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Proof that the de Broglie-Einstein velocity equation is valid for the non-relativistic case

Yusuf Z. Umul

Cankaya University, Electronic and Communication Dept., Yüzüncü Yıl, No:14, Balgat, Ankara, Türkiye

yziya@cankaya.edu.tr

Abstract: We prove that the de Broglie-Einstein velocity equation is also valid for the non-relativistic case. The relativistic energy and momentum relations and the wave properties of matter are used for this proof. Such a behavior is important since the resultant equations are far from the contradictions.

Keywords: Special theory of relativity, quantum theory

1. Introduction

The quantum mechanics has its roots on the invention of Planck, who showed that the radiation was discontinuous by solving the problem of black body [1]. The application of the idea was performed by Einstein. He explained the photoelectric effect successfully by using the quantum formula of Planck [2]. Afterwards, de Broglie proposed that a quantum particle, like an electron, could also have the same dualistic nature like radiation [3, 4]. His ideas were supported by the electron diffraction experiments of Davisson and Germer [5]. One of the most important inventions of de Broglie was the derivation of the de Broglie-Einstein velocity equation [6]. This equation gives the relation between the phase and group velocities of a matter wave in terms of the speed of light.

In this note we will further consider an approach that was mentioned in a recent paper [7]. In the related paper, it was shown that there will be some contradictions if the de Broglie-Einstein relation is only valid for the relativistic cases. However a direct proof of the subject is not given in that study. It is the aim of this note to obtain a rigorous proof of the statement that the de Broglie-Einstein velocity equation is also valid for the non-relativistic speeds.

2. Theory

First of all we will begin by considering the relativistic energy and momentum relations of

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v_g^2}{c^2}}} \tag{1}$$

and

$$p = \frac{m_0 v_g}{\sqrt{1 - \frac{v_g^2}{c^2}}} \tag{2}$$

respectively. m_0 is the rest mass of the quantum particle. v_g and c are the group velocity and the speed of light. The concepts of the group and phase velocities will be considered in the same way as de Broglie [3, 4]. The equations of the energy and momentum that are related with the wave nature of matter can be given by

$$E = \hbar w \tag{3}$$

and

$$p = \hbar k \tag{4}$$

for $\hbar$ is the angular Planck's constant [8]. w and k are the angular frequency and the wave-number, respectively. The relations of

$$\hbar w = \frac{m_0 c^2}{\sqrt{1 - \frac{v_g^2}{c^2}}} \tag{5}$$

and

$$\hbar k = \frac{m_0 v_g}{\sqrt{1 - \frac{v_g^2}{c^2}}} \tag{6}$$

can be defined when Eqs. (1)-(4) are taken into account. It is apparent that the ration of the energy to the momentum of a particle gives

$$\frac{E}{p} = \frac{w}{k} = v_p \tag{7}$$

where v_p is the phase velocity. The velocity equation of de Broglie-Einstein can be obtained as

$$v_g v_p = c^2 \tag{8}$$

when Eqs. (5) and (6) are considered. The most important feature of Eq. (8) is its independency from the relativistic effects which occur because of the term of

$$\sqrt{1-\frac{v_g^2}{c^2}}\ . \tag{9}$$

Now we will take into account the non-relativistic case, which is defined by the condition of $v_g << c$ or alternatively $v_g << v_p$ according to Eq. (8). The energy and momentum relations can be written as

$$\hbar w = m_0 c^2 + \frac{m_o v_g^2}{2} \tag{10}$$

and

$$\hbar k = m_0 v_g + \frac{m_o v_g^3}{2c^2} \tag{11}$$

by using the non-relativistic approximation of Eq. (9). At this point we will define the equations of

$$\hbar w_0 = m_0 c^2 \tag{12}$$

and

$$\hbar w_k = \frac{m_o v_g^2}{2} \tag{13}$$

by following the same path with de Broglie [3, 4]. Equation (12) was first proposed by de Broglie. w_k is the kinetic angular frequency. One also can define the equations of

$$\hbar k_0 = m_0 v_g \tag{14}$$

and

$$\hbar k_k = \frac{m_o v_g^3}{2c^2} \qquad (15)$$

from Eq. (11). The kinetic energy of a particle will be considered as in Eq. (13) by omitting the rest energy. This equation can be rewritten as

$$\hbar w_k = \frac{p_0^2}{2m_0} \qquad (16)$$

where p_0 is equal to $m_0 v_g$. The equation of

$$\hbar k_k v_p = \frac{m_0 v_g^2}{2} \qquad (17)$$

can be obtained by taking into account Eq. (7). A further relation can be written as

$$\frac{m_0 v_g^3}{2c^2} v_p = \frac{m_0 v_g^2}{2} \qquad (18)$$

by using Eq. (15) in Eq. (17). Equation (18) directly yields the de Broglie-Einstein velocity relation. In literature the relation of $v_g = 2v_p$ is found from Eqs. (13) and (14) because of the straightforward usage of these equations. In fact the energy term of $\hbar w$ is not directly equal to the kinetic energy as is considered in the literature [6, 8]. w is equal to $w_0 + w_k$ and w_0 can not be neglected near w_k since w_0 is much more greater than w_k. When these points are considered as in Eqs.(16) and (17), it can be seen that the de Brogile-Einstein relation is directly obtained. These points prove that the velocity equation of de Broglie-Einstein is also valid for the non-relativistic speeds.

3. Conclusion

In this study, we showed that the de Broglie-Einstein velocity equation is also valid in the non-relativistic approximation. This result brings out the necessity to reevaluate the equations, which are obtained by using the kinetic energy equation with the momentum in a straightforward manner. An attempt of regenerating differential equations of quantum mechanics in the light of the analysis, put forward in this paper, will lead to a more consistent theory which has a direct connection between the relativistic and non-relativistic cases.

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