

On the Tides' paradox.

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Abstract

In this paper I analyse the paradox of tides, that claims that the Moon and not the Sun is responsible for them, although the Sun's gravitation is much stronger. I correct the official published calculations of tides that I found in a mainstream book and on the internet, which both appear to be wrong, and I come to the paradox that the Moon, although it has a weaker gravitation, is the only origin of the 12-hours cycled tides. The Sun generates 12-hours cycled tides as well, which are weaker, but which can, added to the Moons' tides, cause spring tides. Theories that refer to centrifugal effects, or to the eccentricity of the Moon-Earth's center of mass can indeed not explain any of the described cyclic tides in a satisfactory way, but they play some role in the global flows of seas.

1. The tides-paradox : Sun versus Moon.

Official literature claim that the Moon's gravitation is responsible for the tides. however, the solar gravitational attraction to the Earth is indeed stronger than that of the Moon, as we see below.

Mass of Sun	:	1.99 . 10 ³⁰ kg	Distance Sun-Earth	:	1.496 . 10 ⁸ km
Mass of Moon	:	7.35 . 10 ²² kg	Distance Moon-Earth	:	3.844 . 10 ⁵ km

The gravitational force g is given by :

$$g = \frac{\gamma M}{r^2} \quad (1.1)$$

wherein γ is representing the universal gravitation constant.

Attraction of Sun:	5.93 . 10 ⁻³ m/s ²	Attraction of Moon :	3.32 . 10 ⁻⁵ m/s ²
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Out of this information, it might seem logic that the Sun is responsible for the tides, and that the Moon is not. But we will see below that this reasoning is not correct. Some publications try to explain the situation without calculations, but with illustrations and lots of explanations. They fail explaining it. Some publications that I found explain it with mathematical calculations, but after a detailed analysis, I discovered several important errors in the author's reasoning.

And this is very frustrating for the few who want to learn new things and who, unfortunately, trust trustfulness maths and physics.

2. The officially published tides theory.

The book of D. Pugh, “Tides, Surges and Mean Sea Level” from the Natural Environment Research Council, Swindon, UK, explains the calculations of the tides in the pages 65 to 67.

Exactly the same explanation is written by R. Stewart in http://oceanworld.tamu.edu/resources/ocng_textbook/chapter17/chapter17_04.htm

We reproduce the latter website's explanation below in dark blue colour. To my best knowledge, these pages are not copyrighted.

In the explanation below, the universal gravitation constant is represented by γ . I prefer to warn the reader in advance that all the expressions after (17.8) are wrong. The explanations follow in the next chapter.

Tidal Potential

Tides are calculated from the hydrodynamic equations for a self-gravitating ocean on a rotating, elastic Earth. The driving force is the small change in gravity due to motion of the moon and sun relative to Earth. The small variations in gravity arise from two separate mechanisms. To see how they work, consider the rotation of moon about Earth.

1. Moon and Earth rotate about the center of mass of the Earth-moon system. This gives rise to a centripetal acceleration at Earth's surface that drives water away from the center of mass and toward the side of Earth opposite moon.
2. At the same time, mutual gravitational attraction of mass on Earth and the moon causes water to be attracted toward the moon.

If Earth were an ocean planet with no land, and if the ocean were very deep, the two processes would produce a pair of bulges of water on Earth, one on the side facing the moon, one on the side away from the moon. A clear derivation of the forces is given by Pugh (1987) and by Dietrich, Kalle, Krauss, and Siedler (1980). Here I follow the discussion in Pugh §3.2.

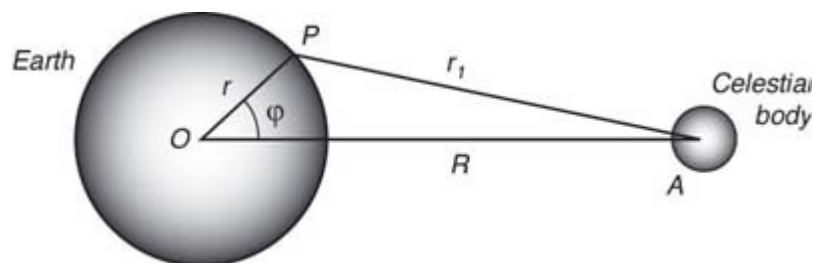


Figure 17.10 Sketch of coordinates for determining the tide-generating potential.

To calculate the amplitude and phase of the tide on an ocean planet, we begin by calculating the forces. The tide-generating potential at Earth's surface is due to the Earth-moon system rotating about a common center of mass. Ignoring for now Earth's rotation, the rotation of moon about Earth produces a potential V_M at any point on Earth's surface

$$V_M = -\frac{\gamma M}{r_1} \quad (17.5)$$

where the geometry is sketched in Figure 17.10, g is the gravitational constant, and M is moon's mass. From the triangle OPA in the figure,

$$r_1^2 = r^2 + R^2 - 2rR \cos \varphi \quad (17.6)$$

Using this in (17.5) gives

$$V_M = -\frac{\gamma M}{R} \left\{ 1 - 2 \left(\frac{r}{R} \right) \cos \varphi + \left(\frac{r}{R} \right)^{1/2} \right\}^{-1/2} \quad (17.7)$$

$r/R \gg 1/60$, and (17.7) may be expanded in powers of r/R using Legendre polynomials (Whittaker and Watson, 1963: §15.1):

$$V_M = -\frac{\gamma M}{R} \left\{ 1 + \left(\frac{r}{R} \right) \cos \varphi + \left(\frac{r}{R} \right)^2 \left(\frac{1}{2} \right) (3 \cos^2 \varphi - 1) + \dots \right\} \quad (17.8)$$

The tidal forces are calculated from the gradient of the potential, so the first term in (17.8) produces no force. The second term produces a constant force parallel to OA. This force keeps Earth in orbit about the center of mass of the Earth-moon system. The third term produces the tides, assuming the higher-order terms can be ignored. The tide-generating potential is therefore:

$$V = -\frac{\gamma M r^2}{2R^3} (3 \cos^2 \varphi - 1) \quad (17.9)$$

The tide-generating force can be decomposed into components perpendicular P and parallel H to the sea surface. The vertical force produces very small changes in the weight of the oceans. It is very small compared to gravity, and it can be ignored. The horizontal component is shown in Figure 17.11. It is:

$$H = -\frac{1}{r} \frac{\partial V}{\partial \varphi} = \frac{2G}{r} \sin 2\varphi \quad (17.10)$$

where

$$G = \frac{3}{4} \gamma M \left(\frac{r^2}{R^3} \right) \quad (17.11)$$

The tidal potential is symmetric about the Earth-moon line, and it produces symmetric bulges.

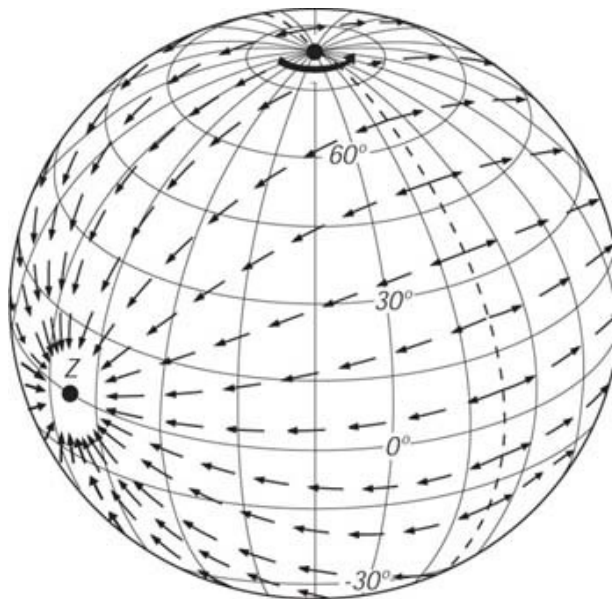


Figure 17.11 The horizontal component of the tidal force on Earth when the tide-generating body is above the Equator at Z. From Dietrich, et al. (1980).

If we allow our ocean-covered Earth to rotate, an observer in space sees the two bulges fixed relative to the Earth-moon line as Earth rotates. To an observer on Earth, the two tidal bulges seems to rotate around Earth because moon appears to move around the sky at nearly one cycle per day. Moon produces high tides every 12 hours and 25.23 minutes on the equator if the moon is above the equator. Notice that high tides are not exactly twice per day because the moon is also rotating around Earth. Of course, the moon is above the equator only twice per lunar month, and this complicates our simple picture of the tides on an ideal ocean-covered Earth. Furthermore, moon's distance from Earth R varies because moon's orbit is elliptical and because the elliptical orbit is not fixed.

Clearly, the calculation of tides is getting more complicated than we might have thought. Before continuing on, we note that the solar tidal forces are derived in a similar way. The relative importance of the sun and moon are nearly the same. Although the sun is much more massive than moon, it is much further away.

$$G_{sun} = G_S = \frac{3}{4}\gamma S \left(\frac{r^2}{R_{sun}^3} \right) \quad (17.12)$$

$$G_{moon} = G_M = \frac{3}{4}\gamma M \left(\frac{r^2}{R_{moon}^3} \right) \quad (17.13)$$

$$\frac{G_S}{G_M} = 0.46051 \quad (17.14)$$

where R_{sun} is the distance to the sun, S is the mass of the sun, R_{moon} is the distance to the moon, and M is the mass of the moon.

3. The results of the official published calculation and the calculation problems.

As we will see below, the officially published calculations on tides appear to be wrong, causing so a total confusion. Let us see what happened.

The definition of the value G is given in (17.11). The result of the publication is the equation (17.14). What is important, is the quotient of the Sun's and the Moon's value of G . The conclusion is that the Sun's influence is lower than that of the Moon by nearly half its value.

But where did the calculations go wrong?

When recalculating the whole chapter "17.4 Theory of Ocean Tides", I come to the following. The equations (17.5) until (17.8) are correct. Let us re-write (17.8) :

$$V_M = -\frac{\gamma M}{R} \left\{ 1 + \left(\frac{r}{R} \right) \cos \varphi + \left(\frac{r}{R} \right)^2 \left(\frac{1}{2} \right) (3 \cos^2 \varphi - 1) + \dots \right\} \quad (17.8)$$

The first error.

The conclusion below (17.8): "The second term produces a constant force parallel to OA." is of course wrong, since the second term contains a term with " $\cos(\varphi)$ ". It is indeed the first term which is generating a constant force parallel to OA, and it is the second term that will have influence on tide-flows, because of its dependence on the angle φ .

Since $r \ll R$, the third term is much smaller than the second term and can be neglected.

Therefore, the contribution of (17.9) is insignificant and consequently, the equations (17.10) until (17.14) are indeed wrong.

That is the reason why I conclude that the tides are generated by this second term and not by the third term.

The second error.

All we have to do is to compare this term for the Sun and the Moon, and come to the right conclusion.

We have to bare in mind that φ and r are constants in this calculation. Only r_1 and R vary. The approach to come to (17.10) is wrong. Let us re-write it down :

wrong equation :

$$H = -\frac{1}{r} \frac{\partial V}{\partial \varphi} = \frac{2G}{r} \sin 2\varphi \quad (17.10)$$

In this equation, the partial derivate of the potential to φ has been taken. However, we should not look for variations of φ because at a certain place φ , which is just an angular location at the Earth's surface, the derivate $\partial \varphi$ just signifies the next infinitesimal part of the Earth's circle. But we have not the intention to integrate over the Earth's circle. Instead, we are looking for the displacement of the water that the gravitational potential will generate, and the force needed to do so. In other words, we search for the gravitational work that the Sun and the Moon generate onto the water.

Moreover, r has wrongly been considered as being constant. Indeed, the Earth's radius is constant, but in (17.10), an attempt was made to describe the force, responsible for the displacement of the water. In that case, r is not the Earth's radius, but represents the location of the water at the initial coordinates (r, φ) . And in that case, the derivation should have been done to $r\varphi$ instead of solely to φ . Since the derivate dr cannot be expressed in terms of other known parameters, we will have to calculate the horizontal forces in another way.

Thus, the second term of (17.8) brings us to the correct force, and the infinitesimal work is given by the force which is exerted over an infinitesimal distance dr_1 . This work corresponds to the infinitesimal change of potential that is given only by the second term of (17.8)).

Let us work this out in the next chapter.

4. How the calculation should be done instead.

We have to bare in mind that the water of the seas can only get in motion when it follows a curve that is parallel to the Earth's surface. Hence, the relevant drawing for this deduction is the one of fig. (4.1).

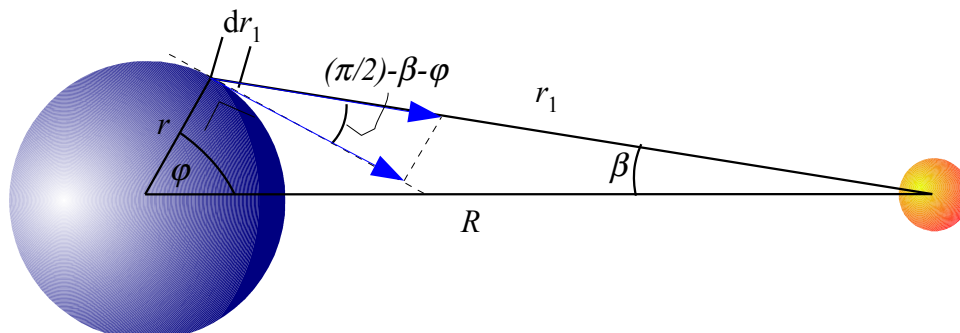


Fig. 4.1 : geometric relationships between Earth (large sphere) and the Moon or the Sun (small sphere).

In the first place, let us look at the fig. 4.1 and write down a goniometric equation: when we derivate (17.6) at both sides and replace r_1 by the expression (17.6), we get :

$$d r_1 = \frac{R - r \cos \varphi}{\sqrt{r^2 + R^2 - 2 r R \cos \varphi}} d R \quad (4.1)$$

For $R \gg r$, which is the case here, (4.1) can be replaced by $d r_1 = d R$. (4.2)

The second term of (17.8).

The second term of (17.8) is :

$$V_2 = - \frac{\gamma M}{R^2} r \cos \varphi \quad (4.3)$$

The work, provided by a displacement of water by a gravitational force causes a change of potential.

The force of the corresponding work is then :

$$F_2 = \frac{d V_2}{d r_1} \quad (4.4)$$

With the help of fig. 4.1, we can easily see that the force which is horizontal to the Earth's surface is then given by :

$$F_{2(H)} = \frac{d V_2}{d r_1} \cos \left(\frac{\pi}{2} - \beta - \varphi \right) = - \frac{d V_2}{d r_1} \sin(\beta + \varphi) \quad (4.5) (4.6)$$

Combining this with (4.2) this brings us to :

$$F_{2(H)} = - \frac{d V_2}{d R} \sin(\beta + \varphi) \quad (4.7)$$

With (4.3), we get :

$$F_{2(H)} = \frac{\gamma M}{R^3} r \cos \varphi \sin(\beta + \varphi) \quad (4.8)$$

The tidal force is inversely related to R^3 . Remark that, as far as we can neglect the angle β , the strongest force occurs at a latitude of 45° , and is oriented towards the intersection of the Earth's equator and the axis Earth-Sun for the Sun's force and the axis Earth-Moon for the Moon's force. This force is axis-symmetric for any φ about this axis, independently from the Earth's rotation axis. Due to the presence of these symmetric bulges at 180° , the tidal cycle is about 12 hours.

The equation (4.8) is valid for the Sun and for the Moon. When comparing these forces, caused by the Moon and by the Sun, the report between both is at any time (if $\beta \ll \varphi$) :

$$\boxed{\frac{M_{Sun} R_{Moon}^3}{M_{Moon} R_{Sun}^3} = 0.46051} \quad (4.9)$$

The conclusion is that, although the officially published reasoning and the calculations are partly wrong, the same result as the published one is found.

5. Discussion and conclusion.

The first term of the Legendre polynomial is constant and is not acting at all on the seas, as far as we do not take in account the spin of the Earth. It is fully compensated by the so-called centrifugal force (which is only a representation of the inertia of mass along a straight line) .

The second term is responsible for the tides, and makes the Moon's influence more important than the Sun's, by a factor of (roughly) two. The tides are cyclic with about 12 hours, due to the symmetric bulges at the opposite sides of the Earth, along the axis Earth-Moon and Earth-Sun. It is clear that the third and the following terms are insignificant to tides.

Very important is the conclusion that, since the first term compensates the centrifugal force, the second term has absolutely nothing to do with centrifugal nor centripetal forces. It does not compensate them, it does not overrule them. The second term just exist as it is. Many publications on tides violate this conclusion.

6. References and other literature.

Wrong theoretical mainstream calculations:

1. Pugh, D., Tides, Surges and Mean Sea Level, *Natural Environment Research Council, Swindon, UK, p. 65-67.*
2. Stewart, R., http://oceanworld.tamu.edu/resources/ocng_textbook/chapter17/chapter17_04.htm

Other literature:

3. De Mees, T., [General insights for the Maxwell Analogy for Gravitation.](#)
[Mercury's perihelion shift and the bending of light grazing the sun.](#)
[Solar-, planetary- and ring-system's dynamics.](#)
[Fast spinning stars' and black holes' dynamics.](#)
[Spherical and disk galaxy's dynamics.](#)