

## Warping of space as caused by new space emitted by the masses

Jesús Sánchez © 2007  
[jesus.sanchez.rebollo@gmail.com](mailto:jesus.sanchez.rebollo@gmail.com)

### Abstract

*“Te quiero convencer y no sé bien de qué”*. Obk. [1]

If you warp an elastic cloth, you increase the surface of the cloth. But also the opposite is true. If you try to add more surface to a cloth, for example, you sew a new piece of cloth in a cut performed in the original cloth, the original cloth will warp (will have waves, creases) to accept this new surface.

Similarly, we will explain here, that the warping of space could be caused by new space (particles that occupy/create new space) emitted by the masses (provoking the space to warp).

In this paper, space-time is defined by the particles that occupy it. If new particles appear, new space is created. If a mass is continuously emitting particles, it will warp space because these particles occupy (create) new space in the surroundings of the mass. This new space (created by the existence of these particles) modifies the metrics of the space, warping it. It will be shown that applying this philosophy to the metrics created by a point mass, we obtain the Schwarzschild metric. Then, we will propose a possible candidate to be these particles that occupy/create space.

### Introduction

*“Little by little, understanding that it is not worth walking only to walk, it is better to walk to keep growing”*. Chambao. [2]

We will follow the next steps:

- First, we will obtain the differential of volume in Schwarzschild metric.
- Then, we will obtain the differential of volume in Euclidean metric.
- Afterwards, we will obtain the metric that corresponds to a warping caused by the masses emitting “new space” in a spherical symmetric way, and check that it is the same as Schwarzschild metric.
- Finally, we will propose a possible candidate to be these particles that occupy/create space emitted by the masses. We will check that the size of these particles increases exactly in the same way as the mass do (when the mass changes with velocity), which make them a plausible candidate.

## Schwarzschild metric

The Schwarzschild metric is defined by [3][4][5][7]:

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dt^2 + \left(\frac{1}{1 - \frac{2m}{r}}\right) dr^2 + r^2 \sin^2 \theta d\vartheta^2 + r^2 d\theta^2 \quad (1)$$

We would remind some of the assumptions made to obtain the Schwarzschild metric [3][4][5][7]:

- The solution is spherically symmetric (this means the metric is the same in each shell of constant  $r$ ).
- $d\theta$  and  $d\vartheta$  are defined in a way that every shell keeps the property that its area is  $4\pi r^2$ . This means, all the non-Euclidean factors are left to  $dr$  and  $dt$
- To obtain the Schwarzschild metric, it is used the weak-field approximation, that approximates to the linearized gravity (it neglects quadratic and higher order elements).

## Differential of volume in Schwarzschild metric

“La elección, el problema es la elección”. Neo. [6]

We will obtain the differential of space volume in the Schwarzschild metric. We consider an instant of time ( $dt = 0$ ). This means, we will work with:

$$ds^2 = \left(\frac{1}{1 - \frac{2m}{r}}\right) dr^2 + r^2 \sin^2 \theta d\vartheta^2 + r^2 d\theta^2 \quad (2)$$

To calculate the differential of volume of the metric (2), we apply definition of differential of volume [7] to the metric tensor  $g_{ij}$  (for space coordinates,  $i, j=1, 2, 3$ ).

$$\begin{aligned} dV_{\text{spaceSchwarzschild}} &= \left| \det(g_{ij})_{i,j=1,2,3} \right|^{\frac{1}{2}} dr d\vartheta d\theta = \left[ \left(\frac{1}{1 - \frac{2m}{r}}\right) r^2 \sin^2 \theta \right]^{\frac{1}{2}} dr d\vartheta d\theta = \\ &= \left[ \left(\frac{1}{1 - \frac{2m}{r}}\right) r^4 \sin^2 \theta \right]^{\frac{1}{2}} dr d\vartheta d\theta = \left(\frac{1}{1 - \frac{2m}{r}}\right)^{\frac{1}{2}} r^2 dr \sin \theta d\vartheta d\theta = \left(1 - \frac{2m}{r}\right)^{-\frac{1}{2}} r^2 dr \sin \theta d\vartheta d\theta \approx \\ &\approx \left(1 + \frac{m}{r} + \frac{3m^2}{2r^2}\right) r^2 dr \sin \theta d\vartheta d\theta \end{aligned} \quad (3)$$

In the last step of (3) we have made an approximation<sup>1</sup> to the quadratic term  $\frac{1}{r^2}$  (more precise approximation than the one used to derive the Schwarzschild metric itself). Anyhow, you can find a table of accuracy of this approximation in Appendix I.

### Differential of volume in Euclidean space (spherical coordinates)

“Escucho tus palabras, ¡Qué bellas y qué caras!” Naty Botero. [8]

In Euclidean space (spherical coordinates) the metric is defined by[9]:

$$ds^2 = dr^2 + r^2 \sin^2 \theta d\varphi^2 + r^2 d\theta^2 \quad (4)$$

So, following same steps as for the Schwarzschild metric, we obtain the differential of volume of Euclidean metric (4) using [7]:

$$\begin{aligned} dV_{Euclidean} &= \left| \det(g_{ij}) \right|_{i,j=1,2,3}^{\frac{1}{2}} dr d\varphi d\theta = [r^2 \sin^2 \theta d\varphi^2 r^2]^{\frac{1}{2}} dr d\varphi d\theta = \\ &= r^2 dr \sin \theta d\varphi d\theta \end{aligned} \quad (5)$$

### Obtaining the metric corresponding to a point mass that emits space

“Me temo que ya sé cómo va a acabar esto”. Prince of Persia. [10]

We will take the following assumptions (similar to Schwarzschild assumptions [3][4][5][7] and demonstrations above):

- The solution is spherically symmetric (this means the metric is the same in each shell of constant  $r$ ).
- $d\theta$  and  $d\varphi$  are defined in a way that every shell keeps the property that its area is  $4\pi r^2$ . This means, all the non-Euclidean factors are left to  $dr$  and  $dt$ .
- We will consider an instant of time ( $dt = 0$ ). This means, the metrics will not depend on  $dt$  and all the non-Euclidean factors are left to  $dr$ .

With these assumptions the metric is as following:

$$ds^2 = F(r)dr^2 + r^2 \sin^2 \theta d\varphi^2 + r^2 d\theta^2 \quad (6)$$

Following same steps as in Schwarzschild metric and calling  $f(r) = F(r)^{\frac{1}{2}}$ , we obtain the differential of volume in this metric as:

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<sup>1</sup>Binomial series [22] approximation  $(1-x)^{-\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \dots$

$$dV = f(r)r^2 dr \text{sen}\theta d\vartheta d\theta \quad (7)$$

We are considering here, that the masses emit particles that occupy (create) space. And this new created space warps space. The distribution of the “emitted” particles will be symmetric spherical. Also, remind that the integration of  $\text{sen}\theta d\vartheta d\theta$  and  $d\vartheta$  in a complete shell is  $4\pi$  [9]<sup>2</sup>.

With this consideration, the differential of volume in each point of space will be increased by the following addend per unit of space:

$$\frac{C}{4\pi f(r)r^2} dr = \frac{k}{f(r)r^2} dr \quad (8)$$

$k$  is a constant that represents the “increment of volume” per unit of volume that the mass emit. We can understand that this  $k$  depends on the number of particles emitted multiplied by their size (the number of particles emitted multiplied by the space occupied/created by each of them).

To know the increment of volume, we multiply (8) (that it is per unit of volume) by the unit of volume:

$$\frac{k}{f(r)r^2} dr f(r)r^2 dr \text{sen}\theta d\vartheta d\theta = k(dr)^2 \text{sen}\theta d\vartheta d\theta \quad (9)$$

Anyhow, if we would not have considered the  $f(r)$  of the metric, and we would have calculated as a spherical symmetric distribution in Euclidean space, we would have obtained the same result (check<sup>3</sup>).

Knowing this, we will go to the next step.

The variation of the differential of volume in this metric (7) should be equal to the variation of the differential of volume in the Euclidean metric (5) plus the increment of volume due to the emitted “space” (9).

As both metrics are spherical symmetric, there will not be any variation depending on  $\theta$  or  $\vartheta$  in a shell of constant  $r$ . So, we will calculate all variations along the  $r$  line. This is made making the differentiation respect to  $r$ .

**First**, we will calculate the variation along  $r$  (the differentiation respect to  $r$ ) of the differential of volume in this metric (7):

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$$2 \int_0^\pi \text{sen}\theta d\theta \int_0^{2\pi} d\vartheta = [-\cos\theta]_0^\pi [\vartheta]_0^{2\pi} = 4\pi$$

<sup>3</sup> Distribution of emitted particles per unit of volume in symmetric spherical Euclidean space:  $\frac{k}{r^2} dr$

Multiplying by unit of volume in Euclidean space:

$$\frac{k}{r^2} dr r^2 dr \text{sen}\theta d\vartheta d\theta = k(dr)^2 \text{sen}\theta d\vartheta d\theta$$

$$\begin{aligned}
dV &= f(r)r^2 dr \sin\theta d\vartheta d\theta \\
d^2V &= f'(r)r^2 (dr)^2 \sin\theta d\vartheta d\theta + f(r)2r(dr)^2 \sin\theta d\vartheta d\theta + f(r)r^2 d^2r \sin\theta d\vartheta d\theta
\end{aligned} \tag{10}$$

The element depending on  $d^2r$  could be directly eliminated according [11] but we will keep it just in case.

**Second step**, we will calculate the variation of the differential of volume in Euclidean metric (5):

$$\begin{aligned}
dV_{Euclidean} &= r^2 dr \sin\theta d\vartheta d\theta \\
d^2V_{Euclidean} &= 2r(dr)^2 \sin\theta d\vartheta d\theta + r^2 d^2r \sin\theta d\vartheta d\theta
\end{aligned} \tag{11}$$

**Third step**, the element depending on  $d^2r$  could be directly eliminated according [11] but we will keep it just in case.

Now, as commented we will make the equation (10)=(11)+(9). This means: The variation of the differential of volume in this metric equal to the variation of the differential of volume in the Euclidean metric plus the increment of volume due to the emitted "space".

$$\begin{aligned}
&f'(r)r^2 (dr)^2 \sin\theta d\vartheta d\theta + f(r)2r(dr)^2 \sin\theta d\vartheta d\theta + f(r)r^2 d^2r \sin\theta d\vartheta d\theta = \\
&= 2r(dr)^2 \sin\theta d\vartheta d\theta + r^2 d^2r \sin\theta d\vartheta d\theta + k(dr)^2 \sin\theta d\vartheta d\theta; \\
&f'(r)r^2 (dr)^2 + f(r)2r(dr)^2 + f(r)r^2 d^2r = 2r(dr)^2 + r^2 d^2r + k(dr)^2; \\
&f'(r) + f(r) \left[ \frac{2}{r} + \frac{d^2r}{(dr)^2} \right] = \frac{2}{r} + \frac{d^2r}{(dr)^2} + \frac{k}{r^2}
\end{aligned} \tag{12}$$

As commented, according [11] the elements depending on  $d^2r$  vanish, you can check a way to demonstrate it<sup>4</sup>.

So we have the differential equation:

$$f'(r) + f(r) \frac{2}{r} = \frac{2}{r} + \frac{k}{r^2} \tag{13}$$

This is a first order linear differential equation, with solution [11]:

$$f(r) = 1 + \frac{k}{r} + \frac{C'}{r^2} \tag{14}$$

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<sup>4</sup>  $\frac{d^2r}{(dr)^2} = \frac{d\left(\frac{dr}{dr}\right)}{dr} = \frac{d(1)}{dr} = 0$

"A esto no se le llamó ejecución. Se le llamó retiro". Blade Runner [23]

being  $C'$  the integration constant. You can check that the solution (14) for  $f(r)$  in this metric is the same as the solution for Schwarzschild metric (3) with:

$$\begin{aligned}
 k &= m \\
 C' &= \frac{3}{2}m^2 \\
 f(r) &= 1 + \frac{m}{r} + \frac{3}{2} \frac{m^2}{r^2}
 \end{aligned} \tag{15}$$

Now, substituting (15) in the differential of volume of the metric (7), we obtain the differential of Volume in Schwarzschild metric (3).

$$dV = f(r)r^2 dr \sin\theta d\theta d\phi = \left(1 + \frac{m}{r} + \frac{3}{2} \frac{m^2}{r^2}\right) r^2 dr \sin\theta d\theta d\phi = dV_{spaceSchwarzschild} \tag{7} (3)$$

As we wanted, we have proved, that considering that the masses emit “space” (particles that occupy/create new space) in a spherical symmetric way, we get the Schwarzschild metric. You can check in Appendix I, that the error of this expression approximated to  $\frac{1}{r^2}$  is practically zero compared to original Schwarzschild expression.

**A possible candidate to be these particles emitted by masses that occupy (create) space**

*“He who controls the frame controls the communication itself”* Mystery et al.[13]

We would remind here that, as explained in paragraph below equation (8),  $k$  is a constant that represents the space emitted by a mass.  $k$  depends on the number of particles emitted, multiplied by their size (this is, depends on the number of particles emitted multiplied by the space occupied/created by each of them).

By (15) we know also that  $k$  depends directly on  $m$ .

So, this means the quantity of space emitted by a mass ( $k$ ) depends directly on the mass ( $m$ ). So, to propose a candidate the conditions that we should request for these candidates would be:

-The number of particles emitted by a mass multiplied by the size of each particle (the total space emitted) must be proportional to the mass that emit them. This means a  $2m$  mass must emit double of the space than a  $m$  mass.

-If a mass changes its value, the number of total space emitted (number of particles multiplied by the size of each of them) must change in the same proportion.

At this point, we will propose the virtual photons as plausible candidates to be these particles. For virtual particles it is very difficult to check the first requirement as the total number of particles emitted and its size (the wavelength) of them is unknown.

Anyhow, we can check the second requirement easily as follows:

If we call  $N_f$  the number of photons emitted by a mass  $m$  and  $\lambda$  the mean wavelength (the mean size) of them we will have according previous comments:

$$\begin{aligned} k &= m \propto N_f \lambda \\ k &= m = C'' N_f \lambda \end{aligned} \tag{16}$$

Being  $C''$  an unknown constant, only used to represent proportionality.

For the equation (16) we will consider two cases: mass at rest (subindex 0) and mass with velocity  $v$  (subindex  $v$ )

We start with the case of mass at rest  $m = m_0$ , so:

$$k_0 = m_0 = C'' N_{f0} \lambda_0 \tag{17}$$

Now, let us suppose that the mass have a velocity  $v$ . The new mass is[14]:

$$m_v = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} m_0 \tag{18}$$

So equation (16) for the case of mass with velocity  $v$  is:

$$k_v = m_v = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} m_0 = C'' N_{fv} \lambda_v \tag{19}$$

Redistributing elements, we have:

$$m_0 = \sqrt{1 - \frac{v^2}{c^2}} C'' N_{fv} \lambda_v \tag{20}$$

Now let us make equal the right terms of (17) and (20), both are equal to  $m_0$ :

$$\sqrt{1 - \frac{v^2}{c^2}} C'' N_{fv} \lambda_v = C'' N_{f0} \lambda_0 \tag{21}$$

$C''$  is constant so it is the same in both terms.

The number of photons  $N_{fv}$  emitted by the mass with velocity  $v$  is the same as the number of photons  $N_{f0}$  when the mass is at rest ( $N_{fv} = N_{f0}$ ). This you can check in [15].

So, eliminating  $C''$ ,  $N_{f_v}$  and  $N_{f_0}$  in both sides of the equation, we get:

$$\sqrt{1 - \frac{v^2}{c^2}} \lambda_v = \lambda_0$$

$$\lambda_v = \frac{\lambda_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (22)$$

So the  $\lambda_v$  should change according (22) for the photons to be a possible candidate of the particles emitted by the masses to occupy/create space (and consequently warp space). But is equation (22) fulfilled?

We know that the frequency and  $\lambda$  of the photons emitted by a mass with a velocity changes according Doppler Effect. Let's check how the frequency vary according Doppler effect [16] and then calculate the variation of  $\lambda$ .  $f_v$  is the frequency observed (by an observer that sees the mass with velocity  $v$ ) and  $f_0$  the frequency emitted by the source (in this case, the mass in its own frame at velocity 0).  $\theta_0$  is the angle of observation from the observer to the source.

$$f_v = \frac{f_0}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left( 1 + \frac{v \cos \theta_0}{c} \right)} \quad (23)$$

As we are calculating the variation of  $\lambda$  of all the photons emitted by the mass (emitted in all directions), we must obtain the mean of  $\cos \theta_0$  from 0 to  $2\pi$  (integration to all the directions) to obtain the variation of the mean  $\lambda$  in all the emitted photons. We use [17] (the mean of any function  $g$ ).

$$\bar{g} = \frac{1}{b-a} \int_a^b g(x) dx = \frac{1}{2\pi-0} \int_0^{2\pi} \cos \theta d\theta = \frac{1}{2\pi} [\sin \theta]_0^{2\pi} = 0 \quad (24)$$

So returning to (23) we get:

$$\bar{f}_v = \frac{f_0}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} (1+0)} = \frac{f_0}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} \quad (25)$$

Transforming the variation of frequency to the variation of  $\lambda$  using  $f = \frac{c}{\lambda}$ , we get:

$$\bar{\lambda}_v = \lambda_v = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \lambda_0 \quad (26)$$

as we wanted to prove (equal to (22)). This means, the photons increase its size in the same proportion as the mass increase its mass with velocity. This is, if the photons transmit the space emitted by the masses, they do increase the space emitted, when the mass increases (and in the same proportion).

*“Todo lo que tiene un principio tiene un final, Neo”* El oráculo en boca del Sr. Smith. [18].

### **Name of the theory**

As this theory propose the photons as being the transmittals of gravitation (by the space they create/occupy, and consequently warp) and also the photons keep being the transmittals of electromagnetic fields (by their energy and interactions), I call it the **photonic system**.

*“Banoa zure barnera, zure egiaren arimara, zure bihotzaren parera, banoa nire bizitzaren amaierara zugar itotzera, itsasoa”* Zea Mays. [19]

According to this theory we live in an ocean of photons that compose space, cause the warping of space (by its non-Euclidean distribution) and cause the electromagnetic interactions.

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*“The trick is to keep breathing”*. Garbage.[21]

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Deseando un mundo mejor para nuestros hijos y nietos,  
En Bilbao, a 7 de Septiembre de 2007,

Jesús Sánchez

Revision 2: 30 de Septiembre de 2007

*“What would you do with your life if you had no chance of failure? Start doing it.”*  
Mystery et al. [13]

*“Volveré pa’ contarte que he soñado colores nuevos y días claros”* Chambao.[2]

If you want to see the mathematics and physics in colour again, please read this wonder [20].

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## Appendix I

In the following table, it is made a comparative of the expression  $\left(1 - \frac{2m}{r}\right)^{\frac{1}{2}}$  and the

expression  $1 + \frac{m}{r} + \frac{3m^2}{2r^2}$ .

The difference correspond to  $\left(1 - \frac{2m}{r}\right)^{\frac{1}{2}} - \left(1 + \frac{m}{r} + \frac{3m^2}{2r^2}\right)$

The error corresponds to  $\frac{\left(1 - \frac{2m}{r}\right)^{\frac{1}{2}} - \left(1 + \frac{m}{r} + \frac{3m^2}{2r^2}\right)}{\left(1 - \frac{2m}{r}\right)^{\frac{1}{2}}}$ . The percentage of error is the same

expression multiplied by 100.

In the right column you can check, that for the examples attached, the error is zero. Only when the relation  $\frac{m}{r}$  is getting large (more than 0,1), the error becomes perceptible. Below the table, you can find attached the data used for the corresponding examples.

$\frac{m}{r}$	$\left(1 - \frac{2m}{r}\right)^{\frac{1}{2}}$	$1 + \frac{m}{r} + \frac{3m^2}{2r^2}$	Difference	Error	%Error	Examples
1E-39	1	1	0	0	0,00%	Proton mass / Proton radius
1,30208E-11	1	1	0	0	0,00%	Earth mass / distance Moon-Earth
1E-10	1	1	0	0	0,00%	
8,33333E-10	1,000000001	1,000000001	0	0	0,00%	Earth mass / Earth Radius
0,000000001	1,000000001	1,000000001	0	0	0,00%	
0,00000001	1,00000001	1,00000001	0	0	0,00%	
1,00267E-08	1,00000001	1,00000001	0	0	0,00%	Sun mass / distance Sun-Earth
2,63158E-08	1,000000026	1,000000026	0	0	0,00%	Sun mass / distance Mercury-Sun
0,0000001	1,0000001	1,0000001	5,107E-15	5,10703E-15	0,00%	
0,000001	1,000001	1,000001	5E-13	5,00044E-13	0,00%	
2,14286E-06	1,000002143	1,000002143	2,296E-12	2,29594E-12	0,00%	Sun mass / Sun radius
0,00001	1,00001	1,00001	5E-11	5,00019E-11	0,00%	
0,0001	1,000100015	1,00010001	5,003E-09	5,002E-09	0,00%	
0,001	1,001001503	1,001001	5,025E-07	5,02002E-07	0,00%	
0,01	1,010152545	1,0101	5,254E-05	5,20165E-05	0,01%	
0,1	1,118033989	1,11	0,008034	0,007185818	0,72%	
0,2	1,290994449	1,24	0,0509944	0,03950013	3,95%	
0,4	2,236067977	1,56	0,676068	0,302346791	30,23%	

The masses are presented in units of length according expression [3][5]:

$$m = \frac{GM}{c^2}$$

The data used are the following:

$$m_{proton} \approx 1 \times 10^{-54} m$$

$$r_{proton} \approx 10^{-15} m$$

$$m_{Earth} = 0,005m$$

$$r_{Earth} = 6 \times 10^6 m$$

$$m_{Sun} = 1,5 \times 10^3 m$$

$$r_{Sun} = 7 \times 10^8 m$$

$$d_{Earth-Sun} = 1,496 \times 10^{11} m$$

$$d_{Mercury-Sun} = 5,7 \times 10^{10} m$$

$$d_{Moon-Earth} = 3,84 \times 10^8 m$$