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Revision of Classical Quantum Mechanics

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Abstract

The author applies his new relativistic mechanics, developed in his previous work, to several basic issues of the classical quantum mechanics. This study shows:

(1) A moving particle's velocity is exactly it's own matter wave's phase velocity so that the particle and its own matter wave move together never separated.

(2) Bohr's quantum structure of the hydrogen atom must be rectified; an electron in the hydrogen atom can have only circular orbits; the electron's transition from an inner orbit to an outer orbit, not the other way round, causes the radiation.

(3) The Compton photon scattering formula violates the principle of elastic collision and must be corrected.

(4) Experiments on the γ-photon scattering and on the diffraction of electrons with near-light velocity may validate the new relativistic mechanics and justify the revision of the classical quantum mechanics.

(5) Schrodinger's wave equation is incorrect and must be replaced by a new equation capable of correctly quantizing rectangular potential well and harmonic oscillator.

(6) There is no such a thing as non-relativistic wave equation; all wave equations are relativistic, but the Klein-Gordon equation is not a correct relativistic wave equation.

(7) "Wave packet" is a misleading concept; moving particles are not wave packets.

(8) Schrodinger's wave function of complex variables is misconceived; wave function is just a useful tool made of real variables.

(9) Born's statistical interpretation of the wave function and Heisenberg's uncertainty principle are questionable.

§1 Brief Review of the New Relativistic Mechanics

In a previous paper entitled "Relativistic Mechanics Based on Variable Speed of Light", the author has revealed many inconsistencies and errors in Einstein's theory of relativity and then developed a new relativistic mechanics. This paper applies the new relativistic mechanics to several issues of the early-year quantum mechanics, clears misunderstandings in classical works and thereby further justifies the new relativistic mechanics. Assuming readers are well familiar with the author's previous paper, we just briefly review key points of the new relativistic mechanics, which are relevant to the discussions in this paper.

In the paper on "Relativistic Mechanics Based on Variable Speed of Light", the author has proven:

(1) The essential difference between relativistic mechanics and non-relativistic mechanics lies in that: a body (**subject**) must use the former to do **relative assessment** of another moving body (**object**) whereas the latter must be used by a body to do **subjective self-assessment** of its own state of motion. **"Relative assessment" is relativistic** whereas **"Self-assessment" is non-relativistic. "Relativistic" or "Non-relativistic" is not determined by the speed of relative motion**. Quantum mechanics is a science, in which we (**subjects**) study relatively moving particles (**objects**). Therefore, all the quantum mechanics ought to be relativistic, no matter how slow particles move. However, Einstein's relativistic mechanics is incorrect and must be replaced by our new relativistic mechanics. Newtonian mechanics, although correct, is non-relativistic and can not be used in the quantum mechanics.

(2) Moving at speed υ, a body with static mass m has moving mass $m' = \dfrac{m}{\sqrt{1+\upsilon^2/c^2}}$.

(3) The new relativistic mechanics has discovered a new physical quantity $E_{km'} = m'\upsilon^2$, which is the kinetic energy possessed by moving mass m' (shortly **kinetic energy of moving mass**). This physical quantity will play a significant role in the revision of classical quantum mechanics.

(4) If a body with static mass m is accelerated by an external force to become moving at speed υ, then the body has **acquired kinetic energy** $E_{km} = mc^2\left(\sqrt{1+\upsilon^2/c^2}-1\right)$.

(5) A body's static mass m and moving mass m' are equivalent to its **static mass-energy** mc^2 and **moving mass-energy** $m'c^2$ respectively. A body's total energy can be expressed in two ways:

Total Energy = Moving mass-energy $m'c^2$ + Kinetic energy of moving mass $m'\upsilon^2$

$$m'c^2 + m'\upsilon^2 = m'c^2\left(1+\frac{\upsilon^2}{c^2}\right) = \frac{mc^2}{\sqrt{1+\upsilon^2/c^2}}\left(1+\upsilon^2/c^2\right) = mc^2\sqrt{1+\upsilon^2/c^2}$$

or Total Energy = Static mass-energy mc^2 + Acquired kinetic energy $mc^2\left(\sqrt{1+\upsilon^2/c^2}-1\right)$

$$mc^2 + mc^2\left(\sqrt{1+\upsilon^2/c^2}-1\right) = mc^2\sqrt{1+\upsilon^2/c^2}\ .$$

(6) Photons have static mass. A moving photon with static mass m and velocity $\upsilon = c$ has moving mass $m' = \dfrac{m}{\sqrt{1+c^2/c^2}} = \dfrac{m}{\sqrt{2}}$, moving mass-energy $m'c^2$, kinetic energy possessed by its moving mass $m'c^2$ and total energy $m'c^2 + m'c^2 = \sqrt{2}mc^2$ or $mc^2\sqrt{1+c^2/c^2} = \sqrt{2}mc^2$.

§2 Interpretation of the de Broglie Matter Wave

A particle moving at velocity υ has moving mass m' and momentum $P = m'v$. The particle's de Broglie matter wave has frequency $\nu = \dfrac{E}{h}$ and wavelength $\lambda = \dfrac{h}{P}$, where h is the Planck constant. The particle's velocity υ must be the same as its matter wave's velocity $\nu\lambda$. Otherwise, the particle would be separated from **its**

own matter wave and the separation would be farther and farther over time. Hence, there must be $\nu\lambda = \frac{E}{P} = \upsilon$ or $E = P\upsilon = m'\upsilon^2$, which is exactly the physical quantity "kinetic energy possessed by moving mass" discovered by the new relativistic mechanics.

Both Newtonian mechanics and Einstein's mechanics do not have this physical quantity $m'\upsilon^2$. Newtonian kinetic energy $m\upsilon^2/2$ and momentum $m\upsilon$ lead to $\nu\lambda = \frac{m\upsilon^2/2}{mv} = \frac{\upsilon}{2} \neq \upsilon$. So, Newtonian mechanics is incompatible with the de Broglie matter wave theory. This is because the Newtonian mechanics is non-relativistic whereas the quantum mechanics, with which we study relatively moving particles, must be relativistic.

If Einstein's kinetic energy $(m' - m)c^2$ is used as E in the matter wave's frequency formula $\nu = \frac{E}{h}$, then $\nu\lambda = \frac{E}{P} = \frac{(m'-m)c^2}{m'\upsilon} = \left(1 - \frac{m}{m'}\right)\frac{c^2}{\upsilon} = \left(1 - \sqrt{1 - \frac{\upsilon^2}{c^2}}\right)\frac{c^2}{\upsilon} \neq \upsilon$, unless $\upsilon = c$. However, according to Einstein's mechanics, no ponderable particles can move as fast as $\upsilon = c$. If Einstein's total energy $m'c^2$ is used as E, then again $\nu\lambda = \frac{m'c^2}{m'\upsilon} = \frac{c^2}{\upsilon} \neq \upsilon$, unless $\upsilon = c$. Obviously, Einstein's mechanics does not conform with the de Broglie matter wave theory either.

Physicists have so far not been able to clearly explain the relationship between a particle's velocity υ and its matter wave's phase velocity $\nu\lambda$ due to the lack of a physical quantity $m'\upsilon^2$. They explain moving particles as wave packets. The explanation is questionable (**for more see §7.1**).

Only our new relativistic mechanics is able to eventually solve this old puzzle: E in the matter wave's frequency formula $\nu = \frac{E}{h}$ is the kinetic energy $m'\upsilon^2$ possessed by a particle's moving mass m' so that the moving particle's velocity and its own matter wave's phase velocity are always the same $\nu\lambda = \frac{E}{P} = \frac{m'\upsilon^2}{m'\upsilon} = \upsilon$. This is precisely true for all kinds of particles, including photons, with any velocity from $\upsilon << c$ to $\upsilon > c$.

A moving particle's total energy is $m'c^2 + m'\upsilon^2$, in which its **moving mass-energy** $m'c^2$ represents its **corpuscularity** and the **kinetic energy possessed by its moving mass** $m'\upsilon^2$ generates its **wave property**. A static particle ($\upsilon = 0$) does not demonstrate any wave property but preserves its corpuscularity. Particles display their wave-corpuscular duality only when they are moving. Moving at sub-light speed ($\upsilon < c$), a particle's corpuscularity overwhelms its wave property ($m'c^2 > m'\upsilon^2$). A particle with super-light speed ($v > c$) has overwhelming wave property ($m'\upsilon^2 > m'c^2$). Photons and any particles moving with the speed of light ($v = c$) have balanced wave-corpuscular duality ($m'\upsilon^2 = m'c^2$).

Moving at the speed $\upsilon = c$, a photon's moving mass m' possesses kinetic energy $m'c^2$ and its matter wave's frequency is $\nu = \frac{m'c^2}{h}$, which is exactly its electromagnetic wave's frequency. Thus, the new relativistic mechanics unites the electromagnetic wave of light with photon's matter wave. The light is both electromagnetic wave and matter wave.

A photon has total energy $m'c^2 + m'c^2 = 2m'c^2$, in which one $m'c^2$ is the photon's moving mass-energy embodying its ponderable corpuscularity and another $m'c^2$ is the photon's kinetic energy possessed by its moving mass to display its wave property. Since $\nu = \frac{m'c^2}{h}$, so **a photon's total energy is** $m'c^2 + m'c^2 = 2m'c^2 = 2h\nu$**, not** $h\nu$**.**

§3 Interpretation of the Hydrogen Atom's Quantum Structure

§3.1. Interpretation Based on the Matter Wave Theory and the New Relativistic Mechanics.

According to the new relativistic mechanics, an electron (static mass m_e) moving with constant speed υ_e has moving mass $m_e' = \frac{m_e}{\sqrt{1+\upsilon_e^2/c^2}}$, momentum $P_e = m_e'\upsilon_e$ and kinetic energy of its moving mass $E = m_e'\upsilon_e^2$. Its matter wave's frequency and wavelength are $\nu_e = \frac{m_e'\upsilon_e^2}{h}$ and $\lambda_e = \frac{h}{m_e'\upsilon_e}$. Its matter wave's velocity $\nu_e\lambda_e = \upsilon_e$, so the electron always moves together with its own matter wave.

Suppose the electron moves along a circular orbit (see §3.3.2) of radius r_e around an hydrogen atom's nucleus. In order to avoid the matter wave's self-interference, the length of one orbital round must be $2\pi r_e = n\lambda_e = n\frac{h}{P_e}$ or $P_e r_e = n\hbar$, where n is the integral number of matter waves on one orbital round and also is the orbit's quantum number.

The electron's orbital period is $T = \frac{2\pi r_e}{\upsilon_e} = \frac{nh}{P_e\upsilon_e}$, its **<u>orbital</u> frequency** is $\nu_T = \frac{1}{T} = \frac{P_e\upsilon_e}{nh}$. Since there are n matter waves on one round of orbit during an orbital period T, so the **<u>matter wave's</u> frequency** is $\nu_e = \frac{n}{T} = n\nu_T$. Thus, $\nu_e = n\frac{P_e\upsilon_e}{nh} = \frac{m_e'\upsilon_e^2}{h} = \frac{E}{h}$, which again proves that the energy E in the matter wave's frequency formula $\nu = \frac{E}{h}$ is exactly the physical quantity $m'\upsilon^2$ discovered by our new relativistic mechanics.

Moving with constant velocity υ_e along a circular orbit inside a hydrogen atom, an electron is under a balanced action of centrifugal force and the Coulomb force of attraction: $\frac{m'_e \upsilon_e^2}{r_e} = \frac{e^2}{r_e^2}$, where e is the electron's charge. Thus, $\upsilon_e = \frac{e^2}{m'_e \upsilon_e r_e} = \frac{e^2}{P_e r_e} = \frac{e^2}{n\hbar}$. The electron's all physical quantities are quantized:

$$\upsilon_e = \frac{e^2}{n\hbar},\ P_e = m'_e\upsilon = \frac{m'_e e^2}{n\hbar},\ E_n = m'_e \upsilon_e^2 = \frac{m'_e e^4}{n^2\hbar^2} = \frac{e^2}{r_e},\ r_e = \frac{n\hbar}{P_e} = \frac{n^2\hbar^2}{m'_e e^2},$$

$$\lambda_e = \frac{h}{P_e} = \frac{2\pi n\hbar^2}{m'_e e^2},\ \nu_e = \frac{E_n}{h} = \frac{m'_e e^4}{2\pi n^2\hbar^3},\ \nu_T = \frac{\nu_e}{n} = \frac{m'_e e^4}{2\pi n^3\hbar^3},\ T = \frac{1}{\nu_T} = \frac{2\pi n^3\hbar^3}{m'_e e^4}.$$

In case of $n = 1$, the electron has minimal orbital radius $r_e = \frac{\hbar^2}{m'_e e^2} \approx 0.529\times 10^{-10}$ *m* (the Bohr radius) and maximal velocity $\upsilon_e = \frac{e^2}{\hbar} \approx 2.188\times 10^6$ *m/s* $<< c$, so that $m'_e \approx m_e$.

The less the quantum number ($n_i < n_j$), the shorter the orbit's radius ($r_i < r_j$) and the larger the electron's velocity and kinetic energy ($\upsilon_{ei} > \upsilon_{ej}$, $E_{n_i} > E_{n_j}$). When the electron transits from an inner orbit n_i into an outer orbit n_j, it releases its extra energy ($E_{n_i} - E_{n_j}$) to radiate a photon of frequency ν. A **photon's total energy is $2h\nu$, not $h\nu$** (see §2). Therefore,

$$2h\nu = E_{n_i} - E_{n_j} = \frac{m_e e^4}{\hbar^2}\left(\frac{1}{n_i^2} - \frac{1}{n_j^2}\right) \quad \text{or} \quad \nu = \frac{m_e e^4}{4\pi\hbar^3}\left(\frac{1}{n_i^2} - \frac{1}{n_j^2}\right).$$

This is the Balmer formula.

§3.2. Mistakes in Classical Quantum Mechanics.

It is well known that **a single body or a single system <u>alone by itself</u> does not possess any potential energy**. A body in a pair with another body has certain potential energy with regard to the other. In a pair as a whole, however, the two bodies' potential energies are mutually offset. **The pair as a whole is a single system and does not have any potential energy available to be released out by the system**.

A hydrogen atom includes a nucleus and an electron moving around the nucleus. The electron has potential energy $-\frac{\kappa}{r_e}$ with regard to the nucleus, where r_e is the distance from the electron to the nucleus and κ is the constant of the Coulomb attraction. The nucleus has potential energy $-\frac{\kappa}{R}$ with regard to the electron, where R is the distance from the nucleus to the electron. Since $R=-r_e$ and $-\frac{\kappa}{R}+\left(-\frac{\kappa}{r_e}\right)=0$, the two potential energies inside of the hydrogen atom are mutually offset so that **the hydrogen atom as a whole system does not have any potential energy available to be radiated**.

A hydrogen atom's total energy includes its mass-energy, which remains constant, and the moving electron's kinetic energy which depends on the electron's velocity. **The energy available for the radiation may come only from the electron's transition from an inner orbit, where it has larger velocity (because of shorter radius) and larger kinetic energy, into an outer orbit where it has less velocity (because of larger radius) and less kinetic energy.**

Classical theory contains two mistakes: (1) It assumes the Newtonian kinetic energy $\frac{1}{2}m_e\upsilon_e^2$ as a part of the energy to be radiated, which causes the electron's separation from its own matter wave; (2) It assumes the electron's potential energy $-\frac{\kappa}{r_e}$, nullified by the potential energy of the nucleus, as another part of the energy to be radiated.

Therefore, $$E=\frac{1}{2}m_e\upsilon_e^2-\frac{k}{r_e}=\frac{1}{2}m_e\upsilon_e^2-\frac{e^2}{r_e}$$ ($\kappa=e^2$ for the hydrogen atom).

Since $\frac{m_e\upsilon_e^2}{r_e}=\frac{e^2}{r_e^2}$ and $r_e=\frac{n^2\hbar^2}{m_e e^2}$, so $E=\frac{1}{2}m_e\upsilon_e^2-\frac{e^2}{r_e}=\frac{1}{2}\frac{e^2}{r_e}-\frac{e^2}{r_e}=-\frac{1}{2}\frac{e^2}{r_e}=-\frac{m_e e^4}{2n^2\hbar^2}<0$. Classical theory has to accept the unreasonable **negative energy** $E<0$, which leads to a **wrong direction of the electron's transition for the hydrogen to be able to radiate energy:** The hydrogen atom radiates as the moving electron releases its energy when it transits from an outer orbit with **less negative** E into an inner orbit with **more negative** E.

Worse, viewed from the de Broglie matter wave theory which appeared later, the moving electron's negative energy $E<0$ means it's matter wave has not only negative frequency $\nu=\frac{E}{h}<0$ but also negative phase velocity $\nu\lambda=\frac{E}{P}<0$ so that the electron and its matter wave would move apart in opposite directions. Moreover, the classical theory has to maintain that a photon's total energy is $h\nu$, not $2h\nu$, which denies a photon of its wave-particle duality (see §2).

Only by this wrong way, Bohr's classical theory can obtain the Balmer formula:

$$h\nu=\left(-\frac{m_e e^4}{2n_j^2\hbar^2}\right)-\left(-\frac{m_e e^4}{2n_i^2\hbar^2}\right)\qquad\text{or}\qquad \nu=\frac{m_e e^4}{4\pi\hbar^3}\left(\frac{1}{n_i^2}-\frac{1}{n_j^2}\right).$$

§3.3. New Interpretation Based on the Correspondence Principle.

Bohr built the hydrogen atom's quantum structure in 1913 by use of his correspondence principle. Because, at that time, the de Broglie matter wave theory had not been born yet. We will first discuss Bohr's approach in a detailed way and then show our new approach based on the new relativistic mechanics,.

§3.3.1. Bohr's Classical Approach.

Bohr assumes the Keplerian elliptic orbit for a charged particle to move around a nucleus and proposes a relationship $E_n = h\nu_T f(n)$ between the **charged particle's energy** E_n and its orbital frequency ν_T on an orbit with the quantum number n. He also defines the **orbit's conservative energy** as $E = \frac{1}{2}mv_e^2 - \frac{k}{r_e}$, where $-\frac{k}{r_e}$ is the charged particle's potential energy at a distance r_e from the nucleus, m is its mass and $\frac{1}{2}mv_e{}^2$ is its non-relativistic Newtonian kinetic energy. The elliptic orbit requires the **orbit's conservative energy** $E < 0$. Because, if $E = \frac{1}{2}m_e v_e^2 - \frac{k}{r_e} = 0$, then $\frac{1}{2}mv_e{}^2 = \frac{\kappa}{r_e}$, the charged particle would move radially with regard to the nucleus; if $E > 0$, then the particle would escape from the nucleus along a hyperbolic path.

It is well known that, given an orbit's conservative energy $E < 0$, the conservation laws of the orbit's angular momentum and the orbit's conservative energy leads to **<u>a whole family</u>** of elliptic orbits with common semi-major axis a, common conservative energy $E = -\frac{\kappa}{2a} < 0$ and common orbital frequency $\nu_T = \frac{1}{2\pi}\sqrt{\frac{\kappa}{m}}a^{-3/2}$. It is important to note, however, E and ν_T are irrelevant to semi-minor axis b. Therefore, the **family** contains countless elliptic orbits with the same a but different b, including orbits with $b \approx 0$. **A charged particle may move along an elliptic orbit with semi-minor axis $b \approx 0$ and appear absurdly close to or even collide with the nucleus!**

Next, Bohr assumes that the orbit's conservative energy E and the charged particle's energy E_n are the same so that the particle has negative energy $E_n = E < 0$! **Actually, however, an orbit's conservative energy E <u>only decides the orbit's form</u>; it has nothing to do with the charged particle's energy E_n**.

Further, by use of his correspondence principle, he deduces $f(n) = -\frac{n}{2}$ (see Appendix §1) and obtains:

$$E_n = E = -\frac{n}{2}h\nu_T = -\frac{nh}{4\pi}\sqrt{\frac{\kappa}{m}}a^{-3/2} < 0.$$

The combination of $E = -\frac{\kappa}{2a}$ with the above equation gives:

$$E_n = E = -\frac{2\pi^2 m\kappa^2}{n^2h^2} = -\frac{m\kappa^2}{2n^2\hbar^2} < 0 \qquad \text{and} \qquad a = \frac{n^2h^2}{4\pi^2 m\kappa} = \frac{n^2\hbar^2}{m\kappa}.$$

For the electron in the hydrogen atom, $m = m_e$ and $\kappa = e^2$; so he has $E_n = E = -\frac{m_e e^4}{2n^2\hbar^2} < 0$.

Bohr's confusing assumption of $E_n = E$ makes the electron having negative energy $E_n < 0$. If $n_j > n_i$, then $E_{n_j} > E_{n_i}$, which forces him to conclude that the hydrogen atom radiates when its electron transits from an outer orbit into an inner orbit, despite the fact that the less the orbit's radius the larger the electron's velocity and kinetic energy. Bohr accepts Einstein's theory of photoelectric quantum, according to which a photon of frequency ν has total energy $h\nu$, not $2h\nu$. Finally, he obtains the Balmer formula:

$$h\nu = E_{n_j} - E_{n_i} = \frac{m_e e^4}{2\hbar^2}\left[\left(-\frac{1}{n_j^2}\right) - \left(-\frac{1}{n_i^2}\right)\right] \qquad \text{or} \qquad \nu = \frac{m_e e^4}{4\pi\hbar^3}\left(\frac{1}{n_i^2} - \frac{1}{n_j^2}\right).$$

Bohr's approach suffers from five shortcomings: (1) His elliptic orbit may cause the electron's collision with the nucleus; (2) He confuses the electron's energy E_n with the elliptic orbit's conservative energy E, which renders the electron to have negative energy $E_n < 0$ because of $E < 0$; (3) He accepts electron's non-relativistic Newtonian kinetic energy, which separates the electron from its own matter wave; (4) He deems the potential energy between the electron and the nucleus, which as a whole is mutually nullified, as releasable energy; (5) He deems that a photon's total energy is $h\nu$, not $2h\nu$, and thus denies photon's wave-particle duality. The five shortcomings lead to the wrong direction of the electron's transition, although superficially he seems to have obtained the Balmer formula.

§3.3.2. Approach Based on the New Relativistic Mechanics and the de Broglie Matter Wave Theory.

Based on the matter wave theory, we can prove that, moving around a nucleus, a charged particle must have circular orbits.

In polar coordinates, the equation of an ellipse is $r = \dfrac{b^2}{a + C\cos\theta}$, where a and b are semi-major and semi-minor axes respectively, $C = \sqrt{a^2 - b^2}$ is semi-focal distance, θ is the argument. Since an elliptic orbit's conservative energy is $E = \dfrac{1}{2}m\upsilon^2 - \dfrac{\kappa}{r} = -\dfrac{\kappa}{2a}$, so

$$\upsilon^2 = \frac{2\kappa}{m}\left(\frac{1}{r} - \frac{1}{2a}\right) \qquad \text{or} \qquad \upsilon^2 = \frac{2\kappa}{m}\left(\frac{C}{b^2}\cos\theta + \frac{a}{b^2} - \frac{1}{2a}\right).$$

According to the new relativistic mechanics, the matter wave's frequency is $\nu = \dfrac{m\upsilon^2}{h}$. As the velocity υ varies along the orbit, the matter wave's frequency ν varies accordingly:

$$\nu = \frac{2\kappa}{h}\left(\frac{C}{b^2}\cos\theta + \frac{a}{b^2} - \frac{1}{2a}\right).$$

For a whole round of an elliptic orbit, the value of the matter wave's frequency is:

$$\nu = \frac{1}{2\pi}\int_0^{2\pi}\frac{2\kappa}{h}\left(\frac{C}{b^2}\cos\theta + \frac{a}{b^2} - \frac{1}{2a}\right)d\theta = \frac{2\kappa}{h}\left(\frac{a}{b^2} - \frac{1}{2a}\right).$$

On the other hand, for an orbit with quantum number n, there must be the same integral n number of matter waves forming a smooth head-tail continuity on a whole round of the orbit in order to avoid the matter wave's self-interference. If a particle's orbital period is T and one round of its orbit has n matter waves, then its **matter wave's frequency** is $\nu = \frac{n}{T} = n\nu_T$, where ν_T is the **orbital frequency**. However, as above mentioned , the family of elliptic orbits has common orbital frequency $\nu_T = \frac{1}{2\pi}\sqrt{\frac{\kappa}{m}}a^{-3/2}$ and common semi-major axis $a = \frac{n^2h^2}{4\pi^2 m\kappa}$.

Therefore,
$$\frac{2\kappa}{h}\left(\frac{a}{b^2} - \frac{1}{2a}\right) = \frac{n}{2\pi}\sqrt{\frac{\kappa}{m}}a^{-3/2} \quad \text{and} \quad \sqrt{a} = \frac{nh}{2\pi\sqrt{m\kappa}}.$$

From these two equations, we can obtain $\frac{a^2}{b^2} - \frac{1}{2} = \frac{1}{2}$ or $a = b$ so that $C = 0$ and $r = \frac{b^2}{a} = a$ so that

$$v^2 = \frac{2\kappa}{m}\left(\frac{a}{b^2} - \frac{1}{2a}\right) = \frac{\kappa}{ma} \quad \text{or} \quad mv^2 - \frac{k}{a} = 0$$

which is exactly a charged particle's kinematic equation of circular motion in a central Coulomb force field. This means that, **moving around a nucleus, a charged particle is destined to have only circular orbits.**

Thus, a correct approach must first assume circular orbits instead of the Keplerian elliptic orbits. All circular orbits have zero conservative orbital energy $E = mv^2 - \frac{\kappa}{r} = 0$. Being always zero, E cannot be the source of energy radiated by an atom due to its electron's transition from one circular orbit to another circular orbit. Let's stress again: **Orbit's conservative energy E (<0, $=0$ or >0) decides only the orbit's form but has nothing to do with the energy radiated by an atom.** The source of the radiated energy is the **electron's kinetic energy** E_n which varies from one circular orbit to another circular orbit. E_n must not be confused with the **orbit's conservative energy** E which is always a constant zero for all circular orbits.

Let a circular orbit with energy level n have radius $r_n = a$. In Appendix §2, by use of the correspondence principle, we have proven $E_n = \frac{nh}{2\pi}\sqrt{\frac{\kappa}{m}}a^{-3/2}$. According to the new relativistic mechanics, on the other hand, the kinetic energy of a moving particle is $E_n = m'v^2 \approx mv^2$ (for $v << c$) and its kinematic equation on a circular orbit of radius a is $mv^2 = \frac{\kappa}{a}$. So, $E_n = \frac{\kappa}{a}$. Thus, the combination of $E_n = \frac{\kappa}{a}$ with $E_n = \frac{nh}{2\pi}\sqrt{\frac{\kappa}{m}}a^{-3/2}$ gives:

$$E_n = \frac{4\pi^2 m\kappa^2}{n^2h^2} > 0 \quad \text{and} \quad a = \frac{n^2h^2}{4\pi^2 m\kappa}.$$

For the electron in the hydrogen atom, $m = m_e$ and $\kappa = e^2$ so that:

$$E_n = \frac{4\pi^2 m_e e^4}{n^2 h^2} \qquad \text{and} \qquad a = \frac{n^2 h^2}{4\pi^2 m_e e^2}.$$

The smaller the quantum number n the smaller the orbit's radius a and, correspondingly, the larger the electron's velocity υ_e and kinetic energy $E_n = m\upsilon_e^2$. Thus, the hydrogen atom radiates energy when its electron transits from an inner n_i orbit into an outer n_j orbit ($i < j$).

According to the new relativistic mechanics, a photon of frequency ν has total energy $2h\nu$. Finally, we obtain the Balmer formula with the correct direction of an electron's transition:

$$E_{n_i} - E_{n_j} = 2h\nu \quad \text{or} \quad \nu = \frac{1}{2h}\left(E_{n_i} - E_{n_j}\right) = \frac{2\pi^2 m_e e^4}{h^3}\left(\frac{1}{n_i^2} - \frac{1}{n_j^2}\right) = \frac{m_e e^4}{4\pi\hbar^3}\left(\frac{1}{n_i^2} - \frac{1}{n_j^2}\right).$$

To sum up, our conclusions are:

(1) The hydrogen atom's electron can only have quantized circular orbits. Only circular orbits can match the correspondence principle with the matter wave theory. Elliptic orbits may cause the electron to break the minimal Bohr radius or even to collide with the nucleus. Historically, Schrodinger tried and failed to find out a non-expansive wave packet of electron moving along a classical Keplerian elliptic orbit in the hydrogen atom. Our approach proves that his failure is destined by his assumption of the elliptic orbit.

(2) The correspondence principle is correct and effective per se. What is wrong, besides the assumption of elliptic orbits with negative **orbital conservative energy** $E < 0$, is Bohr's confused assumption of the **electron's energy** $E_n = E < 0$. The negative energy leads to the wrong direction of electron's transition for the hydrogen atom to radiate energy.

(3) With this issue as an example, we have proven that all the quantum mechanics must be relativistic even in case of $\upsilon << c$. Particularly, a moving particle's moving mass possesses kinetic energy $m'\upsilon^2$, neither Newtonian $\frac{1}{2}m\upsilon^2$ nor Einstein's $(m' - m)c^2$, and a photon's total energy is $2h\nu$, not $h\nu$.

§4 The Photon Scattering

In the Compton photon scattering experiments, incident X-ray photons (wavelength λ and frequency ν) collide with and bounce back from target electrons (static mass m_e). The Compton experiments shows that the relationship between the scattered photon's wavelength λ_1 and the scattering angle θ is $\lambda_1 = \lambda + \lambda_c(1 - \cos\theta)$, where $\lambda_c = \frac{h}{m_e c} \approx 0.024263\dot{A}$ is the so-called Compton wavelength of electron.

The X-ray photon's wavelength is in the range of $\lambda \approx (1\text{~}10)\,\dot{A}$ and, because of $\lambda = \frac{h}{P} = \frac{h}{m'c}$, its moving mass is in the range of $m' = h/\lambda c \approx 2.21\times(10^{-32}\text{~}10^{-33})kg$, much less than electron's $m_e \approx 9.11\times10^{-31}kg$. So, even a hard X-ray's incident photon with $\lambda = 1\dot{A}$ can only give a target electron very small velocity υ_e, while the scattered photon's wavelength increases very little. By use of (3), for example, we can get the following results:

$$\lambda = 1\dot{A}$$

Collision	Oblique		Head-on
θ	60°	90°	180°
λ_1	1.01213 $\dot{A}$	1.02426 $\dot{A}$	1.04582 $\dot{A}$
υ_e/c	0.02408	0.03391	0.04735

Based on Einstein's mechanics, the classical theory explains the Compton photon scattering formula as follows. The laws of conservation of energy and momentum for the photon and the target electron before and after the collision can be expressed as:

$$\left.\begin{aligned} h\nu - h\nu_1 &= E_{ke} \\ P^2 - P_1^2 - 2PP_1\cos\theta &= P_e^2 \end{aligned}\right\} \qquad (1)$$

where ν and ν_1 are the incident photon's and the scattered photon's frequencies respectively, E_{ke} and P_e are the kinetic energy and the momentum acquired by the target electron which becomes moving at velocity υ_e after the collision, $P = \frac{h}{\lambda} = \frac{h\nu}{c}$ and $P_1 = \frac{h}{\lambda_1} = \frac{h\nu_1}{c_1} = \frac{h\nu_1}{c}$ (because of Einstein's constant speed of light $c_1 = c$) are the photon's momentums before and after the collision respectively,.

According to Einstein's mechanics, the target electron with velocity υ_e has moving mass $m_e' = \frac{m_e}{\sqrt{1-\upsilon_e^2/c^2}}$, kinetic energy $E_{ke} = (m_e' - m_e)c^2 = m_e c^2\left(\frac{1}{\sqrt{1-\upsilon_e^2/c^2}} - 1\right)$ and momentum $P_e = m_e'\upsilon_e = \frac{m_e v_e}{\sqrt{1-\upsilon_e^2/c^2}}$. So, the equations in (1) become:

$$\frac{1}{c^2}\left(h\nu - h\nu_1 + m_e c^2\right)^2 = \frac{m_e^2 c^2}{1-\upsilon_e^2/c^2}$$

$$\frac{h^2}{c^2}\left(\nu^2 + \nu_1^2 - 2\nu\nu_1\cos\theta\right) = \frac{m_e^2\upsilon_e^2}{1-\upsilon_e^2/c^2}$$

or

$$\frac{1}{c^2}\left(h\nu - h\nu_1 + m_e c^2\right)^2 - \frac{h^2}{c^2}\left(\nu^2 + \nu_1^2 - 2\nu\nu_1\cos\theta\right) = m_e^2 c^2 \qquad (2)$$

$$\frac{\nu^2 + \nu_1^2 - 2\nu\nu_1\cos\theta}{\left(\nu - \nu_1 + m_e c^2/h\right)^2} = \frac{\upsilon_e^2}{c^2}$$

Therefore, $\nu = c/\lambda$ and $\nu_1 = c/\lambda_1$ lead to the Compton photon scattering formula:

$$\lambda_1 = \lambda + \lambda_c(1-\cos\theta) \qquad (3)$$

$$\frac{v_e^2}{c^2} = 1 - \left(\frac{1}{1 - \lambda_c / \lambda_1 + \lambda_c / \lambda} \right)^2$$

The Compton photon scattering experiment proves the photon's corpuscularity. It seems to be able to justify Einstein's mechanics, too. Actually, it is not so. Based on the Newtonian mechanics, we also can obtain the same formula (3). Indeed, with the Newtonian $E_{ke} = \frac{1}{2} m_e v_e^2$, we can have:

$$h\nu - h\nu_1 = \frac{1}{2} m_e v_e^2$$

$$P^2 + P_1^2 - 2PP_1 \cos\theta = (m_e v_e)^2$$

Placing $P = \frac{h\nu}{c}$, $P_1 = \frac{h\nu_1}{c}$, $\nu = \frac{c}{\lambda}$, $\nu_1 = \frac{c_1}{\lambda_1} = \frac{c}{\lambda_1}$ and $\lambda_c = \frac{h}{m_e c}$ into the above equations, we obtain:

$$\lambda_1 = \lambda + \lambda_c \left(\frac{\lambda_1^2 + \lambda^2}{2\lambda\lambda_1} - \cos\theta \right) = \lambda + \lambda_c (1 - \cos\theta) + \lambda_c \frac{(\lambda_1 - \lambda)^2}{2\lambda\lambda_1}$$

or $$\frac{\lambda_c}{2\lambda\lambda_1}(\lambda_1 - \lambda)^2 - (\lambda_1 - \lambda) + \lambda_c (1 - \cos\theta) = 0 \quad \text{or} \quad \lambda_1 - \lambda = \frac{1 - \sqrt{1 - \frac{2\lambda_c^2}{\lambda\lambda_1}(1 - \cos\theta)}}{\lambda_c / \lambda\lambda_1}.$$

Since $\frac{\lambda_c^2}{\lambda\lambda_1} << 1$ in case of the X-ray's photon ($\lambda >> \lambda_c$ and $\lambda_1 << \lambda_c$), we have:

$$\sqrt{1 - \frac{2\lambda_c^2}{\lambda\lambda_1}(1 - \cos\theta)} = 1 - \frac{\lambda_c^2}{\lambda\lambda_1}(1 - \cos\theta) - \frac{1}{2}\left[\frac{\lambda_c^2}{\lambda\lambda_1}(1 - \cos\theta) \right]^2 - \cdots\cdots \approx 1 - \frac{\lambda_c^2}{\lambda\lambda_1}(1 - \cos\theta).$$

Therefore, $$\lambda_1 - \lambda = \lambda_c (1 - \cos\theta) \quad \text{or} \quad \lambda_1 = \lambda + \lambda_c (1 - \cos\theta).$$

Obviously, the Compton formula (3) cannot support Einstein against Newton.

Moreover, the Compton formula (3) contradicts the principle of elastic collision, according to which if an incident particle's mass is the same as or more than a target particle's mass, then the incident particle will not bounce back and the scattering angle cannot be $\theta > 90°$. Yet, if an incident photon's wavelength is $\lambda = \lambda_c \approx 0.024236\dot{A}$, which is a γ**-photon,** then because of $\lambda = \frac{h}{m'c}$ and $\lambda_c = \frac{h}{m_e c}$ the incident γ**-photon** has moving mass $m' = \frac{h}{\lambda c} = \frac{h}{\lambda_c c} = m_e$. In such a case, the Compton formula (3) contradictorily allows $\theta > 90°$:

$$\lambda = \lambda_c, \; m' = m_e$$

Collision	Oblique		Head-on
θ	90°	120°	180°

λ_1	$0.04853\,\dot{A}$	$0.06066\,\dot{A}$	$0.07279\,\dot{A}$
υ_e/c	0.7454	0.7806	0.800

According to the principle of elastic collision, if $m' = m_e$, a head-on collision will cause the incident particle to stop so that $\theta = 180\,°$ is impossible. The target particle in turn receives all the incident particle's original velocity so that there must be $\upsilon_e = c$. The above table shows that **the Compton formula (3) violates the principle of elastic collision.** It cannot explain the scattering of high-energy photons with short wavelength $\lambda \le \lambda_c$ such as **γ-photons**.

New relativistic mechanics can provide a new photon scattering formula compatible with the principle of elastic collision for photons with any wavelength. According to new relativistic mechanics, photons are ponderable particles and have static mass. An incident photon (static mass m, moving mass m', velocity c, wavelength λ and frequency ν) becomes a scattered photon (static mass m_1, moving mass m_1', velocity c_1, wavelength λ_1 and frequency ν_1) after its collision with a target electron. The target electron (static mass m_e) gets velocity υ_e, momentum $P_e = m_e'\upsilon_e = \dfrac{m_e v_e}{\sqrt{1+\upsilon_e^2/c^2}}$ and kinetic energy $E_{ke} = m_e c^2\left(\sqrt{1+\upsilon_e^2/c^2}-1\right)$.

The incident photon has moving mass $m' = \dfrac{m}{\sqrt{1+c^2/c^2}} = \dfrac{m}{\sqrt{2}}$, moving mass-energy $m'c^2$, kinetic energy possessed by its moving mass $m'c^2$, total energy $m'c^2 + m'c^2 = 2m'c^2 = 2h\nu$ and frequency $\nu = m'c^2/h$. The scattered photon has velocity c_1, moving mass $m_1' = \dfrac{m}{\sqrt{1+c_1^2/c^2}}$, moving mass-energy $m_1'c^2$, kinetic energy possessed by its moving mass $m_1'c_1^2$, total energy $m_1'c^2 + m_1'c_1^2 = m_1'c_1^2\left(\dfrac{c^2}{c_1^2}+1\right) = h\nu_1\left(\dfrac{c^2}{c_1^2}+1\right)$ and frequency $\nu_1 = \dfrac{m_1'c_1^2}{h}$. Therefore,

$$\frac{\nu}{\nu_1} = \frac{m'c^2}{m_1'c_1^2} = \frac{m/\sqrt{2}}{m/\sqrt{1+c_1^2/c^2}} \cdot \frac{c^2}{c_1^2} = \sqrt{\frac{1+c_1^2/c^2}{2}} \cdot \frac{c^2}{c_1^2}.$$

Before the collision, we have $c_1 = c$, the above equation gives exactly $\nu_1 = \nu$. Indeed, our new relativistic mechanics is coherent. After the collision, the scattered photon's total energy can also be expressed as:

$$h\nu_1\left(\frac{c^2}{c_1^2}+1\right) = h\nu_1\frac{c^2}{c_1^2} + h\nu_1 = \sqrt{\frac{2}{1+c_1^2/c^2}}h\nu + h\nu_1.$$

Therefore, the laws of conservation of energy and momentum before and after the collision can be expressed as:

$$2h\nu - \left(\sqrt{\frac{2}{1+c_1^2/c^2}}\,h\nu + h\nu_1\right) = m_e c^2\left(\sqrt{1+\upsilon_e^2/c^2}-1\right)$$

and

$$P^2 + P_1^2 - 2PP_1\cos\theta = \frac{m_e^2\upsilon_e^2}{1+\upsilon_e^2/c^2}$$

or

$$\frac{1}{c^2}\left[\left(2-\sqrt{\frac{2}{1+c_1^2/c^2}}\right)h\nu - h\nu_1 + m_e c^2\right]^2 = m_e^2\left(c^2+\upsilon_e^2\right) \qquad (4)$$

and

$$\left(P^2 + P_1^2 - 2PP_1\cos\theta\right)\left(1+\upsilon_e^2/c^2\right) = m_e^2\upsilon_e^2\,. \qquad (5)$$

$(4)-(5)$ gives:

$$\frac{1}{c^2}\left[\left(2-\sqrt{\frac{2}{1+c_1^2/c^2}}\right)h\nu - h\nu_1 + m_e c^2\right]^2 - \left(P^2 + P_1^2 - 2PP_1\cos\theta\right)\left(1+\upsilon_e^2/c^2\right) = m_e^2c^2\,. \qquad (6)$$

If the incident photon's static mass and the target electron's static mass are the same $m = m_e$, then the incident photon's moving mass and wavelength are:

$$m' = \frac{m}{\sqrt{2}} = \frac{m_e}{\sqrt{2}} \qquad \text{and} \qquad \lambda = \frac{h}{m'c} = \sqrt{2}\,\frac{h}{m_e c} = \sqrt{2}\lambda_c\,.$$

In such a case, according to the principle of elastic collision, a head-on collision must make the incident photon stop ($c_1 = 0$) and the target electron receive all the incident photon's velocity ($\upsilon_e = c$). Indeed, when $c_1 = 0$ so that $\nu_1 = \frac{m_1'c_1^2}{h} = 0$ and $P_1 = m_1'c_1 = 0$, our formula (6) becomes:

$$\left[\left(2-\sqrt{2}\right)\frac{1}{\lambda} + \frac{1}{\lambda_c}\right]^2 - \frac{1}{\lambda^2}\left(1+\frac{\nu_e^2}{c^2}\right) = \frac{1}{\lambda_c^2} \qquad (6')$$

Since $\lambda = \sqrt{2}\lambda_c$, we get exactly $\upsilon_e = c$, which completely **meets the principle of elastic collision.**

On the other hand, if a long-wavelength photon, such as the X-ray photon, collides with a target electron, then the electron gains very little velocity ($\upsilon_e/c \approx 0$) and the photon's velocity almost remains unchanged ($c_1 \approx c$). In such a case, formula (6) becomes formula (2) because of $P = \frac{h\nu}{c}$ and $P_1 = \frac{h\nu_1}{c_1} = \frac{h\nu_1}{c}$. **Indeed, the Compton formula (3) can only approximately explain long-wavelength incident photon's, such as the X-ray photon's, scattering.**

By use of (4) and (5) to eliminate υ_e^2, we can obtain:

$$\left[\left(2-\sqrt{\frac{2}{1+c_1^2/c^2}}\right)\frac{h\nu}{c}-\frac{h\nu_1}{c}+m_ec\right]^2\left(P^2+P_1^2-2PP_1\cos\theta-m_e^2c^2\right)+m_e^4c^4=0\,.$$

Since $\frac{\nu}{c}=\frac{1}{\lambda}$, $\frac{\nu_1}{c}=\frac{\nu_1}{c_1}\cdot\frac{c_1}{c}=\frac{c_1}{\lambda_1 c}$, $P=\frac{h}{\lambda}$, $P_1=\frac{h}{\lambda_1}$, $\lambda_c=\frac{h}{m_ec}$, the above equation becomes:

$$\left[\left(2-\sqrt{\frac{2}{1+c_1^2/c^2}}\right)\frac{1}{\lambda}-\frac{c_1}{\lambda_1 c}+\frac{1}{\lambda_c}\right]^2\left(\frac{1}{\lambda^2}+\frac{1}{\lambda_1^2}-\frac{2\cos\theta}{\lambda\lambda_1}-\frac{1}{\lambda_c^2}\right)+\frac{1}{\lambda_c^4}=0\,.$$

Because of $\lambda=\frac{h}{m'c}$ and $\lambda_1=\frac{h}{m_1'c_1}$, where $m'=\frac{m}{\sqrt{1+c^2/c^2}}=\frac{m}{\sqrt{2}}$ and $m_1'=\frac{m}{\sqrt{1+c_1^2/c^2}}$, we can get

$\frac{\lambda}{\lambda_1}=\frac{\sqrt{2}\,c_1/c}{\sqrt{1+c_1^2/c^2}}$ so that $1+c_1^2/c^2=\frac{2}{2-\lambda^2/\lambda_1^2}$ or $\frac{c_1}{c}=\frac{\lambda/\lambda_1}{\sqrt{2-\lambda^2/\lambda_1^2}}$. Thus, the above equation becomes:

$$\left[\left(2-\sqrt{2-\lambda^2/\lambda_1^2}\right)\frac{1}{\lambda}-\frac{\lambda/\lambda_1}{\sqrt{2-\lambda^2/\lambda_1^2}}\cdot\frac{1}{\lambda_1}+\frac{1}{\lambda_c}\right]^2\left(\frac{1}{\lambda^2}+\frac{1}{\lambda_1^2}-\frac{2\cos\theta}{\lambda\lambda_1}-\frac{1}{\lambda_c^2}\right)+\frac{1}{\lambda_c^4}=0\,.$$

Finally, **our photon scattering formula** is:

$$\left[2\left(1-\frac{1}{\sqrt{2-\lambda^2/\lambda_1^2}}\right)\frac{\lambda_c}{\lambda}+1\right]^2\left[\frac{\lambda_c^2}{\lambda^2}\left(1+\frac{\lambda^2}{\lambda_1^2}-2\frac{\lambda}{\lambda_1}\cos\theta\right)-1\right]+1=0\,. \qquad (7)$$

If the incident photon's mass is the same as the target electron's mass ($m=m_e$), then the incident photon's wavelength is $\lambda=\frac{h}{m'c}=\sqrt{2}\frac{h}{mc}=\sqrt{2}\frac{h}{m_ec}=\sqrt{2}\lambda_c\approx 0.034313\dot{A}$. This is soft γ-ray. For such a γ-photon's scattering case, we can use (7), (6') and $\frac{c_1}{c}=\frac{\lambda/\lambda_1}{\sqrt{2-\lambda^2/\lambda_1^2}}$ to calculate the γ-photon's scattering:

Incident γ-photon: $\lambda=\sqrt{2}\lambda_c$, $m=m_e$.

Collision	Oblique						Head-on
λ/λ_1	0.999	0.99	0.90	0.50	0.10	0.01	0
λ_1/λ_c	1.4156	1.4285	1.5713	2.8284	14.142	141.42	∞
θ	4°18'	13°26'	38°25'	69°11'	86°7'	89°37'	no scattering
c_1/c	0.9980	0.9803	0.8250	0.3780	0.0709	0.0071	0
υ_e/c	0.0532	0.1669	0.4995	0.8997	0.9964	0.9999	1

Indeed, a head-on collision stops the incident γ-photon: $c_1 = 0$ and $\lambda_1 = \dfrac{h}{m'c_1} = \infty$ (the infinite wavelength means no matter wave). The term $\dfrac{\lambda}{\lambda_1}\cos\theta$ in (7) disappears, which means no scattering at all. The target electron fully receives the incident γ-photon's velocity so that $\upsilon_e/c = 1$. Our formula (7) completely conforms with the principle of elastic collision.

In contrast, the Compton formula (3) gives the following results:

Incident γ-photon: $\lambda = \sqrt{2}\lambda_c$, $m = m_e$

θ	4°18’	13°26’	38°25’	69°11’	86°7’	89°37’	90°	120°	180°
λ_1/λ_c	1.4170	1.4416	1.6305	2.0589	2.3464	2.4075	2.4142	2.9142	3.4142
υ_e/c	0.0527	0.1623	0.4052	0.5742	0.6249	0.6330	0.6339	0.6801	0.7071

In case of a head-on collision, the incident γ-photon does not stop but bounces backward diametrically in the opposite direction ($\theta = 180^\circ$) and the target electron does not receive all the incident photon's velocity ($\upsilon_e/c \neq 1$). All these indicate the violation of the principle of elastic collision.

Hopefully, **future experiments on the γ-photon scattering may clarify which mechanics is correct: Ours or Einstein’s .**

§5 The Diffraction of Electron

Suppose an electron with static mass m_e and static mass-energy $E_{me} = m_e c^2$ is accelerated and acquires velocity υ_e and kinetic energy E_{ke}. According to our new relativistic mechanics, the acquired kinetic energy is:

$$E_{ke} = m_e c^2\left(\sqrt{1+\upsilon_e^2/c^2} - 1\right) = E_{me}\left(\sqrt{1+\upsilon_e^2/c^2} - 1\right)$$

so that
$$\frac{\upsilon_e}{c} = \sqrt{\left(E_{ke}/E_{me}\right)^2 + 2\,E_{ke}/E_{me}}\ .$$

The electron’s matter wavelength is: $\lambda_e = \dfrac{h}{m_e'\upsilon_e} = \dfrac{h}{m_e\upsilon_e}\sqrt{1+\upsilon_e^2/c^2} = \dfrac{h}{m_e c}\dfrac{c}{\upsilon_e}\sqrt{1+\upsilon_e^2/c^2}$

or
$$\lambda_e = \lambda_c \frac{E_{ke}/E_{me} + 1}{\sqrt{\left(E_{ke}/E_{me}\right)^2 + 2\,E_{ke}/E_{me}}}\,,$$

where $\lambda_c = \dfrac{h}{m_e c}$ is electron’s Compton wavelength.

According to Einstein’s mechanics, the electron’s kinetic energy is:

$$E_{ke} = (m'_e - m_e)c^2 \qquad \text{or} \qquad E_{ke} = m_e c^2\left(\frac{1}{\sqrt{1-\upsilon_e^2/c^2}} - 1\right)$$

so that
$$\frac{\upsilon_e}{c} = \frac{\sqrt{(E_{ke}/E_{me})^2 + 2E_{ke}/E_{me}}}{E_{ke}/E_{me} + 1}.$$

The electron's matter wavelength is: $\lambda_e = \dfrac{h}{m'_e \upsilon_e} = \dfrac{h}{m_e \upsilon_e}\sqrt{1 - \dfrac{\upsilon_e^2}{c^2}}$

or
$$\lambda_e = \frac{\lambda_c}{\sqrt{(E_{ke}/E_{me})^2 + 2E_{ke}/E_{me}}}.$$

According to the Newtonian mechanics, the electron's kinetic energy is $E_{ke} = \dfrac{1}{2}m_e \upsilon_e^2$ and its matter wavelength is $\lambda_e = \dfrac{h}{m_e \upsilon_e}$ so that $\dfrac{\upsilon_e}{c} = \sqrt{2E_{ke}/E_{me}}$. Therefore,

$$\lambda_e = \frac{\lambda_c}{\sqrt{2E_{ke}/E_{me}}}.$$

If $E_{ke} << E_{me}$ (i.e., $\upsilon_e << c$) so that $E_{ke}/E_{me} << 1$ and $(E_{ke}/E_{me})^2 << 2E_{ke}/E_{me}$, then all the three mechanics give approximately the same $\dfrac{\upsilon_e}{c} \approx \sqrt{2E_{ke}/E_{me}}$ and $\lambda_e \approx \dfrac{\lambda_c}{\sqrt{2E_{ke}/E_{me}}}$. Thus, experiments on the diffraction of slow-moving electrons with low kinetic energy can only display the electron's wave property but cannot judge which one of the three mechanics is true and applicable to the quantum mechanics. In order to clearly judge which mechanics is correct, the diffraction experiments must deal with electrons moving at near-light velocity. Since 1992, physicists have been able to accelerate electrons to $\upsilon_e = 0.9999999994c$.

According to our new relativistic mechanics, electrons with such high velocity have the matter wavelength:

$$\lambda_e = \frac{h}{m'_e \upsilon_e} = \frac{h}{m_e \upsilon_e}\sqrt{1 + \frac{\upsilon_e^2}{c^2}} \approx \sqrt{2}\lambda_c \approx 0.034313\dot{A}.$$

Single-slit diffraction (or double-slit interference) experiments with the slit width (or the width between the double slits) $a = 0.1\dot{A}$ will result in a clearly measurable angle of diffraction:

$$\sin\theta = \lambda_e/a = 0.34313 \qquad \text{or} \qquad \theta \approx 20^\circ 4'.$$

According to Newtonian mechanics, $\lambda_e = \dfrac{h}{m_e \upsilon_e} \approx \dfrac{h}{m_e c} = \lambda_c \approx 0.024263\dot{A}$, the angle of diffraction is about

$$\sin\theta = \lambda_e/a \approx 0.24263 \qquad \text{or} \qquad \theta \approx 14^\circ 25'$$

According to Einstein's mechanics, $\lambda_e = \dfrac{h}{m_e' v_e} = \dfrac{h}{m_e v_e}\sqrt{1-\dfrac{v_e^2}{c^2}} \approx 8.42\times 10^{-7}\dot{A}$ and $\sin\theta = \dfrac{\lambda_e}{a} \approx 0$ or $\theta \approx 0$. No diffraction can be detected.

We suggest physics labs use near-light velocity electrons to do the diffraction experiments in order to judge which mechanics is true.

§6 Questioning Schrodinger's Wave Equation

Schrodinger assumes that the state of a moving particle with momentum P and energy E can be represented by a plane matter wave's function in a form of complex variables:

$$\psi(x,t) = Ae^{i(Px-Et)/\hbar}. \qquad (1)$$

A particle moving with **constant** velocity v has **constant** momentum P and **constant** energy E so that

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = E\psi(x,t), \qquad \frac{\partial^2}{\partial x^2}\psi(x,t) = -\frac{P^2}{\hbar^2}\psi(x,t).$$

To avoid the separation of the particle from its own matter wave, according to **our new relativistic mechanics**, there must be $v = \nu\lambda = \dfrac{E}{P}$ so that $E = Pv = m'v^2 = \dfrac{P^2}{m'}$. If $v << c$, then $m' \approx m$ and $E \approx \dfrac{P^2}{m}$ so that

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{m}\nabla^2\psi(x,t). \qquad (2)$$

This is **our** wave equation for a particle with **constant** velocity. Correspondingly, our stationary wave equation without explicit t is:

$$\nabla^2\psi(x) = -\frac{P^2}{\hbar^2}\psi(x) \qquad \text{or} \qquad \nabla^2\psi(x) = -\frac{mE}{\hbar^2}\psi(x). \qquad (3)$$

Schrodinger's first error is that he assumes the Newtonian kinetic energy $\dfrac{1}{2}mv^2$ as E in the matter wave's frequency formula $\nu = \dfrac{E}{h}$ so that he gets $E = \dfrac{P^2}{2m}$ which , as shown in §2, results in the separation of a particle from its own matter wave. With this wrong $E = \dfrac{P^2}{2m}$, he obtains wrong wave equation for particles with **constant** velocity (see §6.1.1 and §6.1.2):

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\psi(x,t) \qquad \text{and} \qquad \nabla^2\psi(x) = -\frac{2mE}{\hbar^2}\psi(x). \qquad (4)$$

A moving particle with **variable** velocity $v(x)$ has **variable** momentum $P(x) = mv(x)$ and variable energy $E(x) = mv^2(x)$. Its matter wave frequency $\nu(x) = \dfrac{E(x)}{h}$ and wavelength $\lambda(x) = \dfrac{h}{P(x)}$ are all **variable**, too.

Nevertheless, the particle must always move inseparably with its own matter wave. There must always be $\upsilon(x) = \nu(x)\lambda(x) = \dfrac{E(x)}{P(x)}$ or $E(x) = P(x)\upsilon(x) = \dfrac{P^2(x)}{m}$ or $P(x) = \sqrt{mE(x)}$. Keeping in mind that the wave function (1) of a moving particle with **variable** velocity contains **variable** P and E, we have:

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = E\psi(x,t), \qquad \frac{\partial}{\partial x}\psi(x,t) = \frac{i}{\hbar}\left(P + x\frac{\partial P}{\partial x} - t\frac{\partial E}{\partial x}\right)\psi(x,t)$$

$$\frac{\partial^2}{\partial x^2}\psi(x,t) = \frac{i}{\hbar}\left[\frac{i}{\hbar}\left(P + x\frac{\partial P}{\partial x} - t\frac{\partial E}{\partial x}\right)^2 + \left(2\frac{\partial P}{\partial x} + x\frac{\partial^2 P}{\partial x^2} - t\frac{\partial^2 E}{\partial x^2}\right)\right]\psi(x,t).$$

Since $\upsilon = \dfrac{x}{t}$, so $\dfrac{\partial \upsilon}{\partial x} = \dfrac{1}{t}$ and $\dfrac{\partial^2 \upsilon}{\partial x^2} = \dfrac{\partial}{\partial x}\left(\dfrac{\partial \upsilon}{\partial x}\right) = \dfrac{\partial}{\partial x}\left(\dfrac{1}{t}\right) = 0$ (t does not depend on x due to the universal time-synchronism), so we have: $t\dfrac{\partial E}{\partial x} = t\dfrac{\partial}{\partial x}\left(m\upsilon^2\right) = 2\upsilon t\dfrac{\partial}{\partial x}\left(m\upsilon\right) = 2x\dfrac{\partial P}{\partial x}$

$$\frac{\partial^2 P}{\partial x^2} = m\frac{\partial^2 \upsilon}{\partial x^2} = 0$$

$$t\frac{\partial^2 E}{\partial x^2} = \frac{\partial}{\partial x}\left(t\frac{\partial E}{\partial x}\right) = \frac{\partial}{\partial x}\left(2x\frac{\partial P}{\partial x}\right) = 2\left(\frac{\partial P}{\partial x} + x\frac{\partial^2 P}{\partial x^2}\right) = 2\frac{\partial P}{\partial x}$$

$$P + x\frac{\partial P}{\partial x} - t\frac{\partial E}{\partial x} = P + x\frac{\partial P}{\partial x} - 2x\frac{\partial P}{\partial x} = P - x\frac{\partial P}{\partial x}$$

$$2\frac{\partial P}{\partial x} + x\frac{\partial^2 P}{\partial x^2} - t\frac{\partial^2 E}{\partial x^2} = 2\frac{\partial P}{\partial x} - 2\frac{\partial P}{\partial x} = 0.$$

Therefore,
$$\frac{\partial^2}{\partial x^2}\psi(x,t) = -\frac{P^2}{\hbar^2}\left(1 - \frac{x}{P}\frac{\partial P}{\partial x}\right)^2\psi(x,t).$$

Since $i\hbar\dfrac{\partial}{\partial t}\psi(x,t) = E\psi(x,t)$ and $E = \dfrac{P^2}{m}$, our wave equation for a particle with **variable** velocity is:

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{m\left(1 - \dfrac{x}{P}\dfrac{\partial P}{\partial x}\right)^2}\nabla^2\psi(x,t). \qquad (5)$$

Correspondingly, the stationary wave equation without explicit t is:

$$\nabla^2\psi(x) = -\frac{P^2}{\hbar^2}\left(1 - \frac{x}{P}\frac{\partial P}{\partial x}\right)^2\psi(x). \qquad (6)$$

Indeed, in case of a particle with constant velocity and thus constant momentum P so that $\dfrac{\partial P}{\partial x} = 0$, our equations (5) and (6) become exactly (2) and (3) respectively.

Schrodinger has committed three mistakes in obtaining his wave equation for a particle with **variable** velocity in a potential field $V(x)$:

(1) He assumes the Hamiltonian $H = \frac{1}{2}m\upsilon^2 + V(x)$ as E in the matter wave's frequency formula $\nu = \frac{E}{h}$, which inevitably causes the separation of a moving particle from its own matter wave.

(2) Since Hamiltonian $H = constant$ in the non-relativistic Newtonian mechanics, so Schrodinger deems that $E = H = constant$. Actually, however, a particle moving with **variable** velocity $\upsilon(x)$ has **variable** energy $E(x)$.

(3) A particle with **variable** velocity $\upsilon(x)$ also has **variable** momentum $P(x) = m\upsilon(x)$. But, Schrodinger assumes $P = constant$.

Schrodinger's $\frac{\partial E}{\partial x} = 0$, $\frac{\partial P}{\partial x} = 0$ and $E = H = \frac{1}{2}m\upsilon^2 + V = \frac{P^2}{2m} + V = constant$ lead to:

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \left(\frac{P^2}{2m} + V\right)\psi(x,t) \quad \text{and} \quad \frac{\partial^2}{\partial x^2}\psi(x,t) = -\frac{P^2}{\hbar^2}\psi(x,t)$$

or

$$-\hbar^2\nabla^2\psi(x,t) = P^2\psi(x,t)$$

Finally, he gets his wave equation for a particle moving in a potential field:

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(x)\right]\psi(x,t).$$

Correspondingly, Schrodinger's stationary wave equation without explicit t is:

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(x)\right]\psi(x) = E\psi(x). \qquad (4')$$

Below, we will show how Schrodinger's wave equation leads to wrong answers to several basic issues of the classical quantum mechanics (rectangular potential well, hydrogen atom's electron and one-dimensional harmonic oscillator) and what are correct answers given by our wave equation.

§6.1. Particle with Constant Velocity.

A particle moving freely or along an equipotential line has constant velocity. **Our** wave equation for such a **constant-velocity** particle is (3):

$$\nabla^2\psi(x) = -\frac{mE}{\hbar^2}\psi(x),$$

which can be written as:

$$\nabla^2\psi(x) + k^2\psi(x) = 0, \qquad \text{where } k = \frac{\sqrt{mE}}{\hbar} \qquad (7)$$

Schrodinger's wave equation for a **constant-velocity** particle is (4):

$$\nabla^2\psi(x) = -\frac{2mE}{\hbar^2}\psi(x),$$

which can also be written as (7): $\nabla^2\psi(x) + k^2\psi(x) = 0$, <u>**but his**</u> $k = \frac{\sqrt{2mE}}{\hbar}$.

§6.1.1. Rectangular Potential Well.

Suppose a particle makes one-dimensional and uniform motion reciprocally between two rigid walls (potential barriers) at $x=0$ and $x=a$. This **constant-velocity** particle's matter wave equation is (7), which has a general solution:

$$\psi(x)=A\sin kx+B\cos kx\,.$$

The particle's matter wave must be self-compatible at any point x on all of its path in the potential well. So, having traveled a distance of $2a$, the particle returns back to point x where its matter wave function must satisfy the condition $\psi(x)=\psi(x+2a)$. This condition leads to:

$$A\sin kx+B\cos kx=A\sin[k(x+2a)]+B\cos[k(x+2a)]$$

$$A\sin kx+B\cos kx=A(\sin kx\cos 2ka+\cos kx\sin 2ka)+B(\cos kx\cos 2ka-\sin kx\sin 2ka)$$

$$\sin kx[A(1-\cos 2ka)+B\sin 2ka]+\cos kx[B(1-\cos 2ka)-A\sin 2ka]=0\,.$$

Since $\sin kx$ and $\cos kx$ cannot simultaneously be zero, so there must be:

$$A(1-\cos 2ka)+B\sin 2ka=0 \quad \text{and} \quad B(1-\cos 2ka)-A\sin 2ka=0$$

or

$$\left(A^2+B^2\right)(1-\cos 2ka)=0 \quad \text{and} \quad A^2+B^2=1\,.$$

Hence, $1-\cos 2ka=0$ or $\cos 2ka=1$. So, there must be $k=\frac{n}{a}\pi$, where $n=0,1,2,\cdots$.

On the other hand, in the wave equation (7) **we** have $k=\frac{\sqrt{mE}}{\hbar}$ so that:

$$\frac{\sqrt{mE}}{\hbar}=\frac{n}{a}\pi \quad \text{or} \quad E=\frac{n^2\pi^2\hbar^2}{ma^2}=\frac{n^2h^2}{4ma^2}\,.$$

Actually, we can obtain the same result without resorting to the wave equation, provided the following two conditions are observed.

Condition-1: A moving particle's matter wave must be self-compatible $n\lambda=2a$ to avoid its matter wave's self-contradiction.

Condition-2: A moving particle's velocity must be the same as its matter wave's velocity $\nu\lambda=\upsilon$ to avoid the particle's separation from its own matter wave.

From Condition-1, $\lambda=\frac{h}{m\upsilon}=\frac{2a}{n}$ or $\upsilon=\frac{nh}{2ma}$. From Condition-2, $\upsilon=\nu\lambda=\frac{E}{P}$ or $E=P\upsilon=m\upsilon^2$.

The two Conditions together lead to $E=m\left(\frac{nh}{2ma}\right)^2=\frac{n^2h^2}{4ma^2}$, which is exactly **our** solution shown above.

In contrast, in the wave equation (7) **Schrodinger** has $k=\frac{\sqrt{2mE}}{\hbar}$ so that:

$$\frac{\sqrt{2mE}}{\hbar}=\frac{n}{a}\pi \quad \text{or} \quad E=\frac{n^2\pi^2\hbar^2}{2ma^2}=\frac{n^2h^2}{8ma^2}\,.$$

His solution cannot samultaneously satisfy the two Conditions. If the Condition-1 is satisfied $n\lambda = 2a$, then

$$\nu\lambda = \frac{E}{h}\lambda = \frac{n^2 h}{8ma^2}\cdot\frac{2a}{n} = \frac{nh}{4ma} = \sqrt{\frac{n^2h^2}{16m^2a^2}} = \sqrt{\frac{E}{2m}} = \frac{1}{2}\sqrt{\frac{2E}{m}} = \frac{1}{2}\upsilon \neq \upsilon$$

which violates the Condition-2. If the Condition-2 is satisfied $\nu\lambda = \upsilon$, then

$$\lambda = \frac{\upsilon}{\nu} = \sqrt{\frac{2E}{m}}\frac{h}{E} = \sqrt{\frac{2}{mE}}h = \sqrt{\frac{2}{m}\cdot\frac{8ma^2}{n^2h^2}}h = \frac{4a}{n} \quad \text{or} \quad n\lambda = 4a \neq 2a,$$

which violates the Condition-2.

Schrodinger's wrong result $E = \frac{n^2h^2}{8ma^2}$ from his wrong wave equation (4) stems from his wrong assumption of the non-relativistic Newtonian kinetic energy $\frac{1}{2}m\upsilon^2$ as E in the formula $\nu = \frac{E}{h}$. He did not realize that all the quantum mechanics must be relativistic.

§6.1.2. Constant-Velocity Particle in a Central Force Field.

In a central force field, **circular** orbits around the center are equipotential lines. A particle moving along a **circular** orbit has **constant** velocity. With the center of force as the origin of coordinates, we have $x = a\theta$ and $dx = ad\theta$, where x is the length of a circular path, a is the radius of the circle and θ is the argument angle at the center.

Our wave equation (3) in polar coordinates is $\frac{1}{a^2}\frac{d^2}{d\theta^2}\psi(\theta) = -\frac{mE}{\hbar^2}\psi(\theta)$, which can be written as:

$$\frac{d^2}{d\theta^2}\psi(\theta) + k^2\psi(\theta) = 0, \quad \text{where } k = \frac{a\sqrt{mE}}{\hbar}. \tag{8}$$

The general solution of (8) is: $\psi(\theta) = A\sin k\theta + B\cos k\theta$.

Matter wave must be self-compatible at any point along a circular orbit so that $\psi(\theta) = \psi(\theta + 2\pi)$. If we substitute θ for x and π for a, then (8) becomes similar to (7). The general solution of equation (8) is:

$$\psi(\theta) = A\sin k\theta + B\cos k\theta,$$

which is similar to the general solution of equation (7): $\psi(x) = A\sin kx + B\cos kx$. Thus, the matter wave's self-compatibility $\psi(\theta) = \psi(\theta + 2\pi)$ requires $1 - \cos 2k\pi = 0$, which means $k = n = 1,2,3,\cdots$. Since **our** $k = \frac{a\sqrt{mE}}{\hbar}$, so $\frac{a\sqrt{mE}}{\hbar} = n$. Therefore,

$$E = \frac{n^2\hbar^2}{ma^2}. \tag{9}$$

Since **our** $E = m\upsilon^2 = \frac{P^2}{m}$, so $P = \sqrt{mE} = \frac{n\hbar}{a}$ or $Pa = n\hbar$. This is exactly the angular momentum's quantization. Because of $\lambda = \frac{h}{P}$, we get $2\pi a = n\lambda$. This meets the Condition-1. On the other hand, due to $\upsilon = \frac{P}{m} = \frac{n\hbar}{ma}$, so $v\lambda = \frac{E}{P} = \frac{n^2\hbar^2 / ma^2}{n\hbar / a} = \frac{n\hbar}{ma} = \upsilon$. **Our** solution also meets the Condition-2.

Schrodinger's wave equation (4) in polar coordinates is $\frac{1}{a^2}\frac{d^2}{d\theta^2}\psi(\theta) = -\frac{2mE}{\hbar^2}\psi(\theta)$, which can also be written as $\frac{d^2}{d\theta^2}\psi(\theta) + k^2\psi(\theta) = 0$. **But, his** $k = n = \frac{a\sqrt{2mE}}{\hbar}$. So, he has:

$$E = \frac{n^2\hbar^2}{2ma^2}. \tag{10}$$

His Newtonian $E = \frac{1}{2}m\upsilon^2 = \frac{P^2}{2m}$ gives $P = \sqrt{2mE} = \frac{n\hbar}{a}$. Since $P = h\lambda$, so $2\pi a = n\lambda$ meets Condition-1.

However, due to $\upsilon = \frac{P}{m} = \frac{n\hbar}{ma}$, **his** $v\lambda = \frac{E}{P} = \frac{n^2\hbar^2 / 2ma^2}{n\hbar / a} = \frac{n\hbar}{2ma} = \frac{\upsilon}{2} \neq \upsilon$, which **violates** Condition-2. The particle will be separated from its own matter wave.

§6.1.2.1. Electron in the Hydrogen Atom's Coulomb Field.

In a central Coulomb force field, the kinematic equation of a charged particle, moving with a constant velocity υ along a circular orbit of radius a around the center is $\frac{m\upsilon^2}{a} = \frac{\kappa}{a^2}$ or $m\upsilon^2 = \frac{\kappa}{a}$, where κ is the constant coefficient of Coulomb attraction. The particle's constant velocity is $\upsilon = \sqrt{\frac{\kappa}{ma}}$.

As shown above, our equation (3) and Schrodinger's equation (4) give the same $\upsilon = \frac{n\hbar}{ma}$, so $\frac{n\hbar}{ma} = \sqrt{\frac{\kappa}{ma}}$ or $a = \frac{n^2\hbar^2}{m\kappa}$. For an electron in the hydrogen atom $\kappa = e^2$. So, $a = \frac{n^2\hbar^2}{me^2}$, $\upsilon = \frac{n\hbar}{ma} = \frac{e^2}{n\hbar}$ and $P = m\upsilon = \frac{me^2}{n\hbar}$.

Our solution (9) is $E = \frac{n^2\hbar^2}{ma^2} = \frac{me^4}{n^2\hbar^2}$, so $v\lambda = \frac{E}{P} = \frac{me^4 / n^2\hbar^2}{me^2 / n\hbar} = \frac{e^2}{n\hbar} = \upsilon$. The electron's velocity coincides with its matter wave's velocity.

Schrodinger's solution (10) is $E = \frac{n^2\hbar^2}{2ma^2} = \frac{me^4}{2n^2\hbar^2}$. The electron will be separated from its own matter wave, because $\nu\lambda = \frac{E}{P} = \frac{me^4 / 2n^2\hbar^2}{me^2 / n\hbar} = \frac{e^2}{2n\hbar} = \frac{\upsilon}{2} \neq \upsilon$.

§6.1.2.2. Central Elastic Force Field.

In a central elastic force field, the kinematic equation of a particle, linked with the center by an elastic band and moving with a constant velocity υ along a circular orbit of radius a around the center, is $\frac{m\upsilon^2}{a} = Ka$, where K is the band's constant elastic coefficient. The particle's velocity $\upsilon = a\sqrt{\frac{K}{m}}$, period $T = \frac{2\pi a}{\upsilon} = 2\pi\sqrt{\frac{m}{K}}$ and angular frequency $\omega = \frac{2\pi}{T} = \sqrt{\frac{K}{m}}$.

Our wave equation (3) is based on the kinetic energy of moving mass $E = m\upsilon^2$ discovered by the new relativistic mechanics. So, we have $E = Ka^2$ or $a^2 = \frac{E}{K}$. Thus, **our solution (9)** is:

$$E = \frac{n^2\hbar^2}{ma^2} = \frac{n^2\hbar^2 K}{mE} \quad \text{or} \quad E = n\hbar\sqrt{\frac{K}{m}} = n\hbar\omega.$$

The energy gap between two neighboring energy levels is exactly one integral quantum $\hbar\omega = h\nu$. The particle moves together with its own matter wave: $\nu\lambda = \frac{E}{P} = \frac{Ka^2}{m\upsilon} = \frac{Ka^2}{m\sqrt{K/m}a} = \sqrt{\frac{K}{m}}a = \upsilon$.

Schrodinger's wave equation (4) is based on the non-relativistic Newtonian $E = \frac{1}{2}m\upsilon^2$. So, $E = \frac{1}{2}Ka^2$ or $a^2 = \frac{2E}{K}$. Therefore, **his solution (10)** is $E = \frac{n^2\hbar^2}{2ma^2} = \frac{n^2\hbar^2 K}{4mE}$ or $E = \frac{1}{2}n\hbar\sqrt{\frac{K}{m}} = \frac{1}{2}n\hbar\omega$. The energy gap between two neighboring energy levels is a half of one quantum $\frac{1}{2}\hbar\omega = \frac{1}{2}h\nu$! Moreover, the particle will be separated from its own matter wave because of $\nu\lambda = \frac{E}{P} = \frac{Ka^2/2}{m\upsilon} = \frac{Ka^2/2}{ma\sqrt{K/m}} = \frac{1}{2}\sqrt{\frac{K}{m}}a = \frac{\upsilon}{2} \neq \upsilon$.

§6.2. Particle with Variable Velocity: One-dimensional Harmonic Oscillator.

It is well known that an one-dimensional harmonic oscillator moving reciprocally between ($-a, a$) with **variable** velocity corresponds to a particle moving with **constant** velocity in a central elastic force field along a circular orbit of radius a. The circle is commonly known as a "**reference circle**". So, the quantized energy level of the one-dimensional harmonic oscillator must be the same $E = n\hbar\omega$ shown in § 6.1.2.2.

Since one-dimensional harmonic oscillator is a particle with **variable** velocity, we shall use **our wave equation (6)** to deal with it and, hopefully, to get the same $E = n\hbar\omega$.

The kinematic equation of a harmonic oscillator moving reciprocally between ($-a, a$) is:

$$\frac{1}{2}mv^2 + \frac{1}{2}Kx^2 = \frac{1}{2}Ka^2 \qquad \text{or} \qquad mv^2 = K\left(a^2 - x^2\right). \qquad (11)$$

The oscillator's natural frequency ω depends on the constant elastic coefficient: $K = m\omega^2$. The oscillator's momentum is $P = mv = \sqrt{mK\left(a^2 - x^2\right)}$ so that $\frac{\partial P}{\partial x} = -\frac{\sqrt{mK}\,x}{\sqrt{a^2 - x^2}}$. Thus, our wave equation (6) becomes:

$$\nabla^2\psi(x) = -\frac{mK\left(a^2 - x^2\right)}{\hbar^2}\left(1 + \frac{x^2}{a^2 - x^2}\right)\psi(x) \quad \text{or} \quad \frac{d^2\psi}{dx^2} + \frac{mKa^4}{\hbar^2\left(a^2 - x^2\right)}\psi = 0.$$

The oscillator's motion with **variable** velocity from $x = a$ to $x = -a$ and back to $x = a$ corresponds to a particle's motion with **constant** velocity along the **reference circle** of radius a from $\theta = 0$ to $\theta = \pi$ and further to $\theta = 2\pi$. Let $x = a\cos\theta$, so $dx = -a\sin\theta d\theta$ and $a^2 - x^2 = a^2\sin^2\theta$. Our equation becomes:

$$\frac{d^2\psi}{d\theta^2} + k^2\psi = 0, \quad \text{where } k^2 = \frac{mKa^4}{\hbar^2} = \frac{m^2\omega^2a^4}{\hbar^2} \quad \text{or } k = \frac{m\omega a^2}{\hbar}. \qquad (12)$$

The self-compatibility of the oscillator's matter wave demands $\psi(\theta) = \psi(\theta + 2\pi)$. This leads to a general solution $\psi(\theta) = A\sin k\theta$, where $k = n$ is an integral number. Therefore,

$$k = \frac{m\omega a^2}{\hbar} = n \qquad \text{or} \qquad a^2 = \frac{n\hbar}{m\omega}.$$

According to our new relativistic mechanics, the oscillator ($v << c$ and $m' \approx m$) possesses kinetic energy $E = mv^2$. From (11) we have $E = mv^2 = m\omega^2\left(a^2 - x^2\right)$. At $x = 0$, the energy level of the oscillator with **variable** velocity is $E = m\omega^2a^2$ or $E = n\hbar\omega$, which corresponds exactly to that of a particle moving with **constant** velocity along the **reference circle**. $n = 0$ is the oscillator's **static state** with $v = 0$ and $E = 0$. **It does not have zero-point oscillation, nor zero-point energy.** Without the oscillation, it is no longer an oscillator and does not display any wave property ($\nu = \frac{E}{h} = 0$). $n = 1$ is the oscillator's **basic state** with $E_1 = \hbar\omega = h\nu_1$.

Classical quantum mechanics uses **Schrodinger's** wave equation (4'), which is erroneously based on $\frac{\partial P}{\partial x} = 0$ and $\frac{\partial E}{\partial x} = 0$ to deal with the one-dimensional oscillator with **variable** v, P and E. Through a complicated process involving the Hermite polynomial, the deal gives $E = \left(n + \frac{1}{2}\right)\hbar\omega$. An oscillator has non-integral energy quanta at any n-level of energy. At the $n = 0$ level, an oscillator (a static particle) has "zero-point oscillation" and "zero-point energy" $\frac{1}{2}\hbar\omega$. However, the theory of quantized radiation field has to drop the $\frac{1}{2}\hbar\omega$ zero-point energy in order to match the Planck formula.[*] Moreover, for a particle moving with constant velocity along the **reference circle**, Schrodinger's equation (4) gives $E=\frac{1}{2}n\hbar\omega$, not $E=\left(n+\frac{1}{2}\right)\hbar\omega$. The correspondence between an oscillator and a particle moving on the **reference circle** is violated. So, Schrodinger's equations (4) and (4') are inconsistent.

§6.3. On the Relativistic Wave Equation.

It must be stressed again and again that **the difference between "non-relativistic" and "relativistic" has nothing to do with a particle's velocity $\upsilon << c$ or $\upsilon \approx c$**. If a body (**subject**) studies another **relatively** moving body (**object**), which is a **relative assessment**, then it is a "relativistic" case. If a body studies **its own** state of motion, which is a **self-assessment,** then it is a "non-relativistic" case. In the quantum mechanics, we (**subjects**) study (i.e., assess) relatively moving particles (**objects**). Therefore, all the quantum mechanics must always be relativistic, no matter how slow particles move. Particles display their wave property to us only when they are moving in relation to us. Static particles are solely corpuscular without any wave property. We use the wave mechanics to study relatively moving particles. Wave mechanics has nothing to do with static particles. Wave mechanics is always relativistic. There is no such a thing as the "non-relativistic wave mechanics".

The second order partial derivatives of the matter wave function (1) with respect to t and x are:

$$\frac{\partial^2\psi}{\partial t^2}=-\frac{E^2}{\hbar^2}\psi \text{ and } \nabla^2\psi=-\frac{P^2}{\hbar^2}\psi \text{ so that } \frac{P^2}{E^2}\frac{\partial^2}{\partial t^2}\psi=\nabla^2\psi .$$

As stated in §2, to avoid the separation of a moving particle from its own matter wave, there ought to be $E=m'\upsilon^2$. Therefore, $\frac{P^2}{E^2}=\frac{1}{\upsilon^2}$ because of $P=m'\upsilon$. Therefore, $\frac{1}{\upsilon^2}\frac{\partial^2\psi}{\partial t^2}=\nabla^2\psi$. This is the general form of the relativistic wave equation suitable for any moving particle with velocity from $\upsilon << c$ to $\upsilon > c$. Particularly, for a photon moving at $\upsilon = c$, its matter wave equation is $\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}=\nabla^2\psi$. If the wave function ψ represents electric and magnetic fields' intensities respectively, then we get exactly the equations of electromagnetic wave's propagation. Therefore, the physical quantity $E=m'\upsilon^2$ discovered in our new relativistic mechanics unites the electromagnetic wave with the photon's matter wave.

Classical quantum mechanics names the Schrodinger wave equation as non-relativistic and the Klein-Gordon wave equation $-\hbar^2\frac{\partial^2\psi}{\partial t^2}=\left(-\hbar^2c^2\nabla^2+m^2c^4\right)\psi$ as relativistic. Because, for E in the matter wave's frequency formula $\nu=\frac{E}{h}$, the former uses the Newtonian non-relativistic kinetic energy $E=\frac{1}{2}m\upsilon^2$ whereas the latter uses Einstein's relativistic energy $E^2=P^2c^2+m^2c^4$. However, since $P=m'\upsilon$, so Einstein's $E^2=P^2c^2+m^2c^4$

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[*] In July 1926, when Schrodinger introduced his newly-developed wave mechanics to a conference in Munich, Heisenberg challenged him directly on the site: "If it is really as you explain, then the law of Planckian radiation cannot be understood.." But, Heisenberg was unable to explain why and how Schrodinger had gone wrong.

and $m' = \dfrac{m}{\sqrt{1-\upsilon^2/c^2}}$ lead to $\dfrac{E^2}{P^2} = c^2 + \dfrac{m^2c^4}{m'^2\upsilon^2} = c^2 + \left(1-\dfrac{\upsilon^2}{c^2}\right)\dfrac{c^4}{\upsilon^2} = \dfrac{c^4}{\upsilon^2}$ or $\dfrac{E}{P} = \dfrac{c^2}{\upsilon}$, which separates a ponderable moving particle from its own matter wave because of $\nu\lambda = \dfrac{E}{P} = \dfrac{c^2}{\upsilon} \neq \upsilon$ (Einstein's mechanics does not allow a ponderable particle moving at $\upsilon = c$).

To sum up, not only both the Schrodinger and the Klein-Gordon wave equations separate a moving particle from its own matter wave, it is also wrong to label the latter as "relativistic" and the former as "non-relativistic".

§7 On the Matter Wave Function and the Wave Equation

§7.1. Particle Is Not Wave Packet.

Facing the awkward discrepancy between a particle's velocity and its matter wave's **phase** velocity ($\upsilon \neq \nu\lambda$), Schrodinger assumes a particle as a wave packet composed of infinite number of monochromatic plane waves by their superposition. To realize the superposition by means of the Fourier series, Schrodinger has to propose a wave function with **complex form**: $\psi(x,t) = Ae^{i(Px-Et)/\hbar}$.

The **group** velocity of a wave packet is $\upsilon_g = \dfrac{d\omega}{dk}$, where $\omega = 2\pi\nu = \dfrac{E}{\hbar}$ is the angular frequency and $k = \dfrac{2\pi}{\lambda} = \dfrac{P}{\hbar}$ is the wave number, so that $\upsilon_g = \dfrac{dE}{dP}$. The classical quantum mechanics uses non-relativistic Newtonian kinetic energy $E = \dfrac{1}{2}m\upsilon^2$ in the formula $\nu = \dfrac{E}{h}$ to get $\upsilon_g = \dfrac{dE}{dP} = \dfrac{d(m\upsilon^2/2)}{d(m\upsilon)} = \upsilon$, which misleads physicists to believe that a moving particle is a matter wave packet, despite the fact that $E = \dfrac{1}{2}m\upsilon^2$ causes $\upsilon \neq \nu\lambda$.

However, $\upsilon_g = \upsilon$ leads to $\dfrac{d^2\omega}{dk^2} = \dfrac{d}{dk}\left(\dfrac{d\omega}{dk}\right) = \dfrac{d\upsilon_g}{dk} = \hbar\dfrac{d\upsilon}{dk} = \hbar\dfrac{d\upsilon}{dP} = \dfrac{\hbar}{m}\dfrac{d\upsilon}{d\upsilon} = \dfrac{\hbar}{m} \neq 0$, which means **a wave packet will inevitably expand whereas a particle will never "fatten up"**. Therefore, a suspicion about the wave packet concept remains in the physics community. Nevertheless, today's quantum mechanics is still based on Schrodinger's wave function of **complex** variables and holds the coherent superposition of a particle's infinite number of states as one of its basic principles.

The lack of a physical quantity, the kinetic energy possessed by moving mass $m'\upsilon^2$ discovered in the new relativistic mechanics, causes the physics community unable to decisively give up the suspicious "wave packet" concept. Applying our new relativistic $E = m'\upsilon^2$ to the quantum mechanics, we can prove $\nu\lambda = \dfrac{E}{P} = \dfrac{m'\upsilon^2}{m'\upsilon} = \upsilon$ and $\upsilon_g = \dfrac{d\omega}{dk} = \dfrac{dE}{dP} = \dfrac{d(m'\upsilon^2)}{d(m'\upsilon)} = 2\upsilon \neq \upsilon$; i.e., **the matter wave's phase velocity matches the particle's velocity while the wave packet's group velocity does not.** Moreover, the wave packet will still expand:

$$\frac{d^2\omega}{dk^2}=\frac{d\upsilon_g}{dk}=\hbar\frac{d\upsilon_g}{dP}=\hbar\frac{d(2\upsilon)}{d(m'\upsilon)}=\frac{2\hbar}{m'}\neq 0 .$$

Indeed, any wave packets are doomed to expand regardless of $E=\frac{1}{2}m\upsilon^2$ or $E=m\upsilon^2$, because $\frac{d^2\omega}{dk^2}=\frac{d}{dk}\left(\frac{d\omega}{dk}\right)=\frac{d}{dk}\left(\frac{dE}{dP}\right)=0$ demands $\frac{dE}{dP}=constant$, i.e., $\frac{dE}{dP}=\frac{1}{m}\frac{dE}{d\upsilon}=constant$ or $\frac{dE}{d\upsilon}=constant$, which is impossible.

In short, a moving particle is not a matter wave packet. It is needless for Schrodinger to propose a matter wave function of complex variables in order to be able to use the Fourier series for a superposition of infinite number of monochromatic plane waves to compose a wave packet. **We can simply assume a wave function of real variables:**

$$\psi(x,t)=A\cos\left(\frac{Px-Et}{\hbar}\right).$$

Below, we will justify our assumption.

§7.2. Matter Wave Function As a Constant Real Number.

A moving particle displays its wave property in terms of diffraction and interference. But, the particle itself as a ponderable corpuscle does not move in a wavelike manner or ride its own matter wave ups and downs.

Actually, the kinematic equation of a free moving particle with **constant** velocity is $x=\upsilon t$. According to the new relativistic mechanics, $E=m'\upsilon^2$ and $P=m'\upsilon$ so that we always have:

$$Px-Et=m'\upsilon x-m'\upsilon^2 t=m'\upsilon^2 t-m'\upsilon^2 t=0 .$$

Both Schrodinger's wave function and our wave function are nothing else but just a constant real number A:

$$\psi(x,t)=Ae^{i0}=A \quad\text{and}\quad \psi(x,t)=A\cos 0=A .$$

$\psi=A$ testifies that a moving particle does not move in a wavelike manner, although the second order partial derivatives of ψ with respect to t and x lead to a wave propagation equation.

The physical quantity $E=m'\upsilon^2$, discovered in the new relativistic mechanics, not only makes a moving particle inseparable with its own matter wave (because of $\nu\lambda=\upsilon$) but also allows us to use a real constant number to build a matter wave function ψ (because of $Px-Et=0$) to describe a moving particle's wave property in addition to its corpuscularity. **The constant real number A is actually the matter wave's amplitude**.

Historically, Heisenberg deemed that "The wave mechanics is just a useful mathematical tool." Heisenberg is right, although he could not demonstrate why can we create such a "useful mathematical tool". Our new relativistic mechanics gives the answer: $E=m'\upsilon^2$ leads to $Px-Et=0$ and $\psi=A$ so that ψ is just a useful tool.

§7.3. The Continuity Equation.

Schrodinger's wave function $\psi(x,t)=Ae^{i(Px-Et)/\hbar}$ leads to $\frac{\partial\psi}{\partial t}=-i\frac{E}{\hbar}\psi$ and $\frac{\partial\psi}{\partial x}=i\frac{P}{\hbar}\psi$. Our wave function $\psi(x,t)=A\cos\left(\frac{Px-Et}{\hbar}\right)$ gives $\frac{\partial\psi}{\partial t}=A\frac{E}{\hbar}\sin\left(\frac{Px-Et}{\hbar}\right)$ and $\frac{\partial\psi}{\partial x}=-A\frac{P}{\hbar}\sin\left(\frac{Px-Et}{\hbar}\right)$. So, in both cases we can get the same:

$$\frac{\partial\psi}{\partial t}+\frac{E}{P}\frac{\partial\psi}{\partial x}=0. \tag{1}$$

According to the **new relativistic mechanics**, a moving particle possesses kinetic energy $E=m'v^2$ and momentum $P=m'v$ so that $\frac{E}{P}=v$ and the equation (1) becomes $\frac{\partial\psi}{\partial t}+\frac{\partial}{\partial x}(v\psi)=0$. Let $\psi=\rho$ and $v\psi=v\rho=j$. The above equation can be expressed as $\frac{\partial\rho}{\partial t}+\nabla j=0$ which resembles the one-dimensional **continuity equation** in the fluid mechanics with ρ as the number of particles per unit volume (the volume density) and j as the number of particles flowing out in the x-direction through unit area during unit time (the current density).

As shown above in §7.2, the wave function ψ is actually made of a constant real number A. Since $\psi=A$, so $\psi=\rho$ means $A=\rho$ which **represents the number of moving particles per unit volume.** In case of $A=1$, there is only one moving particle with kinetic energy $E=m'v^2$. This is so-called "Normalization". Proceeding from the wave function of real variables, this normalization is deterministic.

Classical quantum mechanics adopts the Newtonian kinetic energy $E=\frac{1}{2}mv^2$ so that $\frac{E}{P}=\frac{1}{2}v$ and the equation (1) becomes $\frac{\partial\psi}{\partial t}+\frac{v}{2}\frac{\partial\psi}{\partial x}=0$ or $\frac{\partial\rho}{\partial t}+\frac{1}{2}\nabla j=0$. **The continuity equation is violated**.

Both Schrodinger's wave function of complex variables and our wave function of real variables lead to:

$$\frac{\partial^2\psi}{\partial t^2}=-\left(\frac{E}{\hbar}\right)^2\psi \quad \text{and} \quad \frac{\partial^2\psi}{\partial t^2}=-\left(\frac{P}{\hbar}\right)^2\psi.$$

Therefore,

$$\frac{\partial^2\psi}{\partial t^2}=\left(\frac{E}{P}\right)^2\frac{\partial^2\psi}{\partial x^2}. \tag{2}$$

According to the **new relativistic mechanics**, we have $\frac{E}{P}=v$ so that the equation (2) becomes:

$$\frac{\partial^2\psi}{\partial t^2}=v^2\frac{\partial^2\psi}{\partial x^2}. \tag{3}$$

This is the moving particle's matter wave propagation equation. The matter wave's velocity is v. In case of $v=c$, we have photon's matter wave propagation equation which coincides with the electromagnetic wave propagation equation, provided ψ represents electromagnetic field's intensities $\vec{E}$ and $\vec{H}$. Therefore, electromagnetic wave is exactly photon's matter wave.

Similarly let $\psi = \rho$ and $\upsilon\psi = \upsilon\rho = j$ in the second order partial derivatives, we have:

$$\frac{\partial^2\psi}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial t}\right) = \frac{\partial}{\partial t}\left(\frac{\partial\rho}{\partial t}\right) \quad \text{and} \quad \upsilon^2\frac{\partial^2\psi}{\partial x^2} = \upsilon\frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}(\upsilon\rho)\right] = \upsilon\nabla(\nabla j).$$

Since $\frac{\partial\rho}{\partial t} + \nabla j = 0$, so $\upsilon\nabla(\nabla j) = \upsilon\nabla\left(-\frac{\partial\rho}{\partial t}\right) = -\nabla\frac{\partial}{\partial t}(\upsilon\rho) = -\nabla\frac{\partial j}{\partial t} = -\frac{\partial}{\partial t}(\nabla j)$. Therefore, **our** wave equation (3) becomes $\frac{\partial}{\partial t}\left(\frac{\partial\rho}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla j)$ or $\left(\frac{\partial\rho}{\partial t} + \nabla j\right) = 0$. **The continuity equation keeps standing.**

On the other hand, since $\frac{E}{P} = \frac{1}{2}\upsilon$ in the **classical quantum mechanics,** so the equation (2) becomes:

$$\frac{\partial^2\psi}{\partial t^2} = \left(\frac{\upsilon}{2}\right)^2\frac{\partial^2\psi}{\partial x^2}. \tag{4}$$

The matter wave moves with velocity $\frac{\upsilon}{2}$ and lags behind the moving particle farther and farther over time. In case of $\upsilon = c$, photon's matter wave equation **does not match** the electromagnetic wave equation.

Moreover, as above-mentioned, classical $\frac{E}{P} = \frac{1}{2}\upsilon$ leads the equation (1) to become $\frac{\partial\rho}{\partial t} + \frac{1}{2}\nabla j = 0$ which in turn leads the right side of the equation (4) to become:

$$\left(\frac{\upsilon}{2}\right)^2\nabla(\nabla\psi) = \frac{\upsilon}{2}\nabla\left[\nabla\left(\frac{1}{2}\upsilon\rho\right)\right] = \frac{\upsilon}{2}\nabla\left(\frac{1}{2}\nabla j\right) = \frac{\upsilon}{2}\nabla\left(-\frac{\partial\rho}{\partial t}\right) = \frac{\partial}{\partial t}\left[\frac{1}{2}\nabla(\upsilon\rho)\right] = -\frac{\partial}{\partial t}\left(\frac{1}{2}\nabla j\right).$$

So, the equation (4) becomes $\frac{\partial}{\partial t}\left(\frac{\partial\rho}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{1}{2}\nabla j\right)$ or $\left(\frac{\partial\rho}{\partial t} + \frac{1}{2}\nabla j\right) = 0$. **Again, the continuity equation is violated**.

The continuity equation is critical and indispensable for any interpretation of a wave function to be valid. In the classical quantum mechanics the non-relativistic Newtonian $\frac{E}{P} = \frac{1}{2}\upsilon$ always violates the continuity equation whereas $\frac{E}{P} = \upsilon$ in our new relativistic mechanics always leads to the correct continuity equation. This testifies once more that the quantum mechanics can only be relativistic and must be based on the new relativistic mechanics. This also proves that the Schrodinger wave equation of complex form is incorrect.

§7.4. Questioning Born's Statistical Interpretation.

As above-mentioned repeatedly, the non-relativistic Newtonian $\frac{1}{2}m\upsilon^2$ causes the separation of a moving particle from its own matter wave. This compels Schrodinger to propose the complex form of the wave function in order to be able to use the superposition by means of the Fourier series to interpret a particle as a wave packet, the group velocity of which matches the moving particle's velocity. Now, since $\frac{E}{P} = \upsilon$ from our new relativistic mechanics can match a particle's velocity with its own matter wave's phase velocity whereas a wave packet is destined to expand, so the complex form of Schrodinger's wave function is needless.

Born adheres to the wave function's complex form and gives the matter wave a **statistical interpretation:** The wave function ψ (actually the product of ψ and its complex conjugate function ψ^*, i.e., $\psi\psi^* = |\psi|^2$) is a moving particle's statistical distribution in space. Born's statistical interpretation is questionable. It is well known that the statistical interpretation cannot give rational explanation to the probabilistic distributions for a particle moving in rectangular potential well, nor for one-dimensional harmonic oscillator. Nevertheless, the physics community has so far ignored these apparent irrationalities. Below, we will show that the statistical interpretation violates the continuity equation which is critical and indispensable for any interpretation to be valid.

Let's begin with the investigation of the wave equation containing the **first order partial derivative** of time. From Schrodinger's complex wave function $\psi(x,t) = Ae^{i(Px-Et)/\hbar}$ and its complex conjugate wave function $\psi^*(x,t) = Ae^{-i(Px-Et)/\hbar}$ we can get:

$$-i\hbar\frac{\partial\psi}{\partial t} = -\hbar^2\frac{E}{P^2}\nabla^2\psi \qquad \text{and} \qquad -i\hbar\frac{\partial\psi^*}{\partial t} = -\hbar^2\frac{E}{P^2}\nabla^2\psi^* .$$

In case of $\upsilon << c$, we have $\frac{E}{P^2} = \frac{m'\upsilon^2}{(m'\upsilon)^2} = \frac{1}{m'} \approx \frac{1}{m}$ in accordance with our **new relativistic mechanics**, so that

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{m}\nabla^2\psi \tag{1}$$

and

$$-i\hbar\frac{\partial\psi^*}{\partial t} = -\frac{\hbar^2}{m}\nabla^2\psi^* . \tag{2}$$

$\psi^* \times (1) - \psi \times (2)$ gives: $i\hbar\left(\psi^*\frac{\partial\psi}{\partial t} + \psi\frac{\partial\psi^*}{\partial t}\right) = -\frac{\hbar^2}{m}\left(\psi^*\nabla^2\psi - \psi\nabla^2\psi^*\right)$

or

$$\frac{\partial}{\partial t}\left(\psi^*\psi\right) - i\frac{\hbar}{m}\nabla\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) = 0 . \tag{3}$$

Since $\nabla\psi = \frac{\partial\psi}{\partial x} = \frac{i}{\hbar}P\psi$ and $\nabla\psi^* = \frac{\partial\psi^*}{\partial x} = -\frac{i}{\hbar}P\psi^*$, so $\psi^*\nabla\psi - \psi\nabla\psi^* = 2\frac{i}{\hbar}P\psi^*\psi$

or

$$i\frac{\hbar}{m}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) = -2\frac{P}{m}\psi^*\psi . \tag{4}$$

Placing (4) into (3), we have:

$$\frac{\partial}{\partial t}\left(\psi^*\psi\right) + 2\frac{P}{m}\nabla\left(\psi^*\psi\right) = 0 . \tag{5}$$

Because of $\frac{P}{m} = \upsilon$ and for a free particle with **constant** υ, the equation (5) becomes:

$$\frac{\partial}{\partial t}\left(\psi^*\psi\right) + 2\nabla\left(\upsilon\psi^*\psi\right) = 0 . \tag{6}$$

According to the statistical interpretation, $\psi^*\psi = |\psi|^2 = \rho$ represents the volume density and $\upsilon\psi^*\psi = \upsilon\rho = j$

represents the current density so that (6) becomes:

$$\frac{\partial \rho}{\partial t} + 2\nabla j = 0 \tag{7}$$

which violates the continuity equation and invalidates the statistical interpretation.

In contrast, we have proven in §7.3 that our **new relativistic mechanics** can directly assure the continuity equation without resort to the statistical interpretation based on Schrodinger's complex form of the wave function.

Classical quantum mechanics adopts Newtonian $E = \frac{1}{2} m v^2$, which separates a particle from its own matter wave. $\frac{E}{P^2} = \frac{1}{2m}$ leads to Schrodinger's wave equation:

$$-i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi \tag{1'}$$

and

$$-i\hbar \frac{\partial \psi^*}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi^* . \tag{2'}$$

$\psi^* \times (1') - \psi \times (2')$ gives:

$$\frac{\partial}{\partial t}\left(\psi^* \psi\right) - i\frac{\hbar}{2m} \nabla\left(\psi^* \nabla \psi - \psi \nabla \psi^*\right) = 0 . \tag{3'}$$

The equation (4) is common for both new relativistic quantum mechanics and classical quantum mechanics, because it does not involve energy E so that it remains the same regardless of $E = m' v^2$ or $E = \frac{1}{2} m v^2$:

$$i\frac{\hbar}{m}\left(\psi^* \nabla \psi - \psi \nabla \psi^*\right) = -2\frac{P}{m} \psi^* \psi . \tag{4}$$

Placing (4) into (3'), we get:

$$\frac{\partial}{\partial t}\left(\psi^* \psi\right) + \frac{P}{m} \nabla\left(\psi^* \psi\right) = 0 \tag{5'}$$

or

$$\frac{\partial}{\partial t}\left(\psi^* \psi\right) + \nabla\left(v \psi^* \psi\right) = 0 \tag{6'}$$

or

$$\frac{\partial \rho}{\partial t} + \nabla j = 0 . \tag{7'}$$

The continuity equation stands and the classical quantum mechanics claims that the statistical interpretation is correct. **<u>It is not so</u>!** Actually, a juxtaposition of the two procedures of deduction from (1') to (7') and from (1) to (7) shows that the coefficient $\frac{1}{2}$ in the Newtonian $E = \frac{1}{2} m v^2$, which causes Schrodinger's wave equations (1') and (2') to have the coefficient $\frac{1}{2}$ in their right side, incidentally offsets the coefficient 2 in the equation (4) so that the equations (5') and (6') can lead to the equation (7') without the coefficient 2. By other words, the continuity equation (7') is barely saved **at the cost of separating a particle from its own matter wave due to the use of the Newtonian** $E = \frac{1}{2} m v^2$. This is not a correct and convincing way to justify Born's statistical interpretation.

Now, let's investigate the wave equation containing the **second order partial derivative** of time. Schrodinger's complex wave function $\psi(x,t) = Ae^{i(Px-Et)/\hbar}$ and complex conjugate wave function $\psi^*(x,t) = Ae^{-i(Px-Et)/\hbar}$

lead to:

$$\frac{\partial^2 \psi}{\partial t^2} = \left(\frac{E}{P}\right)^2 \nabla^2 \psi \tag{8}$$

and $$\frac{\partial^2\psi^*}{\partial t^2}=\left(\frac{E}{P}\right)^2\nabla^2\psi^*. \tag{9}$$

$\psi^*\times(8)-\psi\times(9)$ gives: $$\psi^*\frac{\partial^2\psi}{\partial t^2}-\psi\frac{\partial^2\psi^*}{\partial t^2}=\left(\frac{E}{P}\right)^2\left(\psi^*\nabla^2\psi-\psi\nabla^2\psi^*\right).$$

The left side of the above equation can be written as:

$$\left(\psi^*\frac{\partial^2\psi}{\partial t^2}+\frac{\partial\psi}{\partial t}\frac{\partial\psi^*}{\partial t}\right)-\left(\psi\frac{\partial^2\psi^*}{\partial t^2}+\frac{\partial\psi}{\partial t}\frac{\partial\psi^*}{\partial t}\right)=\frac{\partial}{\partial t}\left(\psi^*\frac{\partial\psi}{\partial t}\right)-\frac{\partial}{\partial t}\left(\psi\frac{\partial\psi^*}{\partial t}\right)=\frac{\partial}{\partial t}\left(\psi^*\frac{\partial\psi}{\partial t}-\psi\frac{\partial\psi^*}{\partial t}\right).$$

The right side can be expressed as: $\left(\frac{E}{P}\right)^2\left[\left(\psi^*\nabla^2\psi+\nabla\psi\nabla\psi^*\right)-\left(\psi\nabla^2\psi^*+\nabla\psi\nabla\psi^*\right)\right]$

$$=\left(\frac{E}{P}\right)^2\left[\nabla\left(\psi^*\nabla\psi\right)-\nabla\left(\psi\nabla\psi^*\right)\right]=\left(\frac{E}{P}\right)^2\nabla\left(\psi^*\nabla\psi-\psi\nabla\psi^*\right).$$

Therefore, $$\frac{\partial}{\partial t}\left(\psi^*\frac{\partial\psi}{\partial t}-\psi\frac{\partial\psi^*}{\partial t}\right)-\left(\frac{E}{P}\right)^2\nabla\left(\psi^*\nabla\psi-\psi\nabla\psi^*\right)=0. \tag{10}$$

Because of $\frac{\partial\psi}{\partial t}=-\frac{i}{\hbar}E\psi$ and $\frac{\partial\psi^*}{\partial t}=\frac{i}{\hbar}E\psi^*$, we have:

$$\frac{\partial}{\partial t}\left(\psi^*\frac{\partial\psi}{\partial t}-\psi\frac{\partial\psi^*}{\partial t}\right)=\frac{\partial}{\partial t}\left(-\frac{i}{\hbar}E\psi\psi^*-\frac{i}{\hbar}E\psi^*\psi\right)=-2\frac{i}{\hbar}E\frac{\partial}{\partial t}\left(\psi^*\psi\right).$$

On the other hand, because of $\nabla\psi=\frac{i}{\hbar}P\psi$ and $\nabla\psi^*=-\frac{i}{\hbar}P\psi^*$, we have:

$$\left(\frac{E}{P}\right)^2\nabla\left(\psi^*\nabla\psi-\psi\nabla\psi^*\right)=\left(\frac{E}{P}\right)^2 2\frac{i}{\hbar}P\nabla\left(\psi^*\psi\right)=2\frac{i}{\hbar}\frac{E^2}{P}\nabla\left(\psi^*\psi\right).$$

Therefore, (10) becomes: $$\frac{\partial}{\partial t}\left(\psi^*\psi\right)+\frac{E}{P}\nabla\left(\psi^*\psi\right)=0.$$

According to Born's statistical interpretation, $\psi^*\psi=|\psi|^2=\rho$ (the volume density) and $\upsilon\rho=j$ (the current density). The above equation becomes: $$\frac{\partial\rho}{\partial t}+\frac{E}{P}\nabla\rho=0 \tag{11}$$

The Newtonian $\frac{E}{P}=\frac{1}{2}\upsilon$ turns (11) into $\frac{\partial\rho}{\partial t}+\frac{1}{2}\nabla(\upsilon\rho)=0$ or $\frac{\partial\rho}{\partial t}+\frac{1}{2}\nabla j=0$, which violates the continuity equation. Born's statistical interpretation fails because it cannot save the continuity equation even at the cost of separating a particle from its own matter wave by use of Newtonian $E=\frac{1}{2}m\upsilon^2$.

The equation (11) would lead to a correct continuity equation, if $\frac{E}{P}=\upsilon$ is used. Unfortunately, as shown above from (1) to (7), with $\frac{E}{P} = \upsilon$ the statistical interpretation cannot provide a correct continuity equation for Schrodinger's wave equation $-i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi$.

In contrast, without resort to Born's statistical interpretation, we have already proven in §7.3 that our new relativistic mechanics can provide the continuity equation for **both** wave equations containing **the first order and the second order partial derivatives of time** respectively.

§7.5. The Complex Matter Wave Function Is Needless.

As above-stated, we can use a constant real number A (the number of particles in unit volume) to build a complex wave function $\psi(x,t) = Ae^{i(Px-Et)/\hbar}$ or a real wave function $\psi(x,t) = A\cos\left(\frac{Px - Et}{\hbar}\right)$. In both cases we can obtain the same wave equations

$$\frac{\partial\psi}{\partial t} + \frac{E}{P}\frac{\partial\psi}{\partial t} = 0 \quad \text{and} \quad \frac{\partial^2\psi}{\partial t^2} = \left(\frac{E}{P}\right)^2\nabla^2\psi.$$

We have proven in §7.3 that, based on our new relativistic mechanics which guarantees a particle moving inseparably with its own matter wave, the continuity equation stands for **both** $\frac{\partial\psi}{\partial t} + \frac{E}{P}\frac{\partial\psi}{\partial t} = 0$ **and** $\frac{\partial^2\psi}{\partial t^2} = \left(\frac{E}{P}\right)^2\nabla^2\psi$ **without resort to Schrodinger's complex wave function and Born's statistical interpretation.**

In §7.4 we have proven that, regardless of $E = \frac{1}{2}m\upsilon^2$ or $E = m'\upsilon^2$, the statistical interpretation cannot provide a correct continuity equation, which is indispensable for any interpretation, for **both** $-i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi$ and $\frac{\partial^2\psi}{\partial t^2} = \left(\frac{E}{P}\right)^2\nabla^2\psi$. So, Born's statistical interpretation is questionable.

Schrodinger proposes the wave function's complex form in order to interpret a particle as a wave packet to bypass the awkward $\nu\lambda \neq \upsilon$. However, particles are not expansion-destined wave packets. Born supports Schrodinger's complex wave function by proposing his statistical interpretation. However, the statistical interpretation does not stand. Therefore, the wave function's complex form is deprived of any rationale for its survival.

§7.6. Questioning Heisenberg's Uncertainty Principle.

Initially, Heisenberg proposed his uncertainty principle only through some "gedanken experiments". Later, based on Born's statistical interpretation, he was able to provide certain mathematical proof. If the statistical interpretation fails, then the uncertainty principle is left only with the "gedanken experiments" and thus turns to be questionable, too. Now, to challenge the uncertainty principle, we need only to analyze some "gedanken experiments".

Heisenberg deems that only observable physical quantities can be prerequisites for any theory in the physics. Yet, he did not clearly defined what kind of physical quantities can be labeled as "**observable**". In fact, in the mechanics only three physical quantities— static mass m, time t and location x— are <u>**directly**</u> **observable (measurable)**. That is why these three constitute the absolute system of units (CGS). All other mechanical quantities (e.g., moving mass, velocity, momentum, kinetic energy, etc.) are derived (calculated according to physical laws) from these three directly measured quantities and can be named as <u>**indirectly**</u> **observable** quantities.

Let's examine the relationship between a particle's location (directly observable) and its momentum (indirectly observable).

<u>Gedanken Experiment-1</u>. A particle moves with a **constant** velocity along the X-axis. The distance from x_1 to x_2 and the particle's static mass m are **directly** measured beforehand. Two **pairs** of laser detectors are aimed at x_1 and x_2 respectively to **directly** measure the time t_1 and t_2 when the particle passes x_1 and x_2. The laser detectors in each pair are identical and installed precisely opposite to each other so that the measurement will not disturb the moving particle. By use of the **directly observed** m, x_1, x_2, t_1, t_2 we can calculate (**indirectly observe**): $\upsilon = \dfrac{x_2 - x_1}{t_2 - t_1}$, $m' = \dfrac{m}{\sqrt{1 + \upsilon^2/c^2}}$, $P = m'\upsilon$. There can't be any uncertainty among all directly or indirectly observed physical quantities.

<u>Gedanken Experiment-2</u>. Let a particle with static mass m be at rest. We, the observers, move uniformly and translationally. In relation to us, the particle seems moving uniformly and translationally. We measure (directly observe) our own state of motion to get x_1, t_1 and x_2, t_2, which are the particle's state of **relative** motion due to the principle of relativity. Our state of motion is not susceptible to any disturbance, because we are macroscopic bodies. The particle remains at rest without any disturbance. We can calculate (indirectly observe) our velocity $\upsilon = \dfrac{x_2 - x_1}{t_2 - t_1}$, which is the seemingly moving particle's relative velocity, and also calculate (indirectly observe) the seemingly moving particle's momentum $P = m'\upsilon = \dfrac{m\upsilon}{\sqrt{1 + \upsilon^2/c^2}}$. There is no uncertainty between the relatively moving particle's x and P, both of which are indirectly observed by our own motion in relation to the particle.

Heisenberg's assertion "location and momentum cannot be **simultaneously** observed with certainty" is a meaningless thesis. There is no such a case as the **simultaneous** observation of location and momentum. Because, location is a **basic** physical quantity which can be **directly** observed whereas the momentum is calculated (**indirectly** odserved). Moreover, a free particle moving with constant velocity has a certain quantity of momentum at any location, known or unknown and measured or unmeasured. No uncertainty between its location and momentum.

"The Lord does not play dice!" Einstein seems right. The physics is deterministic. Unfortunately, Einstein, who is great in the original development of quantum mechanics, could not win his debate against the Copenhagen school only by means of the Einstein-Podolsky-Rosen gedanken experiment.

Appendix. The Correspondence Principle

§1. Bohr's Classical Approach.

Bohr assumes $E_n = h\nu_T f(n)$ or $f(n) = E_n / h\nu_T$ for a charged particle moving on an elliptic orbit. n is the orbit's quantum number, E_n is the particle's energy on the n-th orbit and ν_T is the orbital frequency. Bohr needs to know $f(n)$. He deduces it as follows.

$$f'(n)=\frac{d}{dn}\left(\frac{E_n}{h\nu_T}\right)=\frac{1}{h\nu_T}\frac{dE_n}{dn}+\frac{E_n}{h}\frac{d}{dn}\left(\frac{1}{\nu_T}\right)=\frac{1}{h\nu_T}\frac{dE_n}{dn}-\frac{E_n}{h\nu_T^2}\frac{d\nu_T}{dE_n}\frac{dE_n}{dn}$$

$$=\frac{1}{h\nu_T}\left(1-\frac{E_n}{\nu_T}\frac{d\nu_T}{dE_n}\right)\frac{dE_n}{dn}=\frac{1}{h\nu_T}\left(1-E_n\frac{d\ln\nu_T}{dE_n}\right)\frac{dE_n}{dn}$$

or

$$\frac{dE_n}{dn}=h\nu_T\, f'(n)\Big/\left(1-E_n\frac{d\ln\nu_T}{dE_n}\right).$$

Bohr selects a family of elliptic orbits with $E=\frac{1}{2}m\upsilon^2-\frac{\kappa}{r}<0$ as the family's orbital conservative energy. All elliptic orbits in this family have common characteristic parameters: $E=-\frac{\kappa}{2a}<0$ and $\nu_T=\frac{1}{2\pi}\sqrt{\frac{\kappa}{m}}a^{-3/2}$, where a is their common semi-major axis. Next, Bohr confuses the particle's energy E_n with the orbital conservative energy E to assume $E_n=E$. Since $E<0$, Bohr misunderstands that the shorter the orbit's a (the less the quantum number n), the less (the more negative) the particle's energy E_n. Therefore, he deems that certain energy will be radiated if the charged particle transits from the ($n+1$)-th orbit into the n-th orbit.

According to Einstein's photoelectric theory, a photon of frequency ν has total energy $h\nu$ so that:

$$h\nu=E_{n+1}-E_n=\Delta n\frac{\Delta E_n}{\Delta n}\approx[(n+1)-n]\frac{dE_n}{dn}=\frac{dE_n}{dn}=h\nu_T\, f'(n)\Big/\left(1-E_n\frac{d\ln\nu_T}{dE_n}\right).$$

Bohr's correspondence principle assumes that, as $n\to\infty$, the photon's frequency ν corresponds with the charged particle's orbital frequency ν_T so that $\nu=\nu_T$. Hence,

$$\lim_{n\to\infty}f'(n)=\frac{h\nu}{h\nu_T}\left(1-E_n\frac{d\ln\nu_T}{dE_n}\right)=1-E_n\frac{d\ln\nu_T}{dE_n}.$$

Bohr's confused $E_n=E$ gives $E_n\frac{d\ln\nu_T}{dE_n}=E\frac{d\ln\nu_T}{dE}$. Because of $\nu_T=\frac{1}{2\pi}\sqrt{\frac{\kappa}{m}}a^{-3/2}$, he can obtain:

$$\ln\nu_T=\ln\left(\frac{1}{2\pi}\sqrt{\frac{\kappa}{m}}\right)-\frac{3}{2}\ln a \quad\text{and}\quad \frac{d\ln\nu_T}{dE}=-\frac{3}{2}\frac{d\ln a}{dE}=-\frac{3}{2}\frac{d\ln a}{da}\frac{da}{dE}=-\frac{3}{2a}\frac{da}{dE}.$$

On the other hand, since $E=-\frac{\kappa}{2a}$, so $a=-\frac{\kappa}{2E}$ and $\frac{da}{dE}=\frac{\kappa}{2E^2}=-\frac{a}{E}$. Thus, $\frac{d\ln\nu_T}{dE}=\frac{3}{2E}$ so that:

$$\lim_{n\to\infty} f'(n) = 1 - E\frac{d\ln\nu_T}{dE} = 1 - E\frac{3}{2E} = -\frac{1}{2} \quad \text{or} \quad f(n) = -\frac{n}{2}$$

Finally, Bohr obtains: $$E_n = -\frac{n}{2}h\nu_T = -\frac{nh}{4\pi}\sqrt{\frac{\kappa}{m}}a^{-3/2} < 0\,.$$

§2. Approach Based on New Relativistic Mechanics.

We agree with Bohr's $E_n = h\nu_T f(n)$ and $\frac{dE_n}{dn} = h\nu_T\, f'(n) \Big/ \left(1 - E_n\frac{d\ln\nu_T}{dE_n}\right)$. However, we have proven that an electron's orbit around the hydrogen atom's nucleus must be circular.

The electron's kinematic equation on a circular orbit of radius a is $m\upsilon^2 = \frac{\kappa}{a}$ or $\upsilon = \sqrt{\frac{\kappa}{ma}}$. Its orbital period is $T = \frac{2\pi a}{\upsilon} = 2\pi\sqrt{\frac{m}{\kappa}}a^{3/2}$ and orbital frequency is $\nu_T = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{\kappa}{m}}a^{-3/2}$.

Circular orbit's conservative energy is $E = m\upsilon^2$. According to our new relativistic mechanics, for an electron moving with velocity υ on the n-th circular orbit, the kinetic energy possessed by the electron's moving mass is $E_n = m'\upsilon^2$. Since the electron's velocity $\upsilon << c$, so $m' \approx m$ and $E_n = m\upsilon^2$. Therefore, our $E_n = E$ **is not a confusion but a truth.** The less the quantum number and the radius a, the larger the electron's velocity υ and its kinetic energy E_n. Thus, the electron's transition from an inner n-th orbit into an outer ($n+1$)-th orbit releases energy.

According to our new relativistic mechanics, a photon of frequency ν has total energy $2h\nu$ so that:

$$2h\nu = E_n - E_{n+1} = \Delta n\frac{\Delta E_n}{\Delta n} \approx [n - (n+1)]\frac{dE_n}{dn} = -\frac{dE_n}{dn} = -h\nu_T\, f'(n) \Big/ \left(1 - E_n\frac{d\ln\nu_T}{dE_n}\right).$$

Now, the correspondence principle's $\nu = \nu_T$ leads to $\lim_{n\to\infty} f'(n) = -2\left(1 - E_n\frac{d\ln\nu_T}{dE_n}\right)$. Since we have true $E_n = E$, so $E_n\frac{d\ln\nu_T}{dE_n} = E\frac{d\ln\nu_T}{dE}$ is true, too. Since $\nu_T = \frac{1}{2\pi}\sqrt{\frac{\kappa}{m}}a^{-3/2}$ and $E = m\upsilon^2 = \frac{k}{a}$, we have $\ln\nu_T = \ln\left(\frac{1}{2\pi}\sqrt{\frac{\kappa}{m}}\right) - \frac{3}{2}\ln a$ and $\frac{d\ln\nu_T}{dE} = \frac{3}{2E}$ so that $\lim_{n\to\infty} f'(n) = -2\left(1 - E\frac{3}{2E}\right) = 1$ or $f(n) = n$.

Finally, we obtain: $$E_n = nh\nu_T = \frac{nh}{2\pi}\sqrt{\frac{\kappa}{m}}a^{-3/2} > 0$$