the general science journal

Space created by the Schwarzschild metric

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Abstract

The paper is divided in two parts.

The main concept of the first part is that warping of space is provoked by the particles emitted by the masses, but not because their interactions (not because the energy or momentum of these emitted particles), but instead, due to the space they occupy (they create). The new space occupied/created by these particles warps space.

In the article, it is calculated that the Schwarzschild metric (compared with the Euclidean metric) creates space in a ($1/r^2$) distribution, the same distribution of the particles emitted by the masses. So it is considered the possibility that this new space appears, due to the new space occupied (created) by these emitted particles.

According to this, when a mass increases, it should increase the space occupied by the particles emitted, so the warping is proportional to the mass.

In the second part, it is calculated that the space occupied by the photons emitted by a body is proportional to the mass of the body that emits them. This makes them a possible candidate to be the transmittals of the warping of space. When the mass increases, the space emitted by it should increase in the same proportion (to increase also the warping). In the second part it is shown that the space occupied by the photons emitted by a mass follows this rule. This means, the space occupied by the photons would transmit the space created by the Schwarzschild metric, consequently its warping and consequently the gravitational field. Independently, the energy and momentum of the photons (their interactions) would transmit the electromagnetic field.

1. DISTRIBUTION OF SPACE CREATED BY THE SCHWARZSCHILD METRIC

1.1 Introduction

-First, we will calculate the differential of volume in the Schwarzschild metric and its variation in the radial direction.
-Afterwards, we will calculate the differential of volume in the spherical symmetric Euclidean metric and its variation in the radial direction.
-Then, we will obtain the difference of the variations of volume in both metrics, checking that the Schwarzschild metric indeed does create/increase space. The distribution of space created is the same as whatever distribution of particles emitted by the mass outwards (the type $1/r^2$).
-In the conclusions, it will be commented that this could be understood as if the new space created, was created by particles emitted by the mass (but not by their interactions), only by the space these emitted particles occupy/create.
-Also, it will be mentioned that in the second part, it is proposed a possible candidate to be these particles that occupy/create space.

1.2 Differential of volume in Schwarzschild metric

The Schwarzschild metric is defined by [1][2][3][6]:

$$ds^2 = -\left(1-\frac{2m}{r}\right)dt^2 + \left(\frac{1}{1-\frac{2m}{r}}\right)dr^2 + r^2 sen^2\theta d\vartheta^2 + r^2 d\theta^2 \tag{1}$$

We would remind here that to obtain the Schwarzschild metric [4][5], it is used the weak-field approximation, that approximates to the linearized gravity (it neglects quadratic and higher order elements).

We will obtain the differential of space volume in the Schwarzschild metric. We consider an instant of time $(dt = 0)$. This means, we will work with:

$$ds^2 = \left(\frac{1}{1-\frac{2m}{r}}\right)dr^2 + r^2 sen^2\theta d\vartheta^2 + r^2 d\theta^2 \tag{2}$$

To calculate the differential of volume of the metric (2), we apply definition of differential of volume [6] to the metric tensor g_{ij} (for space coordinates, i,j=1,2,3).

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$$dV_{spaceSwarzschild}=\left|\det\left(g_{ij}\right)_{i,j=1,2,3}\right|^{\frac{1}{2}}drd\vartheta d\theta=\left[\left(\frac{1}{1-\frac{2m}{r}}\right)r^{2}sen^{2}\theta d\vartheta^{2}r^{2}\right]^{\frac{1}{2}}drd\vartheta d\theta=$$

$$=\left[\left(\frac{1}{1-\frac{2m}{r}}\right)r^{4}sen^{2}\theta\right]^{\frac{1}{2}}drd\vartheta d\theta=\left(\frac{1}{1-\frac{2m}{r}}\right)^{\frac{1}{2}}r^{2}drsen\theta d\vartheta d\theta=\left(1-\frac{2m}{r}\right)^{-\frac{1}{2}}r^{2}drsen\theta d\vartheta d\theta \tag{3}$$

1.3 Variation of the differential of volume in Schwarzschild metric with respect to r (in the radial direction

As the Schwarzschild metric is spherical symmetric[1][2][3][6], there will not be any variation depending on θ or ϑ in a shell of constant r. So, we will calculate the variation of the differential of volume with respect to r (in the radial direction), to check how the differential of volume changes depending on r. More precisely, we will calculate the increment of the differential of volume with an infinitesimal increment of r.This is made making the differentiation of (3) with respect to r [11].

$$d^{2}V_{spaceSwarzschild}=\left(-\frac{1}{2}\right)\left(\frac{2m}{r^{2}}\right)\left(1-\frac{2m}{r}\right)^{-\frac{3}{2}}r^{2}(dr)^{2}sen\theta d\vartheta d\theta+$$

$$+\left(1-\frac{2m}{r}\right)^{-\frac{1}{2}}2r(dr)^{2}sen\theta d\vartheta d\theta= \tag{4}$$

$$=(dr)^{2}sen\theta d\vartheta d\theta\left[-m\left(1-\frac{2m}{r}\right)^{-\frac{3}{2}}+2r\left(1-\frac{2m}{r}\right)^{-\frac{1}{2}}\right]$$

There is no element depending on $d^{2}r$ as this element vanishes as you can check in [7].

1.4 Differential of volume in spherical symmetric Euclidean metric

In Euclidean space (spherical coordinates) the metric is defined by[8]:

$$ds^{2}=dr^{2}+r^{2}sen^{2}\theta d\vartheta^{2}+r^{2}d\theta^{2} \tag{5}$$

So, following same steps as for the Schwarzschild metric, we obtain the differential of volume of Euclidean metric (5) using [6]:

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$$dV_{Euclidean}=\left|\det\left(g_{ij}\right)_{i,j=1,2,3}\right|^{\frac{1}{2}} dr d\vartheta d\theta=\left[r^2 sen^2\theta d\vartheta^2 r^2\right]^{\frac{1}{2}} dr d\vartheta d\theta= \tag{6}$$
$$=r^2 dr sen\theta d\vartheta d\theta$$

1.5 Variation of the differential of volume in spherical symmetric Euclidean metric along the r line

As this metric is spherical symmetric [8], there will not be any variation depending on θ or ϑ in a shell of constant r. So, we will calculate the variation of the differential of volume with respect to r (in the radial direction), to check how the differential of volume changes depending on r. More precisely, we will calculate the increment of the differential of volume with an infinitesimal increment of r. This is made making the differentiation of (3) with respect to r [11].

$$d^2V_{Euclidean}=2r(dr)^2 sen\theta d\vartheta d\theta \tag{7}$$

There is no element depending on d^2r as this element vanishes as you can check in [7].

1.6 Difference of the variations of the differential of volume in both metrics

To separate the real increment of space from the apparent increment due to the spherical system of coordinates, we make the subtraction of the variations of the differential of volume in the Schwarzschild metric and in the spherical symmetric Euclidean metric (4)-(7).

$$d^2V_{spaceSwarzschild}-d^2V_{Euclidean}=(dr)^2 sen\theta d\vartheta d\theta\left[-m\left(1-\frac{2m}{r}\right)^{-\frac{3}{2}}+2r\left(1-\frac{2m}{r}\right)^{-\frac{1}{2}}\right]- \tag{8}$$
$$-2r(dr)^2 sen\theta d\vartheta d\theta=(dr)^2 sen\theta d\vartheta d\theta\left[-m\left(1-\frac{2m}{r}\right)^{-\frac{3}{2}}+2r\left[\left(1-\frac{2m}{r}\right)^{-\frac{1}{2}}-1\right]\right]$$

Now, we will use the binomial series [9]. As it is done to obtain the Schwarzschild metric itself, we will make a linearized approximation (we will neglect quadratic elements or higher). This is, we will neglect elements of order higher than $1/r$. More on the validity of this approximation, later.

$$
\begin{aligned}
&d^2V_{spaceSwarzschild} - d^2V_{Euclidean} = \\
&= (dr)^2 sen\theta d\vartheta d\theta \left[-m\left(1-\frac{2m}{r}\right)^{-\frac{3}{2}} + 2r\left[\left(1-\frac{2m}{r}\right)^{-\frac{1}{2}} - 1\right]\right] \approx \\
&\approx (dr)^2 sen\theta d\vartheta d\theta \left[-m\left(1+\frac{3m}{r}\right) + 2r\left(\frac{m}{r} + \frac{3}{2}\frac{m^2}{r^2}\right)\right] = \\
&(dr)^2 sen\theta d\vartheta d\theta \left(-m - \frac{3m^2}{r} + 2m + \frac{3m^2}{r}\right) = m(dr)^2 sen\theta d\vartheta d\theta
\end{aligned}
\tag{9}
$$

As you can check, the terms depending on r cancel. This means, in all the shells of a radius r (independently of the value of r), the space created by the Schwarzschild metric is the same (only depends on m ,not on the distance from the shell to the mass). What this means, will be developed in the conclusions.

To check the accuracy of the approximation used to go from (8) to (9), please check the Appendix I, to check that the error is zero in practically all the cases.

1.7 Conclusions

If you warp an elastic cloth, you increase the surface of the cloth. Interestingly, the opposite is also true. If you try to add more surface to a cloth, it will have to warp to accept this new surface. For example, if you sew a new piece of cloth in a cut performed in the original cloth, the cloth will warp -will have waves, creases- due to this new surface added.

We have checked that the increase of space (9) is the same in every shell of whatever radius r. This is the distribution of increase of space, considering that the mass emits this increase of space outwards. So, each shell "receives" the same increase of space independently of the value of r. This "emitted" increase of space will warp the space as in the example of the cloth.

Explained in another way, if we divide the increase of space by the unit of volume in every shell,

$$
\frac{m(dr)^2 sen\theta d\vartheta d\theta}{r^2 dr sen\theta d\vartheta d\theta} = \frac{m}{r^2} dr \tag{10}
$$

we get a distribution of the increase of space depending on $1/r^2$. This would be, for example, the distribution of whatever particles emitted by the mass. The distribution of the increase of space is the same as if it is sent by the mass.

For further development, in papers related [10][12], space, in general, is considered as composed by the presence of particles. So, the new particles emitted by a mass create/occupy new space. This new space created warps space as in the example of the

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cloth. This new space would not be caused by the energy or the interactions of the particles but only by the new space the particles occupy (create).

In the next part it is proposed a possible candidate to be these particles transmittals of space.

2. PHOTONS AS TRANSMITTALS OF THE SPACE CREATED BY THE SCHWARZSCHILD METRIC

2.1 Introduction

-First, it will be calculated the variation of mass of a body with the velocity of that body.
-Afterwards, it will be calculated the variation of wavelength of the photons emitted by a body with the velocity of that body.
-Then, it will be shown the variation of the number of photons emitted by a body with velocity.
-With all this data, it will be calculated the variation of space occupied by the photons emitted by a body with the velocity of that body. It will vary in the same proportion as the mass of the body.
-In the conclusions, it will be explained that this make them a plausible candidate to be the particles transmittals of space presented in the first part.

2.2 Variation of mass with velocity

First, we would remind here that the size of a photon is directly proportional to its wavelength (more precisely the longitudinal length of a photon is half the wavelength). [13][14]. All the following calculations performed for the wavelength of a photon are directly applicable for the size of a photon

Let us suppose a mass that have at rest the mass m_0. The mass when it moves with velocity v will be [15]:

$$m_v = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} m_0 \qquad (11)$$

2.3 Variation of wavelength of photons emitted by a mass with velocity of the mass

Let us suppose that the mean value of the wavelength of all the photons emitted by a mass is λ_0.

Now, we will calculate the mean wavelength of the photons emitted by the mass when the mass moves at a velocity v. We call this wavelength λ_v.

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The frequency and wavelength of the photons emitted by a mass with velocity v change according Doppler Effect. Let's check how the frequency varies according Doppler effect [16] and then calculate the variation of wavelength. f_v is the frequency observed (by an observer that sees the mass with velocity v)and f_0 the frequency emitted by the source (in this case the mass at its own frame, this means with velocity 0). θ_0 is the angle of observation from the observer to the source.

$$f_v = \frac{f_0}{\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left(1+\frac{v\cos\theta_0}{c}\right)} \qquad (12)$$

As we are calculating the variation of the mean wavelength of all the photons emitted by the mass (emitted in all directions), we must obtain the mean of $\cos\theta_0$ from 0 to 2π (integration to all the directions) to obtain the variation of the mean wavelength in all the emitted photons. We use [17] (the mean of any function g).

$$\overline{g} = \frac{1}{b-a}\int_a^b g(x)dx = \frac{1}{2\pi-0}\int_0^{2\pi}\cos\theta d\theta = \frac{1}{2\pi}\left[sen\theta\right]_0^{2\pi} = 0 \qquad (13)$$

So returning to (12) we get:

$$\bar{f}_v = \frac{f_0}{\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}(1+0)} = \frac{f_0}{\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}} \qquad (14)$$

Transforming the variation of frequency to the variation of wavelength using $f = \frac{c}{\lambda}$, we get:

$$\overline{\lambda}_v = \lambda_v = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\lambda_0 \qquad (15)$$

2.4 Variation of number of photons emitted by a mass with velocity of the mass

Let us call N_0 the number of photons emitted by a mass at rest. We call now N_v the number of photons emitted by the same mass with velocity v.

In [18] it is demonstrated that the number of photons emitted by a source does not vary with velocity (the number of photons emitted by a mass at rest is the same as the number of photons emitted by a mass with velocity v).

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This means:

$$N_v = N_0 \tag{16}$$

2.5 Variation of space occupied by the photons emitted by a mass with velocity of a mass

The space occupied by the photons emitted by a mass at rest is proportional to the number of photons emitted multiplied by its size (its wavelength).

This is, the space occupied by the photons emitted by a mass at rest is proportional to:

$$N_0 \lambda_0 \tag{17}$$

Now, the space occupied by the photons emitted by a mass with velocity v is proportional to the number of photons emitted multiplied by its size (its wavelength). This is, proportional to:

$$N_v \lambda_v \tag{18}$$

Applying equation (15) and (16) to (18) we get, that space occupied by the photons emitted by a mass with velocity v is proportional to:

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} N_0 \lambda_0 \tag{19}$$

We check that the variation of the space occupied by the photons emitted by a mass with velocity (19) is exactly in the same proportion as the variation of mass with velocity (11).

2.6 Conclusions

If we compare expressions (11) and (19), we check that the mass and the space of the photons emitted by it, varies exactly in the same proportion with velocity. This means, when the mass increases, the space occupied by the photons emitted by it varies exactly in the same proportion.

In the first part of this paper, it is presented that the warping of the Schwarzschild metric could be understood as caused by new space emitted continuously by the masses. The higher the mass, the “bigger” the space they emit.

This means, if a mass increases, the space “emitted” by the mass should increase in the same proportion.

In the first part, it is also shown that this space emitted could be created by the particles emitted by the masses, not by their interactions but simply by the space they occupy.

As the mass changes as (11) (the same proportion as the space occupied by the photons (19)), this leads to photons to be a perfect candidate of transmittals of the space "emitted". When the mass changes, the total space occupied/created by the photons changes exactly in the same way.

This means, the space occupied by the photons would transmit the space created by the Schwarzschild metric, consequently its warping and consequently the gravitational field.

Independently, the energy and momentum of the photons (their interactions) would transmit the electromagnetic field.

For a further development regarding this theory, please check [10] and [12].

Acknowledgements

Thanks to my family and friends for their unconditional support. Thanks to the Motionless Motor for being always there.

A Gerardo Mejía, sin su canción "Sueña" hace tiempo que me habría quedado en el camino, a la chiquilla que conocí en el "Mittlestuffe", al canal de música en el que ponen Urban Hits, a los Joses que fueron llamados al "Motionless world". JqashM.

In Bilbao, on 12th of September 2007,
with best wishes,

Jesús Sánchez

Rev. 3: 07/12/2007

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Appendix I

You can find attached the Table 1 to check the accuracy of the approximation made to go from equation (8) to (9).

What we compare is m to the expression:

$$-m\left(1-\frac{2m}{r}\right)^{-\frac{3}{2}}+2r\left[\left(1-\frac{2m}{r}\right)^{-\frac{1}{2}}-1\right] \tag{20}$$

that is the one that we have approximated in (8) to arrive to the solution m in (9).

The column Difference corresponds to:

$$m-\left[-m\left(1-\frac{2m}{r}\right)^{-\frac{3}{2}}+2r\left[\left(1-\frac{2m}{r}\right)^{-\frac{1}{2}}-1\right]\right] \tag{21}$$

And the column Error corresponds to:

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$$\frac{m-\left[-m\left(1-\frac{2m}{r}\right)^{-\frac{3}{2}}+2r\left[\left(1-\frac{2m}{r}\right)^{-\frac{1}{2}}-1\right]\right]}{-m\left(1-\frac{2m}{r}\right)^{-\frac{3}{2}}+2r\left[\left(1-\frac{2m}{r}\right)^{-\frac{1}{2}}-1\right]} \qquad (22)$$

The column %Error is the expression above (22) multiplied by 100.

You can check that for all the examples and for all situations not proximal to singularity $(r > 10m)$ the error is negligible.

r	m	$-m\left(1-\frac{2m}{r}\right)^{-\frac{3}{2}}+2r\left[\left(1-\frac{2m}{r}\right)^{-\frac{1}{2}}-1\right]$	Difference	Error	%Error	Examples
6,00E+06	0,005	0,005000001	-8,15E-10	-1,63E-07	0,0000%	Earth Radius. Earth mass
1,50E+11	1500	1499,999996	3,67E-06	2,44E-09	0,0000%	Distance Sun-Earth. Sun mass.
5,70E+10	1500	1500,000017	-1,66E-05	-1,10E-08	0,0000%	Distance Mercury-Sun. Sun mass
7,00E+08	1500	1500	-1,95E-08	-1,30E-11	0,0000%	Sun radius. Sun mass
1,00E+08	1	1,000000002	-2,25E-09	-2,25E-09	0,0000%	
1,00E+07	1	1,000000003	-3,15E-09	-3,15E-09	0,0000%	
1,00E+06	1	1	-9,47E-11	-9,47E-11	0,0000%	
1,00E+05	1	1	2,46E-10	2,46E-10	0,0000%	
1,00E+04	1	0,999999975	2,50E-08	2,50E-08	0,0000%	
1,00E+03	1	0,999997491	2,51E-06	2,51E-06	0,0003%	
1,00E+02	1	0,999741008	2,59E-04	2,59E-04	0,0259%	
1,00E+01	1	0,963137289	3,69E-02	3,83E-02	3,8274%	
5,00E+00	1	0,758287073	2,42E-01	3,19E-01	31,8762%	

Table 1.

The masses are presented in units of length according expression [2][3]:

$$m = \frac{GM}{c^2}$$

Data used:

$m_{Earth} = 0{,}005m$

$r_{Earth} = 6\times10^{6}\,m$

$m_{Sun} = 1{,}5\times10^{3}\,m$

$r_{Sun} = 7\times10^{8}\,m$

$d_{Earth-Sun} = 1{,}496\times10^{11}\,m$

$d_{Mercury-Sun} = 5{,}7\times10^{10}\,m$