

A Challenging Result in Spherical Geometry

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Abstract

The fifth Euclidean postulate was re-surveyed and an important/challenging result was found.

KeyWords: Euclid, elements, postulates, non-Euclidean geometries, spherical triangles and quadrilaterals

MSC: 51 M04

PACS: 02.40.Dr, 02.40Ky

Result

NA', NB', NC' are the segments of a sphere S whose north pole is N. Choose a point A on NA'. With center N, radius NA, describe an arc cutting AB' at B and AC' at C.

$$\text{So, } NA = NB = NC \text{ -----} \quad (1)$$

Take a point F on NA. With center N, radius NF draw an arc meeting NB at E

$$\text{and NC at D. So, } NF = NE = ND \text{ -----} \quad (2)$$

$$\text{From (1) in triangle NAC, angle NAB} = \text{angle NCB} \quad (3)$$

$$\text{And in triangle, NBC, angle NBC} = \text{angle NCB} \quad (4)$$

$$\text{From (3) and (4) we obtain that angles, NAB=NBC=NCB} = 90 \quad (5)$$

Similarly from (2) we can show that

$$\text{angles, NFE= NEF= NED=NDE} = 90 \text{ degree} \quad (6)$$

From (5) and (6) we get that the sum of the interior angles of the spherical

Quadrilateral BCDE is equal to 360 degrees (7)

Discussion

Since we have derived (7) without assuming Euclid's fifth postulate it is controversial but consistent. How is it possible? What is the mystery? Only further studies will unlock this.

References:

[1] Effimove, NV: Higher Geometry, Mir publishers, Moscow, 1972,pp 1 - 30

[2] Smilga: In the search for the beauty, Mir publishers, Moscow, 1972,pp 1 - 50

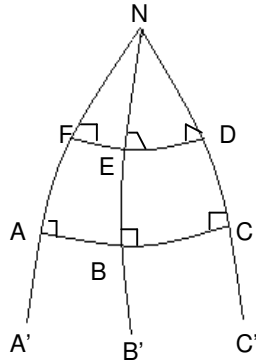


Figure 1 [Spherical]