the general science journal

Sagnac Effect:
The Ballistic Interpretation

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Abstract:

The Sagnac effect and related topics are discussed in detail and expounded within the framework of the Emission Theory of light.

The Sagnac Experiment:

Three plane mirrors, M_1, M_2, and M_3, and a beam splitter P, are mounted on a turntable in a circular configuration of 25 cm radius. A coherent beam of light, from a source S, is made parallel by a lens L, and sent to P.

The initial beam is split by P into two beams, A and B, which traverse the same polygon path in opposite directions. The beam A travels counterclockwise, in the direction of the rotating apparatus, and meets the clockwise beam B, at P. The two beams are focused on a photographic plate O, forming interference fringe bands there.

Both the source of light S and the photographic plate O rotate with the same angular velocity as the apparatus.

For a radius of 25 cm and angular velocity of 2.2π rads^{-1}, a fringe shift of 0.034 is observed. The observed fringe shift varies linearly with the angular velocity, ω, and the area enclosed by the light path, A, [**Ref. #5**].

1. General Considerations

Before applying the Emission Theory of light to the above experiment, the following points have to be clarified and made explicit:

1. The light path, as marked by the three mirrors and the beam splitter, forms a square whose side, by the Pythagoras' theorem, is equal to ($2^{1/2}$) of the radius of the rotating disc.

2. As dictated by the law of reflection, in order for the two beams, A and B, to loop through the Sagnac apparatus, the normal lines of the mirrors, M_1 and M_3, must be at right angles to the normal of the mirror M_2, and at the same time made parallel to the normal of the beam splitter P. For illustration, see [**Ref. #**1]. Moreover, both the direction of the initial beam, from the source S, and the direction of the detector O, must make 45^o and 135^o with the normal of the beam splitter P, respectively.

3. For the Sagnac Experiment to be carried out properly, the rotation of the apparatus must be made as close to uniform circular motion as possible. From this basic requirement, it can be deduced that the tangential velocity of each mirror is perpendicular to its normal, and that of the beam splitter is parallel to its normal.

4. The Sagnac apparatus, in most trials, takes less than one second to make one rotation, in the laboratory. By contrast, the earth takes 86,164.09 seconds to make one rotation relative to the stars. Accordingly, the angular velocity of the Sagnac apparatus is overwhelmingly greater than the angular velocity of the earth. Furthermore, the flight time for both beams is vanishingly small as compared to the earth period of rotation. Therefore, it must be concluded that the effect of the earth rotation on the Sagnac Experiment is well below the apparatus sensitivity. Practically, for any Sagnac loop with radius less than 10 m, the earth tangential velocity is uniform and linear, and as such it cannot be detected through the use of the Sagnac apparatus.

5. In principle, more and more plane mirrors can be added to the Sagnac optical apparatus without altering the experimental results, [**Ref. #6**]. Geometrically, adding more plane mirrors adds more sides to the polygon path of light. As a result, the total length of the light path gets closer to the limit $2\pi r$, where r is the radius of the Sagnac disc. In fact, this increasing approach to the above limit is the optical analogue to the Archimedes way of doing calculus. However, light loops of $2\pi r$ length is easily achieved in modern Sagnac interferometers through the use of circular foils of fiber optics. Based on total internal reflection, a fiber optic is essentially equivalent to a configuration of plane mirrors with infinitely small length.

6. The rotation of the Sagnac apparatus causes the normal lines of the mirrors to rotate with respect to the incident beams. For the counterclockwise beam A, during the travel time from P to M_1, M_1 to M_2, M_2 to M_3, and from M_3 to P once again, the normal lines rotate in the direction of the incident beam A. As a result, the angles of incidence, in the four cases, are reduced, and the total length of the light path is increased. The reverse is true for the clockwise beam B, where the normal lines are shifted away by rotation from the direction of the incident beam, and hence the total length of the light path is decreased. The amount of length change in both cases is directly proportional to the angular velocity of the apparatus ω, and the area, enclosed by the light loop, A. It should be pointed out that within the value range of ω used in the Sagnac Experiment, the mirrors would not get out of alignment enough to break the light loop. Furthermore, the rotation of the normal lines, relative to the incident beams, is the geometrical equivalence to the algebraic representation, which will be discussed later in this exposition.

7. To treat the Sagnac effect according to the Emission Theory, the Stewart-Thomson law of reflection is required. It states that in the reference frame of the laboratory, light is always reflected from a moving reflecting surface with the resultant velocity of its relative velocity with respect to the reflecting surface and the velocity of the reflecting surface relative to the laboratory, [**Ref. #3**]. This important law is a generalization of the Law of Reflection. And it occupies a central position in the treatment of optical phenomena on the basis of the Emission Theory. In its precise mathematical form, the Stewart-Thomson law can be derived and formulated by treating reflection of light as a special case of elastic collision and applying the conservation laws of linear momentum and kinetic energy, for moving bodies, to the incident light and the reflecting surface. However, the quantitative treatment of this subject can be significantly simplified by assuming that the ratio between the mass of the incident light and the mass of the reflecting surface is vanishingly small and practically equal to zero. And therefore, the recoil caused by the incident light on the reflecting surface can be neglected without affecting the precision of the quantitative treatment. Consider, for instance, the simple case of a plane mirror approaching or receding from a stationary light source along the normal to its reflecting surface with a uniform linear velocity, v. If the angle of incidence with the normal, i, is measured counterclockwise with respect to the velocity vector of the mirror, then the magnitude of the relative velocity of the incident light, c', can be computed by applying the law of cosines:

$$c' = \sqrt{c^2 + v^2 + 2cv\cos i} \qquad [1.1],$$

Where c is the Maxwellian speed of light in vacuum. The direction of this relative velocity, i', can be obtained by applying the law of sines to the above arrangement:

$$\sin i' = \left[\frac{c}{c'}\right]\sin i \qquad [1.2].$$

From the law of reflection, the angle of reflection for reflected light must be equal to the direction of the relative velocity of the incident light, i', and hence:

$$\cos i' = \sqrt{1 - \sin^2 i'} = \sqrt{1 - \left[\frac{c^2}{c'^2}\right]\sin^2 i} \qquad [1.3].$$

And by applying the law of cosines once more to compute the speed of the reflected light, we obtain c'':

$$c'' = \sqrt{c'^2 + v^2 + 2c'v\cos i'} \qquad [1.4].$$

By combining Equations [1.1], [1.3], and [1.4], we obtain for the general case:

$$c'' = c\sqrt{1 + 2\frac{v^2}{c^2} + 2\frac{v}{c}\left[\cos i + \sqrt{1 + \frac{v^2}{c^2} + 2\frac{v}{c}\cos i}\,\right]} \qquad [1.5].$$

When a mirror approaches directly a light source along the normal to its reflecting surface, Equation #[1.5] is reduced to:

$$c'' = c + 2v \qquad [1.6];$$

And when it recedes from the source along the same line, we obtain:

$$c'' = c - 2v \qquad [1.7].$$

Equation #[1.5] can be generalized further through the rotation of the normal to the surface of the plane mirror by an angle, *j*, around the velocity vector of the mirror and applying the laws of cosines and sines in each case.

One important case is when ($j = 90^o$), In this special case, the speed of the reflected light is always equal to the speed of the incident light, i.e., ($c'' = c$), regardless of its angle of incidence; and its direction is governed by the law of reflection.

It should be mentioned that, in terms of incidence and reflection angles, the law of reflection from moving mirrors is established as an empirical law [**Ref. #3**]:

$$\frac{\sin i_1}{\cos i_1 + \frac{v}{c}} = \frac{\sin i_2}{\cos i_2 - \frac{v}{c}} \qquad [1.8];$$

Where v is the velocity of the mirror, i_1 and i_2 are the angles of incidence and reflection, respectively.

Furthermore, the fact that ($\sin i_1 = \sin i_2$), for a mirror rotating about its normal, is experimentally verified, [**Ref. #4**].

8. The Stewart-Thomson law can be extended to include change of velocities upon refraction by moving media as follow:
Light always moves, in a moving medium of refractive index *n*, with the resultant of its refracted velocity relative to the refractive medium and the velocity of the refractive medium with respect to the laboratory, [**Ref. #3**]. Accordingly, upon emerging from the refractive medium, light always restores its initial velocity of incidence as measured in the reference frame of the laboratory.

Taking into account the above remarks, we can now work out the details of the Sagnac effect within the context of the Emission Theory of light. The two cases of rotating and stationary source and detector have to be considered in this discussion.

2. The Case of Rotating Source and Detector

The initial beam, emitted by the source S towards the beam splitter P, moves with the resultant velocity c',

$$c' = \sqrt{c^2 + v^2 + 2cv\,c\cos\theta} \qquad [2.1],$$

Where v is the tangential velocity of S, and θ is the direction of the incident beam relative to the normal of P, and equal to 45^o. The incident beam is split by P into beams, A and B. Beam A is transmitted by P towards the mirror M_1 with the resultant velocity c_A ,

$$c_A = c' = \sqrt{c^2 + v^2 + 2cv\cos\theta} \qquad [2.2].$$

Beam B is reflected by P towards the mirror M_3 with the vectors sum of the velocity of the initial beam with respect to P and the tangential velocity of P relative to the laboratory,

$$c_B = \sqrt{c^2 + v^2 - 2cv\cos\theta} \qquad [2.3].$$

Because M_1, M_2, and M_3 have normal lines at right angles to the vectors of their tangential velocity, they would not change the speeds of incident beams upon reflection. This passive role of the mirrors simplifies calculations considerably. Only S, P, and O, have to be taken into account in computations based on the Emission Theory.

Let t_A and t_B denote the total travel time for the beam A and the beam B, respectively.

$$t_A = \frac{l + t_A v\cos\theta}{c_A} = \frac{l}{c_A - v\cos\theta} \qquad [2.4],$$

Where l is the length of the total path, and ($t_A v\cos\theta$) the projection of the detector displacement, during the travel time t_A, onto the polygon path of the beam A.
For the beam B,

$$t_B = \frac{l - t_B v\cos\theta}{c_B} = \frac{l}{c_B + v\cos\theta} \qquad [2.5];$$

Where ($t_B\, v\cos\theta$) is the projection of the detector displacement, during the travel time t_B, onto the polygon path of the beam B.

Using Equations #[2.4] and #[2.5], we compute the time difference Δt,

$$\Delta t = t_A - t_B = \frac{l}{c_A - v\cos\theta} - \frac{l}{c_B + v\cos\theta} = l\left[\frac{c_B - c_A + 2v\cos\theta}{(c_A - v\cos\theta)(c_B + v\cos\theta)}\right] \qquad [2.6].$$

For ($v << c$), we can use as an approximation, ($c_A = c_B$) and [$(c^2 - v^2 cos^2\theta) \approx c^2$], in the above equation:

$$\Delta t = 2l\left[\frac{v\cos\theta}{c^2}\right] \qquad [2.7].$$

From the design of the Sagnac Experiment, we have,
($l = 4[2]^{1/2}r$), ($v = \omega r$), and ($\theta = 45^o$), where r is the radius of the Sagnac loop. By substituting in Eq. #[2.7], we obtain,

$$\Delta t = \frac{8\omega r^2}{c^2} \qquad [2.8].$$

Using ($A = 2r^2$) in the last equation, we obtain,

$$\Delta t = \frac{4\omega A}{c^2} \qquad [2.9];$$

Where (A) is the area enclosed by the light path.
Finally, we multiply (Δt) by the factor (c/λ) to calculate the interference fringe shift Δz,

$$\Delta z = \frac{4\omega A}{c\lambda} \qquad [2.10];$$

Where (λ) is the wavelength of the light used in the experiment.

3. The Case of Stationary Source and Detector

Given the feasibility of executing each run of the experiment in sufficiently short period of time, it's possible to calculate the Sagnac effect in the case of stationary source and detector. Let a source of light S, at rest in the laboratory, emit an initial beam in the direction of a rotating beam splitter P. Since the tangential velocity of P is too small compared to the velocity of light, the effect of light aberration on the angles of incidence and reflection as required by the Stewart-Thomson law, can be omitted without affecting the precision of the calculations.
Let A denote the part of the initial beam transmitted by the beam splitter P towards the mirror M1, with the velocity of the initial beam, c, and hence,

$$c_A = c \qquad [3.1];$$

Where (c_A) is the velocity of the transmitted beam A.

Let B denote the part of the initial beam reflected by P towards the mirror M_3, with the vector sum of the velocity of the initial beam relative to P, and the tangential velocity of P in the laboratory, and therefore for ($v << c$),

$$c_B \approx (c - 2v\cos\theta) \qquad [3.2];$$

Where v is the tangential velocity of the splitter P.
We now compute the travel time for each beam,

$$t_A = \frac{l}{c_A} = \frac{l}{c} \qquad [3.3];$$

Where t_A is the travel time for the beam A, and l is the length of the light path. For the beam B, the travel time t_B,

$$t_B = \frac{l}{c_B} = \frac{l}{c - v\cos\theta} \qquad [3.4].$$

Let ($\Delta t = t_B - t_A$), and for ($v << c$), [$c(c - 2v\cos\theta) \approx c^2$], and hence,

$$\Delta t = \frac{8\omega r^2}{c^2} \qquad [3.5].$$

Where ω is the angular velocity of the beam splitter P, and r is the radius of the Sagnac disc. Noting that ($A = 2r^2$), and multiplying by (c/λ), we obtain the phase shift Δz,

$$\Delta z = \frac{4\omega A}{c\lambda} \qquad [3.6];$$

Where λ is the wavelength of the light used in the experiment.
We conclude, therefore, that for the range of angular velocities used in the Sagnac Experiment, stationary and rotating detectors show the same interference fringe shift.

4. The Michelson-Gale Experiment

This experiment is identical to the Sagnac Experiment in the case of rotating source and detector, except the earth here plays the role of the rotating Sagnac disc.

Because the earth angular velocity is several orders of magnitude smaller than that of the Sagnac disc, the area enclosed by the light path must be corresponding greater. For this reason, a rectangle of (613m x 340m) area is chosen by the experimenters as path. For experimental setup horizontal to the earth surface, only the component of the earth tangential velocity perpendicular to its surface is effective. Consequently, the Michelson-Gale Experiment would work best at the North and the South Poles, and would not work at all at the Equator.

However, for a rectangular light path vertical to the earth surface, the opposite is true, i.e. the experiment would give its best results at the Equator, and no results at the earth Poles.

The Michelson-Gale Experiment, which was set up horizontally to the earth surface somewhere between 30-Parallel and 45-Parallel in the United States, performed rather well. The expected displacement was 0.236 of a band. The value obtained as the mean of 269 readings was 0.230 of a band with a probable error of ±0.005. It is a very good result, considering the large area enclosed by the rectangular path and the loss of definition caused by air currents [**Ref. #5**].

5. The Relativistic Interpretation

It's appropriate at this point to review the conventional explanation for the Sagnac effect. Let *x*, *y*, *z*, *t* and *x'*, *y'*, *z'*, *t'* be the usual Relativistic axes in the reference frame of the rotating Sagnac disc and that of the laboratory, respectively.

In the plane of the light loop, ($z = z' = 0$), and the other co-ordinates,

$$x' = x\cos(\omega t) - y\sin(\omega t) \qquad [5.1];$$

$$y' = y\sin(\omega t) + y\cos(\omega t) \qquad [5.2];$$

$$t' = t \qquad [5.3];$$

Where (ω) is the angular velocity of rotation.

General Relativity gives, in the reference frame of the laboratory, the interval s between two events,

$$ds^2 = c^2dt^2 - dt^2 = c^2dt^2 - dx^2 - dy^2 \qquad [5.4];$$

When both events are in the plane ($z = z' = 0$).

Substituting x' and y' as defined above, we put

$$dx' = \left[dx\sin(\omega t) - dy\cos(\omega t)\right] - \left[x\cos(\omega t) - y\sin(\omega t)\right]\omega dt \qquad [5.5];$$

And similar expression for (dy'), with a total of 33 terms for the interval (s)!

However, when terms involving (ω^2) are neglected, the number of terms is significantly reduced,

$$ds^2 = c^2dt^2 - 2\omega(xdy - ydx)dt - dt^2 \qquad [5.6].$$

Equation #[5.4] gives an interval of zero for any two points on a ray of light, and hence,

$$c^2dt^2 - 4\omega dA - dt^2 = 0 \qquad [5.7];$$

Where {$d_A = 1/2[xdy - ydx]$} for the triangle whose apex is at the origin and whose base is the element of the light loop [**Ref. #4**].

By integrating over the light loop, we get the approximations,

$$t_1 = \frac{dl}{c} + \frac{2\omega A}{c^2} \qquad [5.8];$$

Where (t_1) is the travel time for the counterclockwise beam.

The travel time for the clockwise beam, (t_2),

$$t_2 = \frac{dl}{c} - \frac{2\omega A}{c^2} \qquad [5.9];$$

And therefore,

$$\Delta t = t_1 - t_2 = \frac{4\omega A}{c^2} \qquad [5.10].$$

Now we must note the weak points in the above explanation.

Due to the dubious step of explaining away the less obscure, the Sagnac effect, by something far more obscure, the Pseudo-Riemannian geometry of General Relativity, the conventional interpretation is completely devoid of any insight into the true nature of the Sagnac effect. In addition, the use of two different co-ordinate systems in the quantitative treatment of the Sagnac-type experiments is redundant and unnecessary. Only the rotating co-ordinate system is important.

As a matter of fact, in the Michelson-Gale Experiment, the observer does not even need to be aware of the existence of the Solar System, let alone setting up stationary co-ordinates relative to the stars.

The problems, posed by rotation to the Relativity Theory, run even deeper than this.

Recall that, in Special Relativity, clocks in relative motion cannot be synchronized, and length of measuring rods varies with states of motion. Well, on a rotating disc of iron for instance, every point has a unique motion relative to every point else on the disc. This also applies to every point on rotating clocks and measuring rods. Here, ‘point’ really means ‘point’ in the strict geometrical sense of the term, i.e. all the way down across the infinitesimals. The net result of all this is the utter annihilation of the measuring functions of clocks and measuring rods on rotating platforms.

It's easy, therefore, to see how Einstein was forced to model his Four-dimensional Space on the Riemannian Geometry whose co-ordinate systems have no physical meaning at all. In order to make contact with the world of measurements; the so-called Post-Newtonian Approximation is invented. It's lengthy and arbitrary. But even then, it works in only few cases, including the case of Sagnac effect.

6. Concluding Remarks

It should be clear from this discussion that the Sagnac and related experiments are more readily handled and explained away by theories based on variable speed of light including the Emission Theory. In fact, experiments of the Sagnac type are part of the evidence against Special Relativity [**Ref.** #2], and any other theory built upon constant speed of light with respect to moving observers.

Explanations based on General Relativity, like the one given earlier, cannot change this fact. That is because, even though not always clear, the latter theory implies varying speed of light with position. And in any case, those explanations are contrived and unconvincing.

Furthermore, the ability of observers to detect and measure the angular velocities of their platforms, without any reference to anything else, must worry all those who believe that the Principle of Relativity is valid under all conceivable circumstances. Mere familiarity and a bit of collective wizardry may lessen this concern. But, really, if they think about it more carefully, rotation must be deeply troubling. It's, above all, the strongest evidence, as demonstrated by Newton's Rotating-Pail Experiment, for absolute space and time.

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