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A Great Battle in Spherical Geometry

S. Kalimuthu

SF 211 , Kanjampatti P.O, Pollachi Via, Tamil Nadu 642003, India

Abstract: In this study, without assuming the fifth Euclidean postulate, the following theorem was established: There exists a spherical triangle whose interior angle sum is equal to four right angles.

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A Journey Through Euclid V

Euclid (circa 300 BC) produced the definitive treatment of Greek geometry and number theory in the 13 volume Elements.

- Ptolemy (circa 130 AD) assumed that there was at least one line parallel to a line through a given point which is equivalent to Euclid's postulate-circular reasoning
- Proclus (410-485) assumed parallel lines are always equidistance which is an added assumption about parallel lines.
- Wallis (1616-1703) proved the Parallel Postulate assuming a postulate about Similar Triangles which is equivalent to Euclid's postulate-circular reasoning.
- Saccheri (1667-1733) worked with quadrilaterals, now called Saccheri quadrilaterals, where the base angles are rights angles and the sides adjacent to the base are congruent.

The question is: what can be proven about the summit angles, <D and <C? Without assuming the Parallel Postulate, it can be proven that the two summit angles are congruent. Then, there are three distinct possibilities:

- The summit angles are acute angles.
- the summit angles are right angles.
- the summit angles are obtuse angles.

What Saccheri Finally Wrote Was: "The hypothesis of the acute angle is absolutely false, because [it is] repugnant to the nature of the straight line!" (Greenberg, p.155)

- C Clairaut (1713-1765) proved the Parallel Postulate assuming a postulate about the Existence of Rectangles which is equivalent to Euclid's postulate-circular reasoning.
- C Legendre (1752-1833) worked with the Parallel Postulate assuming a postulate about angle sum of a triangle being equal to 180o which is equivalent to Euclid's postulate-circular reasoning.
- C Lambert (1728-1777) worked with quadrilaterals, now called Lambert quadrilaterals, which have three right angles. The question is what can be said about the fourth angle?
- C Since so many mathematicians had tried to prove Euclid's Parallel Postulate, Klügel did his doctoral thesis in 1763 finding the flaws in 28 different proofs of this postulate. The thesis led d'Alembert to call Euclid's Parallel Postulate "the scandal of geometry." (Greenberg, p.161)
- C The Hungarian Farkas Bolyai wrote to his son János:

You must not attempt this approach to parallels. I know this way to its very end. I have traversed this bottomless night, which extinguished all light and joy in my life. I entreat you, leave the science of parallels alone. I thought I would sacrifice myself for the sake of truth. I was ready to become a martyr who would remove the flaw from geometry and return it purified to mankind. I turned back when I saw that no man can reach the bottom of the night. I turned back unconsoled, pitying myself and all mankind.

I have traveled past all reefs of this infernal Dead Sea and have always come back with broken mast and torn sail. The ruin of my disposition and my fall date back to this time. I thoughtlessly risked my life and happiness. (Greenberg, pp: 161-162)

C The son János Bolyai (1802-1860)wrote back:

It is now my definite plan to publish a work on parallels as soon as I can complete and arrange the material. When you, my dear Father, see them, you will understand; at present I can only say nothing except this: that out of nothing I have created a strange new universe. All that I have sent you previously is like a house of cards in comparison to a tower.(Greenberg, p. 163)

- C When János's father send his work to Gauss (1777-1855), Gauss wrote back that he, in essence, had done this work but would never publish it since:

Most people have not the insight to understand our conclusions and I have encountered only a few who received with any particular interest what I communicated to them.

(Greenberg, p. 178)

- C Lobachesky (1792-1656) was the mathematician first to publish an account of non-Euclidean geometry in 1829. However, the original was published in Russian. It was not until 1840 that the work was published in German and received some recognition. Since his work openly challenged Kant's view of space as "a priori" knowledge, he was fired 1846 from his university post.
- C In 1868, Beltrami settled the question about Euclid's Parallel Postulate by proving that no proof was possible.
- C Riemann (1826-1866) developed elliptic geometry starting in 1854.
- C Klein, Beltrami and Poincaré worked in the last half of the 19th century in developing models for hyperbolic geometry.
- C In 1882, Pasch developed one of the first modern set of axioms for Euclidean geometry.
- C In 1902, Hilbert, a great champion of the axiomatic method, published a set of axioms which filled the gaps for Euclidean geometry.
- C In 1932, Birkhoff developed a new set of axioms for geometry, based totally on the connections between geometry and real numbers and include distance and angle as undefined terms.
- C Gödel, in 1940, proved that no mathematical system can be complete.

Equivalent Statements for Hyperbolic Geometry:

- C Given a line and a point P not on, there are at least two distinct lines through P parallel to.
- C Every triangle has angle sum less than 180o
- C If two triangles are similar, then the triangles are congruent.
- C There exist an infinite number of lines through a given point P parallel to a given line.
- C In the Saccheri quadrilateral, the summit angles are congruent and less than 90o
- C In the Lambert quadrilateral, the fourth angle is less than 90o.
- C Rectangles do not exist.

RESULT

NA’, NB’, NC’ are the meridians of a sphereS whose north pole is N.Choose a point. A on NA’. With center N, radius NA, describe an arc cutting AB’ at B AD at C.

So, NA = NB = NC
(1)

Take a point F on NA. With center N, radius NF draw an arc meeting NB at E And NC at D. So, NF = NE = ND

(2

From (1) in triangle NAC, angle NAB = angle NCB (3)

And in triangle, NBC, angle NBC = angle NCB (4)

From (3) and (4) we obtain that angles NAB=NBC=NCB = 90 degree (5)

Similarly from (2) we can show that angles, NFE= NEF= NED=NDE = 90 degree (6)
From (5) and (6) we get that the sum of the interior angles of spherical quadrilateral BCDE is equal to 360 degrees (7)

DISCUSSION

Since we have derived (7) without assuming Euclid's parallel postulate, it is controversial but consistent.How is it possible?What is the mystery?There is something hidden treasure of physical geometry.The classical geometry is widely used in mechanics.The principles of non-Euclidean geometries are applied in quantum mechanics and general theory of relativity.A turning pointin geometry always influenced theoretical physics.There are many burning problems in physics.Further investigations to be devoted to this peculiar mathematical analysis will unlock this problematic problem and give rise to a new field

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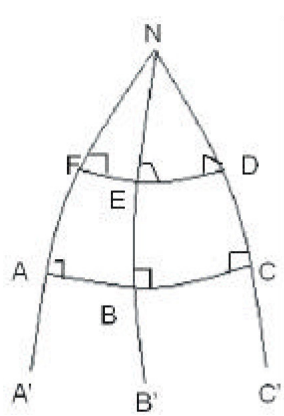

Fig. 1: Spherial