2^2} \quad \Leftrightarrow$$

$$\Delta t = \frac{2DF^2}{c}\frac{f_1^2 - f_2^2}{(F^2 + f_1^2)(F^2 + f_2^2)}$$

This is the exact formula of the dispersion in the interstellar vacuum.
It's easy to verify that this formula is equivalent to the formula actually used:

$$F^2 << f_1^2 \quad \text{and} \quad F^2 << f_2^2 \quad \Leftrightarrow \quad \Delta t = \frac{2DF^2}{c}\frac{f_1^2 - f_2^2}{f_1^2 f_2^2}$$

If F is the cold electron plasma frequency of the medium:

$$F^2 = \frac{n_e q_e^2}{m_e \varepsilon_0}$$

q_e -- electron charge ; m_e -- electron mass ; ε_0 -- vacuum permittivity ;
n_e -- number of electrons per volume.

$$\Delta t = 2.12\times 10^{-5}\ \frac{f_1^2 - f_2^2}{f_1 2 f_2^2} D n_e \qquad \text{(SI units)}$$

The usual formula is:

$$\Delta t = 4.15\times 10^{3}\frac{f_1^2 - f_2^2}{f_1^2 f_2^2} DM \qquad \text{(not SI)}$$

But the exact formula that we have derived allows the direct calculation of the distance D if we do two different measures with two pairs of frequencies.

Calculation of the distance

$$\begin{cases} \Delta t_A = \dfrac{2DF^2}{c} \dfrac{f_1^2 - f_2^2}{(F^2 + f_1^2)(F^2 + f_2^2)} \\ \Delta t_B = \dfrac{2DF^2}{c} \dfrac{f_3^2 - f_4^2}{(F^2 + f_3^2)(F^2 + f_4^2)} \end{cases} \Leftrightarrow$$

$$a = \frac{\Delta t_B (f_1^2 - f_2^2)}{\Delta t_A (f_3^2 - f_4^2)}$$

$$F^2 = \frac{a(f_3^2 + f_4^2) - (f_1^2 + f_2^2) - \sqrt{(f_1^2 + f_2^2 - a(f_3^2 + f_4^2))^2 - 4(1-a)(f_1^2 f_2^2 - a f_3^2 f_4^2)}}{2(1-a)}$$

$$D = \frac{\Delta t_A c}{2F^2} \frac{(F^2 + f_1^2)(F^2 + f_2^2)}{f_1^2 - f_2^2}$$

Thus, it's possible to measure the distance of a pulsar, or any other object with variable emission, by direct observation. Practically the accurate calculation takes some limits of the values of frequencies and bandwidths. (For Vela pulsar $F = 1.95 \times 10^3$)