Fractal Method for Forecasting Solar Events Data and the Like

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Abstract: With the fractal method (including the rescaled range analysis (R/S analysis) method), some important questions for forecasting solar events data and the like are discussed. Two forecasting manners are proposed: single-value-forecasting (according to the given data to forecast the next one datum only), and multiple-value-forecasting (according to the given data to forecast the next several data). The examples for forecasting sunspot number, 2800MHz solar flux, cosmic ray index, typhoon track, and stock index are presented. As forecasting the cosmic ray index, the concept of “warning interval” is presented to cooperate with the concept of “confidence limit”. Finally the improved Newton’s formula of universal gravitation with the form of variable dimension fractal is briefly introduced to calculate (forecast) the gravitation in solar system.

Key words: Fractal method, variable dimension fractal, rescaled range analysis (R/S analysis), forecasting, solar events data

1 Introduction

As well-known, it is a very important problem to forecast the solar events and the like. For solving this problem, based on some published papers [1-9], this paper discusses how to forecast the solar events data and the like with the fractal method (including the rescaled range analysis (R/S analysis) method),

2 Constant and Variable Dimension Fractal Prediction Methods

Recently, fractal method has been successfully used in many fields, and it is used for finding out the deeply hidden organized structure in complex phenomena. The quantity for reflecting the character of organized structure is called the fractal dimension, expressed with the value of D. In the fractal methods for general application at present, the fractal dimension D is a constant. For example the values of fractal dimension D for different coastlines may be taken as 1.02, 1.25 and so on. The fractal model[1] reads

$$N = \frac{C}{r^D} \qquad (1)$$

where r is the characteristic scale, such as time, length, coordinates and so on; N is the object number or quantity related with the value of r, such as sunspot number, solar flux, cosmic ray index, price, temperature, the value to be predicted and so on; C is a constant to be determined, D is the fractal dimension.

In the recent application of fractal method, D is the constant, may be called constant dimension fractal. It is a straight line in the double logarithmic coordinates. According to arbitrary two data points (N_i, r_i) and (N_j, r_j) on this straight line, the fractal parameters of this straight line, i.e., the fractal dimension D_{ij} and the constant C_{ij}, can be determined; Substituting the coordinates of the two data points into Eq.(1), they can be solved

$$D_{ij} = \frac{\ln(N_i / N_j)}{\ln(r_j / r_i)} \qquad (2)$$

$$C_{ij} = N_i r_i^{D_{ij}} \qquad (3)$$

For the straight line functional relation in the double logarithmic coordinates, it is able to process prediction and calculation with the constant dimension fractal directly.

But for the non-straight line functional relation in the double logarithmic coordinates, it is unable to process the prediction and calculation with the constant dimension fractal. Many questions belong to this situation. In order to overcome this difficulty, the concept of variable dimension fractal in references [2] ~ [4] is introduced, namely the fractal dimension D is the function of characteristic scale r .

$$D = F(r) \qquad (4)$$

Now how to carry on prediction and calculation with this fractal model is to be discussed.

For the sake of convenience, let r denote the serial number of time, it will stipulate some year for the first year, then $r_1 = 1$, for the second year, $r_2 = 2$, and so on. Let N denote the given value and the value to be predicted, for example, taking N_1 as the value of the first year, N_2 as the value of the second year, and so on.

Now supposing that n data points are given, the values for the first year to the n^{th} year are known, thereupon the question becomes how to predict the values for the $(n+1)^{th}$ year, $(n+2)^{th}$ year and so on.

As a result of the n^{th} data point, namely the values of N_n and r_n for the n^{th} year are given $(r_n = n)$, and the value of r_{n+1} for $(n+1)^{th}$ year is also known $(r_{n+1} = n+1)$, if the fractal dimension $D_{n,n+1}$ of the constant dimension fractal decided by the n^{th} data point and $(n+1)^{th}$ data point is known, then the value for the $(n+1)^{th}$ year can be solved from Eq.(2)

$$N_{n+1} = N_n \left(\frac{r_n}{r_{n+1}}\right)^{D_{n,n+1}} \qquad (5)$$

To this analogizes, the values for the $(n+2)^{th}$ year and the like can be solved.

As for how to decide the fractal dimension $D_{n,n+1}$, it needs the information given by D_{12} (decided by the given first data point and second data point), $D_{23}\cdots D_{n-1,n}$ (decided by other given data points). But in general case, it is very difficult to discover the changing rule for these values of fractal dimension.

In this case, the above method cannot be used directly. The transformation of accumulated sum for the given values have to be carried on firstly, then the above method can be used to forecast the values of accumulated sum for the $(n+1)^{th}$ year, $(n+2)^{th}$ year and so on. Finally the values to be predicted are solved by the values of accumulated sum.

The advantage for using accumulated sum is that a sequence with increasing and decreasing can be changed into a monotone increasing sequence.

This method may be introduced as follows.

The first step, plotting the original data points $(N_i, r_i)(i=1\sim n)$ in the double logarithmic coordinates. In the ordinary circumstances they cannot fairly agree with a constant dimension fractal model, $N_i\ (i=1,2\cdots n)$ may be arranged to a fundamental sequence, namely it can be written as

$$\{N_i\}=\{N_1,N_2,N_3\cdots\}\quad (i=1,2\cdots n)$$

Other sequences may be constructed according to the fundamental sequence. For example, for $S^{(1)}$, i.e., the sequence of first order accumulated sum, $S_1^{(1)}=N_1$, $S_2^{(1)}=N_1+N_2$, $S_3^{(1)}=N_1+N_2+N_3$, …; according to analogize, the sequence of second order accumulated sum, the sequence of third order accumulated sum, and the like can be constructed, namely it can be written as

$$\{S_i^{(1)}\}=\{N_1,N_1+N_2,N_1+N_2+N_3,\cdots\}\quad (i=1,2\cdots n) \tag{6}$$

$$\{S_i^{(2)}\}=\{S_1^{(1)},S_1^{(1)}+S_2^{(1)},S_1^{(1)}+S_2^{(1)}+S_3^{(1)}\cdots\}\quad (i=1,2\cdots n) \tag{7}$$

$$\{S_i^{(3)}\}=\{S_1^{(2)},S_1^{(2)}+S_2^{(2)},S_1^{(2)}+S_2^{(2)}+S_3^{(2)}\cdots\}\quad (i=1,2\cdots n)$$

$$\{S_i^{(4)}\}=\{S_1^{(3)},S_1^{(3)}+S_2^{(3)},S_1^{(3)}+S_2^{(3)}+S_3^{(3)}\cdots\}\quad (i=1,2\cdots n)$$

It needs to point out that $S_i^{(2)}$ denote second order accumulated sum, instead of the second power of S_i. $S_i^{(3)}$ and the like should be comprehended similarly.

The second step, establishing the fractal models for various order accumulated sum. Taking the second order accumulated sum as an example. Plotting the data points $(S_i^{(2)}, r_i)(i=1\sim n)$ in the double logarithmic coordinates, linking these points one by one，it may result in the sectioned constant dimension fractal model. For example, according to n data points, the sectioned constant dimension fractal model composed from $n-1$ straight lines (for different straight line, its fractal dimension is also different, this also is the simplest variable dimension fractal model), and the fractal parameters $D_{ij}^{(2)}, (i = 1 \sim n-1, j = i+1)$ and $C_{ij}^{(2)}$ for each straight line can be calculated according to Eq.(2) and Eq. (3) (in which the value of N_i is replaced by $S_i^{(2)}$) Which means

$$D_{ij}^{(2)} = \ln(S_i^{(2)} / S_j^{(2)}) / \ln(r_j / r_i) \qquad (8)$$

$$C_{ij}^{(2)} = S_i^{(2)} r_i^{D_{ij}^{(2)}} \qquad (9)$$

The third step, choosing the best transformation and determining its corresponding fractal parameters. Separately drawing various order accumulated sum's data points in the double logarithmic coordinates, then choosing the best transformation (its values of fractal dimension are even increased or even decreased) and determining its corresponding fractal parameters. Because in the ordinary circumstances, the second order accumulated sum is the best, the case of second order accumulated sum will be discussed only.

After choosing the fractal model, the suitable method should be used for deciding the fractal dimension $D_{n,n+1}^{(2)}$ firstly, then uses the reconstructive Eq.(5) to carry on the forecast for accumulated sum. Because the values of fractal dimension are evenly increased or evenly decreased, using the following linear interpolation formula can solve the fractal dimension $D_{n,n+1}^{(2)}$

$$D_{n,n+1}^{(2)} = 2D_{n-1,n}^{(2)} - D_{n-2,n-1}^{(2)} \qquad (10)$$

For the second order accumulated sum, Eq.(5) can be expressed by

$$S_{n+1}^{(2)} = S_n^{(2)} \left(\frac{r_n}{r_{n+1}}\right)^{D_{n,n+1}^{(2)}} \qquad (11)$$

For the reason that $S_1^{(1)} \sim S_n^{(1)}$, $S_1^{(2)} \sim S_n^{(2)}$ are already calculated, then the forecasting first order accumulated sum can be obtained from the forecasted second order accumulated sum, which means

$$S_{n+1}^{(1)} = S_{n+1}^{(2)} - S_n^{(2)} \qquad (12)$$

Then the forecasting value can be obtained from the forecasted first order accumulated sum, which means

$$N_{n+1} = S_{n+1}^{(1)} - S_n^{(1)} \qquad (13)$$

According to analogize similarly, N_{n+2}, N_{n+3} and so on can be obtained.

This forecasting manner is named for multiple-value-forecasting, because according to n given data $(N_1, N_2, N_3 \cdots N_n)$, the next several data（$N_{n+1}$，$N_{n+2}$，$N_{n+3}$，and so on）are forecasted.

According to the practice, the multiple-value-forecasting is only suitable for the cases that the data are on the increase or decrease, otherwise other manners should be used.

Another forecasting manner proposed by this paper is named for single-value-forecasting, because according to n given data $(N_1, N_2, N_3 \cdots N_n)$, only the next one datum N_{n+1} is forecasted. After the real value of N_{n+1} is given, taking the real values of $(N_1, N_2, N_3 \cdots N_{n+1})$ as the given data to forecast the next one datum N_{n+2}. To this analogizes, N_{n+3} and the like can be forecasted one by one.

In order to compare with the fractal method, the following linear forecasting method is used.

$$N_{n+1} = 2N_n - N_{n-1} \qquad (14)$$

3 Confidence limit

Suppose that, according single-value-forecasting or multiple-value-forecasting, n data $(Y_1', Y_2', Y_3' \cdots Y_n')$ are forecasted already, the corresponding real values are $(Y_1, Y_2, Y_3 \cdots Y_n)$, then for the $(n+1)^{\text{th}}$ forecasting value Y_{n+1}', the confidence limit corresponding to $(1-\alpha)\times 100\%$ is as follows

$$Y_{n+1}' \pm t_\alpha S_Y \qquad (15)$$

where：$S_Y = \sqrt{\dfrac{\sum (Y - Y')^2}{n-2}}$

In this paper, the value of α will be taken as $\alpha = 0.05$.

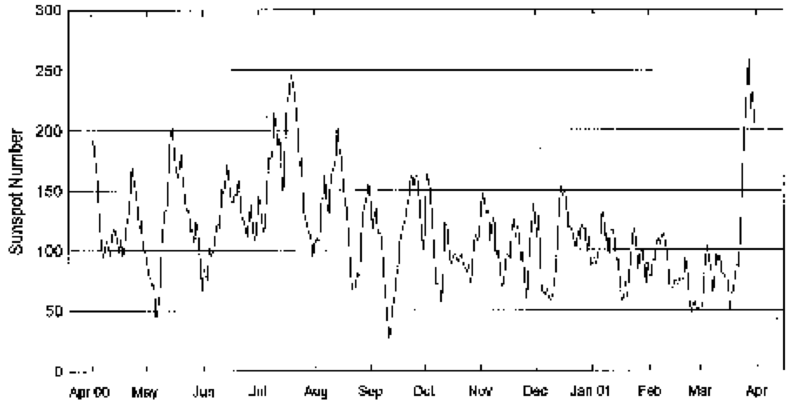
3 Examples of single-value-forecasting

Example 1, according to the sunspot numbers from April 1 to 20, 2000, to forecast the sunspot numbers from April 21 to June 9, 2000 with single-value-forecasting.

The given data are shown in the following table (downloaded from：http://www.ngdc.noaa.gov/ngdc.html， http://sgd.ngdc.noaa.gov/sgd/jsp/solarfront.jsp).

International Relative Sunspot Numbers
Apr 2000 - Mar 2001

27
Mar 01



Day	Apr 00	May	Jun	Jul	Aug	Sep	Oct*	Nov*	Dec*	Jan 01*	Feb*	Mar*
1	187	91	65	145	106	142	115	135	124	89	78	52
2	193	80	79	141	110	118	164	147	109	94	78	53
3	177	76	75	124	107	128	159	141	128	86	92	75
4	164	71	101	114	110	134	150	130	66	96	91	92
5	129	71	95	127	144	114	128	133	65	101	105	104
6	108	42	99	154	143	114	97	108	63	130	110	91
7	94	52	105	177	164	110	71	122	68	131	105	85
8	100	64	120	177	140	85	72	127	61	105	111	63
9	108	99	122	179	126	55	71	95	58	115	114	79
10	102	120	119	215	154	42	57	101	62	95	105	97
11	96	133	151	202	165	26	82	90	72	115	100	90
12	113	133	147	186	170	35	122	72	89	117	71	95
13	118	181	156	194	176	63	121	70	114	111	71	80
14	114	193	171	164	204	60	104	84	135	92	68	80
15	105	205	158	148	183	77	89	98	153	92	75	75
16	98	189	142	197	178	85	92	95	145	75	73	75
17	110	170	139	224	152	108	97	94	151	59	71	51
18	94	161	147	228	140	112	95	116	136	60	76	65
19	103	167	145	246	133	121	90	125	118	73	75	66
20	121	180	159	241	106	124	94	110	120	61	76	80
21	128	163	147	231	77	137	97	120	116	81	94	88
22	145	143	127	216	67	142	89	113	107	93	81	65
23	170	132	124	199	67	160	85	91	102	112	59	113
24	160	134	119	171	77	163	82	98	115	118	48	149
25	151	115	111	177	81	153	86	74	108	106	56	186
26	136	117	129	133	79	161	73	58	121	84	58	218
27	118	106	138	126	113	162	80	79	118	97	50	241
28	124	124	115	120	132	142	106	106	118	102	51	258
29	100	117	109	113	138	119	113	123	99	90		218
30	100	93	114	112	144	100	108	138	111	70		231
31		67		93	157		111		87	93		205
Mean	125.5	121.6	124.9	170.1	130.5	109.7	100.1	106.5	104.5	95.1	80.1	114.2

* = Provisional.

Take April 1, 2000 as the first day and let $r_1 = 1$, that day's sunspot number is N_1, and $N_1 = 187$. According to analogize similarly, for April 20, 2000, $r_{20} = 20$, $N_{20} = 121$.

According to the given sunspot numbers from April 1 to 20, 2000, with the Eqs. (10) - (13) and single-value-forecasting, all the forecasting results are shown in Table 1.

Table 1 Forecasting results for the sunspot numbers from April 21 to June 9, 2000

No.	Month	day	real value	FM*	FM error	LM**	LM error
1	4	21	128	124	-4	139	11
2		22	145	132	-13	135	-10
3		23	170	151	-19	162	-8
4		24	160	180	20	195	35
5		25	151	167	16	150	-1
6		26	136	151	20	142	6
7		27	118	138	20	121	3
8		28	124	117	-7	100	-24
9		29	100	124	24	130	30
10		30	100	97	-3	76	-24
11	5	1	91	97	6	100	9
12		2	80	88	8	82	2
13		3	76	76	0	69	-7
14		4	71	72	1	72	1
15		5	71	67	-4	66	-5
16		6	42	67	25	71	29
17		7	52	36	-16	13	-39
18		8	64	48	-16	62	2
19		9	99	61	-38	76	-23
20		10	120	99	-21	134	14
21		11	133	122	-11	141	8
22		12	133	136	3	146	13
23		13	161	136	-25	133	-28
24		14	193	160	-27	189	-4
25		15	205	200	-5	225	20
26		16	189	213	24	217	28
27		17	170	195	25	179	3
28		18	161	174	13	151	-10
29		19	167	164	-3	152	-15
30		20	180	171	-9	173	-7
31		21	163	185	22	193	30
32		22	143	166	23	146	3
33		23	132	144	12	123	-9
34		24	134	132	-2	121	-13
35		25	115	134	19	136	21
36		26	117	114	-3	96	-21
37		27	106	116	10	119	13
38		28	124	104	-20	95	-29
39		29	117	124	7	142	25
40		30	93	116	23	110	17
41		31	67	91	24	69	2
42	6	1	85	63	-22	41	-44

43	**2**	**79**	**83**	**4**	**103**	**24**
44	**3**	**75**	**76**	**1**	**73**	**-2**
45	**4**	**101**	**72**	**-29**	**71**	**-30**
46	**5**	**95**	**100**	**5**	**127**	**30**
47	**6**	**99**	**94**	**-5**	**89**	**-10**
48	**7**	**105**	**98**	**-7**	**103**	**-2**
49	**8**	**120**	**104**	**-16**	**111**	**-9**
50	**9**	**122**	**120**	**-2**	**135**	**13**

***FM = Fractal forecasting method**

****LM = Linear forecasting method**

From the results of Table 1 it is known that in the above continual 50 days' predictions given by fractal forecasting method, there are 22 days that the error less than 10, the proportion is 44%; 14 days the error is greater than or equal to 11, and less than or equal to 20, the proportion is 28%; 13 days the error is greater than or equal to 21, and less than or equal to 30, the proportion is 26%; only 1 day the error greater than 31, the proportion is 2%. The maximum error is equal to 38.

While for the results given by linear forecasting method, there are 23 days that the error less than 10, the proportion is 46%; 9 days the error is greater than or equal to 11, and less than or equal to 20, the proportion is 18%; 15 days the error is greater than or equal to 21, and less than or equal to 30, the proportion is 30%; 3 days the error greater than 31, the proportion is 6%. The maximum error is equal to 44.

The results of confidence limit of fractal forecasting method are as follows.

For April 30, the forecasting value is equal to 97 (the real value is also equal to 97), $S_Y = 19.26$， $t_{0.05}(7) = 2.365$， according to Eq.(15), it gives

Confidence limit：97 ± 45.5

For May 10, the forecasting value is equal to 99 (the real value is equal to 120), $S_Y = 17.71$， $t_{0.05}(17) = 2.110$， according to Eq.(15), it gives

Confidence limit：99 ± 37.0

For May 20, the forecasting value is equal to 171 (the real value is equal to 180), $S_Y = 17.94$， $t_{0.05}(27) = 2.052$， according to Eq.(15), it gives

Confidence limit：171 ± 36.8

For May 30, the forecasting value is equal to 116 (the real value is equal to 93), $S_Y = 17.12$， $t_{0.05}(37) = 2.028$， according to Eq.(15), it gives

Confidence limit：116 ± 34.7

For June 9, the forecasting value is equal to 120 (the real value is equal to 122), $S_Y = 17.01$， $t_{0.05}(47) = 2.014$， according to Eq.(15), it gives

Confidence limit：120 ± 34.3

It should be noted that, in this example, the time interval is equal to one day, therefore the sunspot number of the next day can be forecasted only. If the time interval is equal to one week, one month, one year and the like, then the sunspot number of the next week, next month, next year and the like can be forecasted.

For example, the time interval is equal to one month, if the given data are as follows

total number in Jan., total number in Feb., ⋯ total number in Nov.

Then the total number in Dec. can be forecasted.

While, for other case, if the given data are selected as follows

number of Jan. 15, number of Feb. 15, ⋯ number of Nov. 15

Then the number of Dec. 15 can be forecasted one month ahead of time.

Example 2, according to the Penticton 2800MHz solar flux from April 1 to 20, 2000, to forecast the Penticton 2800MHz solar flux from April 21 to June 9, 2000 with single-value-forecasting.

The given data are also downloaded from: http://www.ngdc.noaa.gov/ngdc.html, http://sgd.ngdc.noaa.gov/sgd/jsp/solarfront.jsp .

Similar to Example 1, the forecasting results are shown in Table 2.

Table 2 Forecasting results for the 2800MHz solar flux from April 21 to June 9, 2000

No.	Month	day	real value	FM*	FM error
1	4	21	189.2	185.7	-3.5
2		22	204.1	193.3	-10.8
3		23	208.4	210.1	1.7
4		24	208.1	214.4	-6.3
5		25	205.1	213.3	-8.2
6		26	192.4	209.3	16.9
7		27	186.0	194.2	8.2
8		28	186.0	186.7	0.7
9		29	177.5	186.6	9.1
10		30	172.0	177.0	5.0
11	5	1	160.1	170.8	10.7
12		2	155.3	157.6	2.3
13		3	139.6	152.5	12.9
14		4	136.8	135.4	-1.4
15		5	132.1	132.7	0.6
16		6	129.1	128.6	-1.1
17		7	133.4	125.1	-8.3
18		8	139.6	130.3	-9.3
19		9	152.5	137.3	-15.1
20		10	182.8	151.6	-31.2
21		11	181.3	184.8	3.5
22		12	194.4	183.1	-11.3
23		13	222.0	197.1	-24.9

24		**14**	**237.6**	**227.0**	**-10.6**
25		**15**	**249.9**	**243.7**	**-6.2**
26		**16**	**264.5**	**256.7**	**-7.8**
27		**17**	**268.1**	**272.1**	**4.0**
28		**18**	**258.8**	**275.6**	**16.8**
29		**19**	**260.4**	**265.0**	**4.6**
30		**20**	**251.6**	**266.3**	**14.7**
31		**21**	**238.0**	**256.5**	**18.5**
32		**22**	**220.3**	**241.5**	**21.2**
33		**23**	**209.5**	**222.1**	**12.6**
34		**24**	**194.3**	**210.4**	**16.1**
35		**25**	**177.4**	**193.9**	**16.5**
36		**26**	**172.4**	**175.7**	**3.3**
37		**27**	**166.2**	**170.6**	**4.4**
38		**28**	**160.2**	**163.8**	**3.6**
39		**29**	**153.1**	**157.8**	**4.7**
40		**30**	**150.5**	**150.3**	**-0.2**
41		**31**	**158.7**	**147.7**	**-11.0**
42	**6**	**1**	**152.3**	**156.6**	**4.3**
43		**2**	**192.7**	**150.0**	**-42.7**
44		**3**	**170.7**	**193.0**	**22.3**
45		**4**	**174.7**	**169.7**	**-5.0**
46		**5**	**176.1**	**174.1**	**-2.0**
47		**6**	**192.0**	**175.4**	**-16.6**
48		**7**	**185.8**	**193.3**	**6.5**
49		**8**	**179.9**	**185.8**	**5.9**
50		**9**	**174.1**	**179.6**	**5.5**

***FM = Fractal forecasting method**

From the results of Table 2 it is known that in the above continual 50 days' predictions given by fractal forecasting method, there are 30 days that the error less than 10, the proportion is 60%; 15 days the error is greater than 10, and less than or equal to 20, the proportion is 30%; 3 days the error is greater than 20, and less than or equal to 30, the proportion is 6%; only 2 days the error greater than 30, the proportion is 4%. The maximum error is equal to 42.7.

While for the results given by linear forecasting method, the maximum error is equal to 62.4.

The results of confidence limit of fractal forecasting method are as follows.

For April 30, the forecasting value is equal to 177.0 (the real value is equal to 172.0), $S_Y = 9.82$, $t_{0.05}(7) = 2.365$, according to Eq.(15), it gives

Confidence limit：177.0 ± 23.2

For May 10, the forecasting value is equal to 151.6 (the real value is equal to 182.8), $S_Y = 9.00$, $t_{0.05}(17) = 2.110$, according to Eq.(15), it gives

Confidence limit: 151.6 ± 19.0.

For May 20, the forecasting value is equal to 266.3 (the real value is equal to 251.6), $S_Y = 11.60$, $t_{0.05}(27) = 2.052$, according to Eq.(15), it gives

Confidence limit: 266.3 ± 23.8.

For May 30, the forecasting value is equal to 150.3 (the real value is equal to 150.5), $S_Y = 12.08$, $t_{0.05}(37) = 2.028$, according to Eq.(15), it gives

Confidence limit: 150.3 ± 24.5.

For June 9, the forecasting value is equal to 179.6 (the real value is equal to 174.1), $S_Y = 13.24$, $t_{0.05}(47) = 2.014$, according to Eq.(15), it gives

Confidence limit: 179.6 ± 26.7.

The concept of "warning interval" presented in this paper is as follows.

As well-known, the concept of "confidence limit" has already obtained the widespread application. However it has a minor defect in something otherwise perfect that, the existing biggest forecasting error isn't emphasized. For this reason, the concept of "warning interval" is used to cooperate with the concept of "confidence limit", and its upper limit or lower limit may be closer to the real value.

Supposing the $(n+1)^{th}$ forecasting value is N_{n+1}, while within the n forecasting values before N_{n+1}, the biggest forecasting error is E_{max}, then the "warning interval" for the $(n+1)^{th}$ forecasting value is as follows

$$N_{n+1} \pm E_{max}$$

Example 3, according to the THULE cosmic ray indices from March 1 to 19, 2001, to forecast the THULE cosmic ray indices from March 20 to 31, 2001 with single-value-forecasting.

The given data are also downloaded from: http://www.ngdc.noaa.gov/ngdc.html, http://sgd.ngdc.noaa.gov/sgd/jsp/solarfront.jsp .

All the forecasting results are shown in Table 3.

Table 3 Forecasting results for the cosmic ray indices from March 20 to 31, 2001

No.	Month	day	real value	FM*	FM error	warning interval	confidence limit
1	3	20	4079.7	4172.9	93.2		
2		21	4094.7	4069.0	-25.7	4069.0 ± 93.2	
3		22	4136.1	4089.4	-46.7	4089.4 ± 93.2	
4		23	4146.5	4139.3	-7.2	4139.3 ± 93.2	4139.3 ± 1304.9

5	**24**	**4140.0**	**4151.7**	**1,7**	**4151.7 ± 93.2**	**4151.7 ± 314.6**
6	**25**	**4147.1**	**4143.8**	**-3.3**	**4143.8 ± 93.2**	**4143.8 ± 191.8**
7	**26**	**4156.5**	**4152.5**	**-4.0**	**4152.5 ± 93.2**	**4152.5 ± 145.4**
8	**27**	**4058.4**	**4162.7**	**104.3**	**4162.7 ± 93.2**	**4162.7 ± 120.5**
9	**28**	**4001.3**	**4050.0**	**48.7**	**4050.0 ± 104.3**	**4050.0 ± 148.8**
10	**29**	**4003.7**	**3986.9**	**-16.8**	**3986.9 ± 104.3**	**3986.9 ± 133.2**
11	**30**	**4066.0**	**3991.3**	**-74.7**	**3991.3 ± 104.3**	**3991.3 ± 125.4**
12	**31**	**3975.2**	**4063.7**	**88.5**	**4063.7 ± 104.3**	**4063.7 ± 126.6**

***FM = Fractal forecasting method**

From the results of Table 3 it is known that in the above continual 12 days' predictions given by fractal forecasting method, there are 4 days that the error less than 10, the proportion is 33%; 1 day the error is greater than 10, and less than or equal to 20, the proportion is 8%; 1 day the error is greater than 20, and less than or equal to 30, the proportion is 8%; 5 days the error greater than 30, the proportion is 42%. The maximum error is equal to 104.3.

While for the results given by linear forecasting method, the maximum error is equal to 153.1.

From the results of Table 3 it is also known that for this example, all the upper limits or lower limits of "warning interval" are closer to the real values than that of "confidence limit".

Example 4, forecasting the stock index with single-value-forecasting.

From November 1, 2000 to February 7, 2001, the program "Daily Finance and Economics" of Beijing wired television station conducted the competition of stock index prediction. Before 13 o'clock of every business day, the participants were requested to deliver their predictions for the closing index of the same day to the television station by telephone, 2 winners of the first prize (one for Shanghai market, another for Shenzhen market), 8 winners of the second prize and 10 winners of the third prize were awarded every day. We obtained the news on November 17 and began to participate. Until the competition end on February 7, 2001, we won the first prize twice (one for Shanghai market, another for Shenzhen market), the second prize twice and the third prize seven times.

In this example, the time interval is equal to two hours, the forecasting results of the stock index of Shanghai market are as follows.

Table 4 Prediction results of the stock index of Shanghai market（1A0001）

No.	**Date**	**Prediction Value**	**Real value**	**Error**	**Award**
1	**Nov. 16, 2000**	**2087.87**	**2095.98**	**-8.11**	
2	**Nov. 17, 2000**	**2104.99**	**2093.23**	**11.76**	
3	**Nov. 20, 2000**	**2103.48**	**2101.38**	**2.10**	**Third prize**
4	**Nov. 21, 2000**	**2115.14**	**2097.98**	**17.16**	
5	**Nov. 22, 2000**	**2109.29**	**2113.30**	**-4.01**	
6	**Nov. 23, 2000**	**2125.61**	**2119.43**	**6.18**	
7	**Nov. 24, 2000**	**2131.43**	**2053.37**	**78.06**	

8	**Nov. 27, 2000**	**2048.50**	**2049.67**	**-1.17**	**Third prize**
9	**Nov. 28, 2000**	**2071.23**	**2079.39**	**-8.16**	
10	**Nov. 29, 2000**	**2082.63**	**2067.49**	**15.14**	
11	**Nov. 30, 2000**	**2063.54**	**2070.61**	**-7.07**	
12	**Dec. 1, 2000**	**2082.96**	**2081.84**	**1.12**	
13	**Dec. 4, 2000**	**2092.32**	**2092.13**	**0.19**	**Second prize**
14	**Dec. 5, 2000**	**2099.49**	**2091.66**	**7.83**	
15	**Dec. 6, 2000**	**2095.93**	**2075.62**	**20.31**	
16	**Dec. 7, 2000**	**2065.51**	**2075.04**	**-9.53**	
17	**Dec. 8, 2000**	**2085.09**	**2073.16**	**11.93**	
18	**Dec. 11, 2000**	**2044.21**	**2046.07**	**-1.86**	**Third prize**
19	**Dec. 12, 2000**	**2047.74**	**2059.05**	**-11.31**	
20	**Dec. 13, 2000**	**2057.62**	**2056.12**	**1.50**	
21	**Dec. 14, 2000**	**2055.93**	**2051.07**	**4.86**	
22	**Dec. 15, 2000**	**2041.31**	**2039.36**	**1.95**	
23	**Dec. 18, 2000**	**2026.44**	**2044.54**	**-18.10**	
24	**Dec. 19, 2000**	**2052.43**	**2049.03**	**3.40**	
25	**Dec. 20, 2000**	**2058.43**	**2071.26**	**-12.83**	
26	**Dec. 21, 2000**	**2084.98**	**2076.89**	**8.09**	
27	**Dec. 22, 2000**	**2079.10**	**2069.77**	**9.33**	
28	**Dec. 25, 2000**	**2071.63**	**2068.17**	**3.46**	
29	**Dec. 26, 2000**	**2075.03**	**2076.26**	**-1.23**	
30	**Dec. 27, 2000**	**2070.66**	**2058.24**	**12.42**	
31	**Dec. 28, 2000**	**2057.65**	**2053.70**	**3.95**	
32	**Dec. 29, 2000**	**2070.41**	**2073.47**	**-3.06**	
33	**Jan. 2, 2001**	**2095.00**	**2103.46**	**-8.46**	
34	**Jan. 3, 2001**	**2121.09**	**2123.89**	**-2.80**	
35	**Jan. 4, 2001**	**2123.90**	**2117.40**	**6.50**	
36	**Jan. 5, 2001**	**2125.34**	**2125.30**	**0.04**	**First prize**
37	**Jan. 8, 2001**	**2108.06**	**2102.06**	**6.00**	
38	**Jan. 9, 2001**	**2098.75**	**2101.13**	**-2.38**	
39	**Jan. 10, 2001**	**2120.91**	**2125.61**	**-4.70**	
40	**Jan. 11, 2001**	**2132.74**	**2119.14**	**13.60**	
41	**Jan. 12, 2001**	**2106.41**	**2104.74**	**1.67**	
42	**Jan. 15, 2001**	**2054.82**	**2032.44**	**22.38**	
43	**Jan. 16, 2001**	**2003.01**	**2006.88**	**-3.87**	
44	**Jan. 17, 2001**	**2035.48**	**2034.58**	**0.90**	
45	**Jan. 18, 2001**	**2043.70**	**2043.10**	**0.60**	**Third prize**
46	**Jan. 19, 2001**	**2063.47**	**2065.60**	**-2.13**	
47	**Feb. 5, 2001**	**2036.62**	**2008.03**	**28.59**	
48	**Feb. 6, 2001**	**1960.85**	**1995.31**	**-34.46**	
49	**Feb. 7, 2001**	**1979.34**	**1979.93**	**-0.59**	**Second prize**

In the above continual 49 days' actual predictions, there were 2 days that the error less than 0.5, the proportion is 4%; 5 days the error less than 1.0, the proportion is 10%; 12 days the error less than 2.0, the proportion is 25%; 24 days the error less than 5.0, the proportion is 49%; 35 days the error less than 10.0, the proportion is 71%; and 14 days the error greater than 10.0, the proportion is 29%.

Obviously, this method also may be used to predict the stock price.

4 Application of multiple-value-forecasting

Example 5, until 08 o'clock, July 20, 1980, the tracks of No. 8007 typhoon (JOE) are given in Table 5, with multiple-value-forecasting try to forecast predict its future tracks [5].

Table 5 the given tracks of No. 8007 typhoon

No	me/m–d –h			North latitude/(°)	East longitude /(°)
1	7	16	14	10.0	147.0
2			20	11.0	146.0
3		17	02	12.0	145.0
4			08	12.7	143.8
5			14	12.3	143.2
6			20	12.5	142.0
7		18	02	13.1	140.2
8			08	13.5	138.9
9			14	14.0	137.5
10			20	14.2	136.2
11		19	02	14.3	134.7
12			08	14.7	133.1
13			14	15.0	131.7
14			20	15.2	130.1
15		20	02	15.7	128.1
16			08	16.1	126.7

With the multiple-value-forecasting method, the future latitudes and longitudes may be obtained respectively.

It should be noted that, the typhoon track is a two-dimensional problem, while with fractal method it may be changed into one-dimensional problems, this is another advantage of fractal method.

All the forecasting results of this paper, the real vales and the forecasting results [6] are shown in Table 6 and Table 7.

Table 6 forecasting result for the latitudes of No. 8007 typhoon

No.	Time /m –d –h			Real value	Prediction value of this paper	Prediction value of Ref. [6]
17	7	20	14	16.3	16.4	
18			20	16.4	16.7	17.0
19		21	02	17.1	17.1	
20			08	17.4	17.4	18.2
21			14	18.1	17.8	
22			20	18.7	18.1	19.2
23		22	02	19.1	18.5	
24			08	19.5	18.8	20.0
25			14	20.1	19.2	
26			20	20.2	19.5	
27		23	02	20.4	19.9	
28			08	20.9	20.3	
29			14	20.9	20.6	
30			20	20.5	21.0	

Table 7 forecasting result for the longitudes of No. 8007 typhoon

No	Time /m –d –h			Real value	Prediction value of this paper	Prediction value of Ref. [6]
17	7	20	14	125.3	125.3	
18			20	123.8	123.8	123.6
19		21	02	122.1	122.4	
20			08	120.8	121.0	120.9
21			14	119.0	119.6	
22			20	117.2	118.2	118.4

23	**22**	**02**	**115.3**	**116.8**	
24		**08**	**113.6**	**115.5**	**116.0**
25		**14**	**112.2**	**114.1**	
26		**20**	**110.3**	**112.8**	
27	**23**	**02**	**108.4**	**111.4**	
28		**08**	**106.7**	**110.1**	
29		**14**	**105.3**	**108.9**	
30		**20**	**103.0**	**107.5**	

From the results of Table 6 and Table 7 it is known that although the result of longitudes is not so good, generally speaking the prediction results of this paper are satisfying.

5 Improved Rescaled Range (R/S) Forecasting Method

The energies released by natural disaster, such as fire, earthquake and so on, are huge. If these energies can effectively use, even if a small part, the humanity also will obtain the rich energy. For the prediction to the situation of natural disaster, the improved rescaled range (R/S) analysis presented is effective[7].

H. E. Hurst is a renowned hydrology scientist. He proposed the R/S analysis or the rescaled range analysis method [8]. In which the most important work was the calculation of the Hurst exponent H.

Two new data grouping methods will be adopted to calculate the Hurst exponent H.

The first data grouping method: The number of the data in an interval is increased progressively. For example we may make the first interval to contain the data of 1950–1955 (there are 6 years' data altogether), the second interval contains the data of 1950–1956 (there are 7 years' data altogether), the rest may be deduced by analogy, the last interval contains the data of 1950–2000 (there are 51 years' data altogether). It should be noted that, the Hurst exponent calculated by the data of the first interval is taken as the Hurst exponent of the year 1955, the rest may be deduced by analogy, and the Hurst exponent calculated by the data of the last interval is taken as the Hurst exponent of the year 2000.

The second data grouping method: The number of the data in an interval is fixed. For example we may make the first interval to contain the data of 1950–1955, the second interval contains the data of 1951–1956, the rest may be deduced by analogy, the last interval contains the data of 1995–2000, namely each interval contains 6 years' data altogether. Similar to the first method, the Hurst exponent calculated by the data of any interval is taken as the Hurst exponent of the last year of this interval.

Now the application example of the first data grouping method is presented.

Example 6, according to the fire numbers of China from 1950 to 1999, to forecast the fire number of year 2000.

Firstly calculate the Hurst exponent H from 1955 to 1999. Then according to these Hurst exponents to calculate the Hurst exponent of year 2000 (H_{2000}) with the fractal method, it gives

$$H_{2000} = 0.7750$$

According to this calculated H_{2000}, the fire number of year 2000 may be predicted with the shooting method. Finally it gives that as the fire number of year 2000 is equal to 199960, H_{2000} =0.7750. While the real value is 189185, the predicted error is just 5.7 %.

In order to effectively carry on the analysis and prediction to the statistical data [7], the concept of high order Hurst exponent is also introduced. For all the calculated Hurst exponents, taking these values of H as the ordinary given statistical data, thereupon the R/S analysis to these data may be carried on, thus obtain a group of new Hurst exponents, and name it H_1, which is the Hurst exponent of Hurst exponent. The rest may be deduced by analogy, and may give the higher order Hurst exponents H_2, H_3 and so on. With the help of the Hurst exponent and high order Hurst exponent, whether the next year's fire number will be increased sharply can be judged.

6 Calculate (Forecast) the Gravitation in Solar System According to the Improved Newton's Formula of Universal Gravitation with the Form of Variable Dimension Fractal

In reference [9], the following improved Newton's formula of universal gravitation was presented

$$F = -\frac{GMm}{r^2} - \frac{3G^2M^2mp}{c^2r^4}$$

where: G —gravitational constant; M and m —masses of the two bodies; r —the distance between the two bodies; c —velocity of light; p —half normal chord for body m moving around the body M with a curve, and the value of p reads

$p = a\ (1-e^2)$ (for ellipse)

$p = a\ (e^2-1)$ (for hyperbola)

$p = y^2/2x$ (for parabola)

For the problem of gravitational deflection of photon orbit around the sun and the problem of advance of planet perihelion, by using the improved formula of universal gravitation, the same results as given by general relativity can be reached.

The improved Newton's formula of universal gravitation can be transformed into the form of variable dimension fractal

$$F = -\frac{GMm}{r^D}$$

Then it gives

$$D = 2 - \ln(1 + \frac{3GMp}{c^2r^2}) / \ln r$$

The values of D are different for different problems.

For the problem of gravitational deflection of photon orbit around the sun

$1.954997 \leqslant D \leqslant 2$

For the problem of advance of planet perihelion

Suppose $D=2-\varepsilon$, then it gives: $2.018165 \times 10^{-9} \leqslant \varepsilon \leqslant 4.935239 \times 10^{-9}$

7 Conclusions

This paper discusses some important questions for forecasting solar events data and the like with the fractal method (including the rescaled range analysis (R/S analysis) method). The examples presented in this paper indicate that, these forecasting and calculating methods will possibly have the good application prospect in the future.

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