

# Abandoning the Ideas of Length Contraction and Time Dilation

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This paper demonstrates that including the 3-vector Lorentz force law within the framework of Special Relativity Theory allows derivation of the fundamental relativistic equations pertaining to a charged particle without using Lorentz transformation, and hence without using its kinematic effects; *i.e.* length contraction and time dilation. Besides that, the invariance of light speed can be interpreted in a way that does not depend on the properties of space-time. **Key words:** Lorentz force law, special relativity, length contraction, time dilation.

## 1. Introduction

The Lorentz transformation (LT) embedded in Special Relativity Theory (SRT) [1] was first derived by H.A. Lorentz [2] using his theory of electrons, which in accord with prevailing wisdom included the concept of an absolutely still aether. Lorentz demonstrated that under Galilean transformation from the absolute rest frame  $k$  (the aether frame) to another inertial frame  $k'$  moving with constant velocity  $\mathbf{u}$  with respect to  $k$ , Maxwell's equations change form, acquiring additional terms of first and second degree in  $u/c$ . But all experiments conducted to detect the aether failed, so Lorentz diverted his attention from the aether to the kinematics of moving material bodies. His endeavors led him to the observation that the term at first order in  $u/c$  could easily be eliminated by the following alteration to the temporal part of Galilean transformation:

$$t' = t \rightarrow t' = \gamma(t - ux/c^2)$$

where  $\gamma = 1/\sqrt{1 - u^2/c^2}$ . To get rid of the terms at second order in  $u/c$ , Lorentz proposed another alteration, one that had originally been proposed by Fitzgerald to explain the negative result of the Michelson-Morley experiments, where no aether was detected. Lorentz combined the Lorentz-Fitzgerald contraction proposition with his first alteration to get what we now know as the Lorentz transformation LT:

$$t' = \gamma(t - ux/c^2), \quad x' = \gamma(x - ut), \quad y' = y, \quad z' = z$$

Under LT, Maxwell's equations keep their form in all inertial frames, showing in the process the equivalence of all inertial frames.

Through his two postulates [1] Einstein was able to do away with the aether, and to derive the LT differently from Lorentz, thereby demonstrating the special importance of these transformations. Since Einstein's treatment of special relativity theory (SRT) is based on a discussion of space and time, and since any velocity, including that of light, is by definition derived from distance and time, it follows that the dynamics of a moving particle (or photon) are built to accommodate the LT. Through this process, SRT suffers three major flaws [3]:

- 1) Interpreting the invariance of light speed by using LT
- 2) Length contraction as a kinematic effect.
- 3) Time dilation as a kinematic effect.

To overcome the above-mentioned flaws, alternative relativity theories have been proposed. For example, H.E. Wilhelm [4,5] proposed one that essentially completes the work initiated by Fitzgerald, Larmor, and Lorentz, where length contraction and time dilation are regarded as dynamic rather than kinematic effects. Also, O.D. Jefimenko [6,7,8] proposed another theory, where relativistic electromagnetism and relativistic mechanics are derived without any reference to the two postulates of SRT or ever invoking the concepts of length contraction and time dilation, from his "Theory of Electromagnetic Retardation". Therefore, the experiments that are interpreted as proofs of length contraction and time dilation have alternative interpretation in terms

of velocity-dependent interactions present in the systems under consideration [9].

Historically, the connection between SRT and electromagnetism is very close, and this close relation could be emphasized even more by including into the main body of SRT the Lorentz force law (LFL); *i.e.*

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (1)$$

and by applying to the LFL the relativity principle. The present paper develops this approach, to demonstrate that the fundamental relativistic equations can be derived without using LT, hence, without using the idea of kinematic length contraction and time dilation (flaws 2 and 3). Likewise curing flaw (1), light speed invariance is reinterpreted in a way that does not depend on the properties of space-time.

## 2. Force, Velocity, & Field Transformations

This Section includes derivations of the 3-Vector Relativistic Force and Velocity Transformations, as well as the Relativistic Electromagnetic Field Transformation. Let there be a particle of mass  $m$  and charge  $q$  moving with velocity  $\mathbf{v}$  in the frame  $s$ , subject to an electric field  $\mathbf{E}$  and a magnetic flux density  $\mathbf{B}$ . The LFL experienced by this particle has Cartesian components expressed in frame  $s$  as

$$F_x = q(E_x + v_y B_z - v_z B_y) \quad (2a)$$

$$F_y = q(E_y + v_z B_x - v_x B_z) \quad (2b)$$

$$F_z = q(E_z + v_x B_y - v_y B_x) \quad (2c)$$

In frame  $s'$  moving with velocity  $\mathbf{u}$  parallel to their common  $x$  axis, they are expressed as follows

$$F'_x = q(E'_x + v'_y B'_z - v'_z B'_y) \quad (3a)$$

$$F'_y = q(E'_y + v'_z B'_x - v'_x B'_z) \quad (3b)$$

$$F'_z = q(E'_z + v'_x B'_y - v'_y B'_x) \quad (3c)$$

Taking the scalar produce of Eq. (1) with  $\mathbf{v}$  gives

$$\mathbf{F} \cdot \mathbf{v} = q\mathbf{E} \cdot \mathbf{v} \quad (4)$$

Now if we multiply the Eq. (4) with  $-u/c^2$  and add it to (2a), after rearrangements we get

$$\begin{aligned} F_x \left( 1 - \frac{uv_x}{c^2} \right) - \frac{u}{c^2} F_y v_y - \frac{u}{c^2} F_z v_z \\ = q \left[ E_x \left( 1 - \frac{uv_x}{c^2} \right) + v_y \left( B_z - \frac{u}{c^2} E_y \right) - v_z \left( B_y + \frac{u}{c^2} E_z \right) \right] \end{aligned}$$

Dividing this relation by  $(1 - uv_x/c^2)$ , and, at the same time, multiplying and dividing the last two terms by the scalar factor  $\gamma$ , we get

$$F_x - \frac{F_y u v_y / c^2}{(1 - u v_x / c^2)} - \frac{F_z u v_z / c^2}{(1 - u v_x / c^2)}$$

$$= q \left[ E_x + \frac{v_y \gamma (B_z - u E_y / c^2)}{\gamma (1 - u v_x / c^2)} - \frac{v_z \gamma (B_y + u E_z / c^2)}{\gamma (1 - u v_x / c^2)} \right]$$

Comparing the last equation with (3a), we see that in order for the LFL to hold true in frame  $s'$ , we should have

$$F'_x = F_x - \frac{u F_y v_y / c^2}{(1 - u v_x / c^2)} - \frac{u F_z v_z / c^2}{(1 - u v_x / c^2)} \quad (5a)$$

$$v'_y = \frac{v_y}{\gamma (1 - u v_x / c^2)} \quad (6a), \quad v'_z = \frac{v_z}{\gamma (1 - u v_x / c^2)} \quad (6b)$$

and

$$E'_x = E_x \quad (7a)$$

$$B'_y = \gamma (B_y + u E_z / c^2), \quad B'_z = \gamma (B_z - u E_y / c^2) \quad (7b,c)$$

According to the relativity principle, the inverse of the 3-vector relativistic velocity transformations can be obtained, by replacing every  $\mathbf{u}$  with  $-\mathbf{u}$  and interchanging the primed and non-primed quantities; *i.e.*,

$$v_y = \frac{v'_y}{\gamma (1 + u v'_x / c^2)}, \quad v_z = \frac{v'_z}{\gamma (1 + u v'_x / c^2)} \quad (8a,b)$$

Hence we can substitute from Eq. (6a) into Eq. (8a) and obtain

$$v_y = \frac{v_y}{\gamma^2 (1 + u v'_x / c^2) (1 - u v_x / c^2)}$$

This equation is algebraic identity, and must hold for all  $v_y$ ; *i.e.*,

$$\gamma^2 (1 + u v'_x / c^2) (1 - u v_x / c^2) = 1 \quad (9)$$

Eq. (9) helps to identify  $\gamma$ . For simplicity, take the special case that the charged particle  $q$  is at rest in frame  $s'$ . Since

$$v'_x = 0 \quad \text{i.e.} \quad v_x = u$$

These results, when substituted into Eq. (9), lead to the determination of  $\gamma$ ; *i.e.*,

$$\gamma^2 (1 - u^2 / c^2) = 1 \quad (10)$$

or

$$\gamma = 1 / \sqrt{1 - u^2 / c^2} \quad (11)$$

Starting now from Eq. (2b), using Eq. (10), we have

$$F_y = q \left[ \gamma^2 (1 - u^2 / c^2) E_y + v_z B_x - \gamma^2 (1 - u^2 / c^2) v_x B_z \right] c^2$$

Adding and subtracting the two terms

$$\gamma^2 u B_z, \quad \gamma^2 \frac{u v_x}{c^2} E_y$$

we obtain

$$F_y = q \left[ \gamma^2 \left( 1 - \frac{u v_x}{c^2} \right) (E_y - u B_z) + v_z B_x - \gamma^2 (v_x - u) \left( B_z - \frac{u}{c^2} E_y \right) \right]$$

Dividing the last relation by  $\gamma (1 - u v_x / c^2)$ , we have

$$\frac{F_y}{\gamma (1 - u v_x / c^2)} = q \times$$

$$\left[ \gamma (E_y - u B_z) + \frac{v_z}{\gamma (1 - u v_x / c^2)} B_x - \left( \frac{v_x - u}{1 - u v_x / c^2} \right) \gamma \left( B_z - \frac{u}{c^2} E_y \right) \right]$$

Comparing the last equation with (3b), we deduce

$$F'_y = \frac{F_y}{\gamma (1 - u v_x / c^2)}, \quad v'_x = \frac{v_x - u}{1 - u v_x / c^2} \quad (6b,c)$$

$$E'_y = \gamma (E_y - u B_z), \quad B'_x = B_x \quad (7d)$$

In a similar way, starting from Eq. (2c), we have

$$F'_z = \frac{F_z}{\gamma (1 - u v_x / c^2)}, \quad E'_z = \gamma (E_z + u B_y) \quad (7e)$$

The transformations (5a), (5b) and (5c) that we derived also apply for the 3-vector force given by the equation

$$\mathbf{F} = d\mathbf{P} / dt \quad (12)$$

That means the LFL transforms in the appropriate way to agree with the relativistic relations between force and momentum. Also, Eq. (12) reflects the fact that the equation

$$d\mathbf{P} / dt = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (13)$$

is the law of motion for a charged particle moving at relativistic velocities.

As for the 3-vector relativistic velocity transformations (6a), (6b) and (6c), they were derived as a result of mathematical consideration only, without taking into account the definition of velocity as a rate of change of position with time. As obtained by Einstein [1], the relativistic electromagnetic field transformations, are a consequence of LT. Therefore, to derive the Eqs. (7), we introduced an alternative path that is not based on LT.

The aim in this Section was to demonstrate the importance of the LFL, and, in addition, to explain that there is an alternative path not using LT in the derivation of the 3-vector relativistic force and velocity transformations appertaining to a charged particle, as well as the relativistic electromagnetic field transformation.

### 3. Momentum, Energy, & Mass Transformations

This Section derives the transformation equations for momentum, energy and relativistic mass. Within relativistic electromagnetism we know what dynamic variables are correct for a charged particle. To find these dynamic variables in the present work, we will extend the classical dynamical variables in such a way as to make them compatible with the relativistic relations in Section 1.

Therefore it could be expected that starting from the conventional definition of momentum in frames ( $s$ ) and ( $s'$ ) as a product of total mass times velocity, which in vector form are

$$\mathbf{P} = m \cdot \mathbf{v}, \quad \mathbf{P}' = m' \cdot \mathbf{v}' \quad (14a,b)$$

would lead to relativistic dynamic variables. Eq. (14a) could be written as follows:

$$v_x = P_x / m, \quad v_y = P_y / m, \quad v_z = P_z / m \quad (15a-c)$$

Now let us demonstrate that the definition (15) of velocity combined with Eqs. (6a, 6b, 6c) will lead to the 3-vector relativistic transformation for momentum and mass. Taking Eqs. (15a, 15b) and substituting them in (6a), we see:

$$v'_y = \frac{P_y / m}{\gamma(1 - uP_x / c^2 m)} = \frac{P_y}{\gamma(m - uP_x / c^2)}$$

To make the right-hand side of the last Equation compatible with the definition of the velocity in frame  $s'$ , we should have:

$$m' = \gamma(m - uP_x / c^2), \quad P'_y = P_y \quad (16a,b)$$

Similarly, substituting (15a, 15c) in (6b) gives:

$$P'_z = P_z \quad (16c)$$

Let us now take Eqs. (15) and substitute into (6c):

$$v'_x = \frac{P_x / m - u}{1 - uP_x / c^2 m} = \frac{\gamma(P_x - um)}{\gamma(m - uP_x / c^2)}$$

Using Eq. (16a), and in order for the definition of velocity Eqs. (15) to hold true in frame ( $s'$ ), we should have:

$$P'_x = \gamma(P_x - um) \quad (16d)$$

On the other hand, from Eqs. (6a,6b,6c) we see that these are equivalent to:

$$\frac{1}{\sqrt{1 - v'^2 / c^2}} = \frac{1 - uv_x / c^2}{\sqrt{1 - u^2 / c^2} \sqrt{1 - v^2 / c^2}} \quad (17a)$$

$$\frac{v'_y}{\sqrt{1 - v'^2 / c^2}} = \frac{v_y}{\sqrt{1 - v^2 / c^2}}, \quad \frac{v'_z}{\sqrt{1 - v'^2 / c^2}} = \frac{v_z}{\sqrt{1 - v^2 / c^2}} \quad (b,c)$$

$$\frac{v'_x}{\sqrt{1 - v'^2 / c^2}} = \frac{v_x - u}{\sqrt{1 - u^2 / c^2} \sqrt{1 - v^2 / c^2}} \quad (17d)$$

Multiplying Eqs. (17) by  $m_0$  and comparing the result with Eq. (16), we deduce:

$$m = \frac{m_0}{\sqrt{1 - v^2 / c^2}}, \quad m' = \frac{m_0}{\sqrt{1 - v'^2 / c^2}} \quad (18a,b)$$

Eqs. (18) are the relativistic mass of the particle when its velocity is  $\mathbf{v}$  in frame  $s$ , and  $\mathbf{v}'$  in frame  $s'$ , and  $m_0$  is the rest mass of the particle.

The next step is to explore the energy. Starting from Eq. (4) and using Eq. (12), we have

$$q\mathbf{E}\mathbf{v} = \mathbf{F} \cdot \mathbf{v} = \mathbf{v}d(m\mathbf{v}) / dt \quad (19)$$

Multiplying Eq. (19) by  $dt$ , using Eq. (18), and integrating, we have:

total energy=

$$\int \mathbf{v}d(m\mathbf{v}) = v \frac{m_0 v}{\sqrt{1 - v^2 / c^2}} - \int dv \frac{m_0 v}{\sqrt{1 - v^2 / c^2}} = mc^2 \quad (20)$$

Call the total energy ( $\epsilon$ ), we then have

$$\epsilon = mc^2 \quad (21)$$

Eq. (21) is the relativistic definition of energy in frame  $s$ . Using (21) in Eqs. (16a, 16d), we get

$$\epsilon' = \gamma(\epsilon - uP_x), \quad P'_x = \gamma\left(P_x - \frac{u}{c^2} \epsilon\right) \quad (22a,b)$$

$$P'_y = P_y, \quad P'_z = P_z \quad (22c,d)$$

where

$$\epsilon' = m'c^2 \quad (23)$$

is the relativistic energy in frame  $s'$ . Eqs. (22) are just the 3-vector relativistic transformation for momentum and energy. Now by taking the square of relations for relativistic mass, we get

$$m^2 = \frac{m_0^2}{(1 - v^2 / c^2)} \quad (a), \quad m'^2 = \frac{m_0^2}{(1 - v'^2 / c^2)} \quad (b) \quad (24)$$

Eqs. (24) could be written as follows

$$m'^2(1 - v'^2 / c^2) = m^2(1 - v^2 / c^2) \quad (25)$$

According to Eqs. (14), (21) and (23), Eq. (25) could be written as follows

$$\epsilon'^2 - c^2 \mathbf{P}'^2 = \epsilon^2 - c^2 \mathbf{P}^2 \quad (26)$$

Multiplying Eq. (24) by  $c^4$ , and rearranging

$$\varepsilon^2 = c^2 P^2 + m_0^2 c^4 \quad (27)$$

$$\varepsilon'^2 = c^2 P'^2 + m_0^2 c^4 \quad (28)$$

as expected.

The rest mass  $m_0$  is constant, independent of velocity, and so can be expressed in terms of the invariant of the quantity of energy and momentum, as we see in Eq. (26). The external electrical force transfers energy to the moving charged particle, which in turn expresses itself as an increase in the mass, Eq. (18), of the particle.

Now we can express the definition of velocity by the momentum and energy rather than by a time derivative of the coordinate. From Eq. (14) and Eq. (21), we have

$$\mathbf{v} = c^2 \mathbf{P} / \varepsilon \quad (29)$$

This definition takes into account the dynamical quantities of the particle. Thus enables us to understand how a moving charged particle responds to the relative motion, through the change in the energy and momentum as in Eqs. (22), which grants the validity of Eq. (26). Therefore the similarity between the space/momentum and time/mass transformations, *i.e.*

$$\begin{aligned} x' &= \gamma(x - ut) & P'_x &= \gamma(P_x - um) \\ y' &= y, \quad z' = z & P'_y &= P_y, \quad P'_z = P_z \\ t' &= \gamma(t - ux / c^2) & m' &= \gamma(m - uP_x / c^2) \end{aligned}$$

is not coincidental. This similarity entails more investigation in order to decide which quantity must be used for the definition of the velocity of a moving charged particle.

#### 4. The Physical Meaning of $\mathbf{F} \cdot \mathbf{v} = q\mathbf{E} \cdot \mathbf{v}$

Eq. (4) reduces to the well-known physical law

$$d\varepsilon / dt = q\mathbf{E} \cdot \mathbf{v} \quad (30)$$

This can be demonstrated as follows. The left-hand side of Eq. (4) together with Eqs. (12) and (29) implies

$$\mathbf{F} \cdot \mathbf{v} = \frac{d\mathbf{P}}{dt} \cdot \frac{c^2 \mathbf{P}}{\varepsilon}$$

The last equation could be written as follows

$$\mathbf{F} \cdot \mathbf{v} = \frac{1}{2} \frac{d(c^2 P^2)}{\varepsilon dt}$$

Using Eq. (27) in the last equation gives

$$\mathbf{F} \cdot \mathbf{v} = \frac{1}{2} d(\varepsilon^2) / \varepsilon dt$$

or

$$\mathbf{F} \cdot \mathbf{v} = d\varepsilon / dt$$

Comparing the last equation with Eq. (4) leads to Eq. (30).

In relativistic mechanics, a similar relation ( $d\varepsilon / dt = \mathbf{F} \cdot \mathbf{v}$ ) was derived using on the assumption that space-time possesses

field-like properties, resembling those of the electromagnetic field. By contrast, in our formulation the change in the total energy of the moving particle is brought about due to the work performed by the external electric field.

In tensor formalism [10], the power  $d\varepsilon / dt$  is the partner of the LFL in the 4-force. We have preferred to work in the ordinary 3-vector formulation, in order to demonstrate that even in such a formulation it is possible to derive the relation (30).

#### 5. Reinterpretation of Light Speed Invariance

It is known that the electromagnetic field possesses momentum density  $\mathcal{P}$ , which is related to the energy density of the field  $\mathcal{E}$  through the relation  $\mathcal{E} = c\mathcal{P}$ . According to the classical relation, we are able also to derive this relation in our work, if we assume, as Einstein did, that electromagnetic field consist of photons with proper mass ( $m_0 = 0$ ). Let us assume also that in frame ( $s$ ) the Eq. (27) holds true for a photon, we then have

$$\mathcal{E}^2 - c^2 \mathcal{P}^2 = 0$$

$$\text{or} \quad \mathcal{E} = c\mathcal{P} \quad (31)$$

Substituting Eq. (31) in Eq. (26) gives

$$\mathcal{E}'^2 - c^2 \mathcal{P}'^2 = 0$$

$$\text{or} \quad \mathcal{E}' = c\mathcal{P}' \quad (32)$$

From Eqs. (31) and (32), we see that

$$c = \mathcal{E} / \mathcal{P} = \mathcal{E}' / \mathcal{P}' \quad (33)$$

The invariance of light speed and the momentum-energy relation resemble the two faces of one coin. This is obvious from Eq. (33), where the validity of one leads to the validity of the other. Also Eq. (33) leads to reinterpretation of the invariance of light speed by using the material properties of the photon (energy, momentum). Thus let us demonstrate that Eq. (31), in addition to the Eqs. (6), leads to this invariance.

Assume that the photon travels along the  $x$ -axis, which means that it carries momentum  $\mathcal{P}_x = \mathcal{E} / c_x$  in the  $x$ -direction. The invariance light speed could be verified by substituting Eq. (29) into Eq. (6a):

$$v'_x = \frac{c^2 \mathcal{P}_x / \mathcal{E} - u}{1 - u\mathcal{P}_x / \varepsilon}$$

Using the Eq. (31) in the last equation gives

$$v'_x = \frac{c(1 - u/c)}{1 - u/c} = c, \quad c_x \equiv c$$

A similar result can be obtained, if it is assumed that in frame  $s$  the photon travels along the  $y$ -axis. In this case  $v'_x = -u$ , and from Eq. (6b), we see

$$v'_y = c^2 \mathcal{P}_y / \gamma \mathcal{E} = c / \gamma, \quad c_y \equiv c$$

which in turn means

$$v_x'^2 + v_y'^2 = u^2 + c^2 / \gamma^2 = c^2$$

$$f = -m_0 c^2 \sqrt{1 - v^2 / c^2}$$

Therefore, the magnitude of the speed of light is again  $c$ .

The definition (29) is in accord with 3-vector velocity transformations (6) when used in the case of electromagnetic fields (photons), where the invariance of light speed could be interpreted depending on the photon's dynamic quantities (energy, momentum), and not on the kinematics of the LT as in SRT. This invariance is the result of the response of energy-momentum to the relative motion, through the change in energy and momentum.

## 6. System Relativistic Lagrangian

It is known from relativistic mechanics that the relativistic Lagrangian of a charged particle moving in an electromagnetic field is derived depending on the relativistic mass. The relativistic mass is obtained in this formulation, and, depending on that mass, the relativistic Lagrangian of the system could be derived.

Let us assume that the Lagrangian of the system is equal to the sum of the usual potential energy part and an unknown function  $f$  of  $\mathbf{v}$  velocity and mass  $m$ ; *i.e.*,

$$L = f(\mathbf{v}, m) + q\mathbf{v} \cdot \mathbf{A}(\mathbf{r}, t) - q\Phi(\mathbf{r}, t) \quad (34)$$

where  $\mathbf{A}$  and  $\Phi$  are the vector and scalar potentials of the applied field. Now to determine the unknown function  $f$ , we use Lagrange's Eqs.

$$\frac{d}{dt}(\partial L / \partial \mathbf{v}) - \partial L / \partial \mathbf{r} = 0 \quad (35)$$

Computing the first and the second terms of the above equations for the Lagrangian (34) gives

$$\partial L / \partial \mathbf{v} = \partial f / \partial \mathbf{v} + q\mathbf{A}$$

*i.e.*

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \mathbf{v}} \right) = \frac{d}{dt} \left( \frac{\partial f}{\partial \mathbf{v}} \right) + q\nabla(\mathbf{A} \cdot \mathbf{v}) - q\mathbf{v} \times (\nabla \times \mathbf{A}) + q\partial \mathbf{A} / \partial t \quad (36)$$

$$\text{and} \quad \partial L / \partial \mathbf{r} = -q\nabla\Phi + q\nabla(\mathbf{A} \cdot \mathbf{v}) \quad (37)$$

Substituting (36) and (37) in (35) gives

$$\frac{d}{dt} \left( \frac{\partial f}{\partial \mathbf{v}} \right) + q\nabla(\mathbf{A} \cdot \mathbf{v}) - q\mathbf{v} \times (\nabla \times \mathbf{A}) + q\frac{\partial \mathbf{A}}{\partial t} + q\nabla\Phi - q\nabla(\mathbf{A} \cdot \mathbf{v}) = 0$$

$$\text{or} \quad \frac{d}{dt}(\partial f / \partial \mathbf{v}) = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Using Eq. (13), we then have

$$\frac{d}{dt}(\partial f / \partial \mathbf{v}) = \frac{d}{dt}(m\mathbf{v})$$

$$\text{or} \quad \partial f / \partial \mathbf{v} = m\mathbf{v} \quad \text{i.e.} \quad f = \int_0^v m v dv$$

Replacing  $m$  by its value from (18a) and integrating we get

which means that the relativistic Lagrangian takes the form

$$L = -m_0 c^2 \sqrt{1 - v^2 / c^2} + q\mathbf{v} \cdot \mathbf{A} - q\Phi$$

## Conclusion

In recent years important theories alternative to Einstein's, such as Wilhelm's theory and Jefimenko's theory, have been developed. Wilhelm's theory is based upon accepting the world aether as a carrier of electromagnetic effects. According to this theory, length contraction and time dilation are dynamical rather than kinematics effects. But by means of his "theory of electromagnetic retardation", Jefimenko's works reject the length contraction and time dilation, so that the reality of these concepts becomes doubtful. It is also possible to explain the disappearance of length contraction and time dilation on the basis of theoretical considerations, without using the "theory of electromagnetic retardation". This was demonstrated in our work by including the LFL within the framework of SRT, and analyzing the result according to the relativity principle. This re-derivation proves the ability by the LFL to represent reality, and hence the need to study more adequate basic laws, such as the LFL. It is not at all a drawback to take the LFL as a cornerstone in deriving the Eqs. of SRT and relativistic electromagnetism.

In the development of the SRT, Einstein required the use of measuring rods and clocks. Time dilation would occur as clocks run more slowly, and we know that the phenomenon of time dilation has received direct and indirect evidence, especially in the case of the transverse Doppler effect. In spite of that, we will demonstrate in our next work that, by using the present formulation, it is possible to derive the transverse Doppler effect without using the concept of time dilation.

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# Correspondence

**Resolving the Twins Paradox (cont. from p. 82)**

Begin with clocks A and B at rest relative to one another and separated by a distance  $r$ , with  $r$  being 300,000 km. Let us further stipulate that these clocks have been synchronized according to the method described above. At a definite time, let clock A begin a journey toward clock B at velocity  $\mathbf{v}$ , with speed  $v$  being  $(2/3)c$ . At this same instant, let a stationary observer located at clock B test the synchronization of the two clocks using this same method.

Under classical mechanics, if the speed of a light signal were  $c$  in the stationary system, its velocity would be  $c \pm v$  in the moving system. In this case we would have

$$t_b = 0, t_a = \frac{r}{c+v} = 0.6 \text{ sec}, t'_b = \frac{2r}{c+v} = 1.2 \text{ sec} \quad (1)$$

so that we would have

$$t_a - t_b = \frac{r}{c+v} - 0 = 0.6 \text{ sec}, \quad (2)$$

$$t'_b - t_a = \frac{2r}{c+v} - \frac{r}{c+v} = 0.6 \text{ sec}$$

Under classical mechanics, once the two clocks have been properly synchronized, introducing relative motion between the clocks would not be expected to affect their synchronization. This will not be the case if we accept the second postulate of SRT, which states that the velocity of light is the same in all inertial frames. In this case we have

$$t_b = 0, t_a = r/c = 1 \text{ sec}, t'_b = 2r/(c+v) = 1.2 \text{ sec} \quad (3)$$

so that we have

$$t_a - t_b = r/c = 1 \text{ sec}, \quad (4)$$

$$t'_b - t_a = 2r/(c+v) - r/c = 0.2 \text{ sec}$$

Under the second postulate of SRT, if distant clocks are properly synchronized while at rest relative to one another, they will not

retain their synchronization if relative motion between the clocks is introduced. In this example, we have  $t_a - t_b > t'_b - t_a$ , so that a stationary observer located at clock B would now perceive clock A to be advanced in comparison to clock B.

**The Desynchronization Factor**

To quantify this desynchronization, we need only calculate the time discrepancy between the clocks per unit distance for a specified velocity. We do this by stipulating clock B as the origin of both the stationary and moving coordinate systems, so that  $x$  represents both the location of clock A in the stationary system and its distance from clock B at the instant the light signal arrives. In this way,  $t$  properly represents the travel time of the light signal between clock B and clock A in the stationary system so that

$$t = x/c \quad (5)$$

For the example given above, an observer moving with clock A will perceive the light signal to reach clock A at point  $x'$  and at time  $t'$ , where

$$x' = -300,000 \text{ km}, t' = x'/c = +1 \text{ sec} \quad (6)$$

By contrast, the stationary observer at clock B will record the location and time of this event as

$$x = x' + vt = -180,000 \text{ km}, t = x'/(c+v) = 0.6 \text{ sec} \quad (7)$$

The desynchronization of the clocks turns out to be equal to the travel time of the light signal from clock B to clock A, as measured in the stationary system, multiplied by the velocity ratio  $v/c$ , in that we have

$$t' - t = x'/c - x/c = (v/c)(x/c) = vx/c^2 \quad (8)$$

**(concluded on p. 100)**