Einstein’s Logical Errors

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This paper follows on the theme of the comments by Antonio Saraiva,

Abstract

Einstein’s derivation of the transformation equations in the Special Theory of Relativity contains multiple leaps of faith in the logic which cannot be justified.

Introduction

The basis of Einstein’s Special Theory of Relativity is the Lorenz transformation. His derivation of this is shown in Appendix 1 of his book "Relativity, The Special and The General Theory". The first stages of his reasoning are correct. He then makes multiple logical errors.

Einstein’s Correct reasoning

In Appendix 1 of his book, starting with

$x = ct$ (Einstein1)

$x^1 = ct^1$ (Einstein2)

Considering light rays going along the positive and negative x axes

$(x^1 - ct^1) = \lambda(x - ct)$ (Einstein3)

$(x^1 + ct^1) = \mu(x + ct)$ (Einstein4)

He introduces for convenience the constants a and b in the place of the constants λ and μ where

$a = ½ (\lambda + \mu)$

$b = ½(\lambda - \mu)$

This gives the equations

$x^1 = a x - bct$ (Einstein5a)

$ct^1 = act - bx$ (Einstein5b)

Further analysis of the equations

Considering the first of these equations

$x^1 = a x - bct$ (Einstein5a)

when $t = 0$, $x^1 = ax$ (8)

when $x^1 = 0$, $x = (bc/a)t$ (9)

when x = 0, x^1 = - bct(10)

The second equation at (5) in Einstein's reasoning is

ct^1 = act – bx (Einstein5b)

when t = 0, t^1 = – (b/c) x (11)

when t^1 = 0, t = (b/ac) x (12)

when x= 0, t^1 = at (13)

He first uses

when t = 0, x^1 = ax (8)

"Two points of the x^1 axis which are separated by the distance Δ x^1=1 when measured in the K^1 system are thus separated in our instantaneous photograph by the distance

Δ x = 1/a" (Einstein7)

There is no problem with the logic of this step.

The Logical Errors

He then uses

when t^1 = 0, t = (b/ac)x (12)

in equation

x^1 = a x – bct (Einstein5a)

To give

x^1 = a x – (b^2/a)t (15)

He then uses

when x^1 = 0, x = (bc/a)t (9)

To get, defining v as x/t

v=bc/a (Einstein6)

Using (Einstein6) in (15) gives

x^1 = a (1 – v^2 / c^2) x (Einstein, no number, between 7 and 7a)

He summarises these above steps as follows

"But if a snapshot be taken from K^1 (t^1 = 0), and if we eliminate t from the equations (5), taking into account the expression (6) we obtain

x^1 = a (1 – v^2 / c^2) x"

There are a number of points to bring out. He first uses when $t^1 = 0$, $t = (b/ac)x$ (12)

this could be written as

when $t^1 = 0$, $x/t = ac/b$ (16)

He then uses

when $x^1 = 0$, $x = (bc/a)t$ (9)

Which can be written as

when $x^1 = 0$, $x/t = bc/a$

So (x/t) can be either (bc/a) or (b/ac) in the same equation!

This confirms the error of using derivations from $t^1 = 0$ and $x^1 = 0$ in the same equation

Einstein then goes on to say

"From this we conclude that two points on the x axis separated by the distance 1 (relative to K) will be represented on our snapshot by

$\Delta x^1 = a\,(1 - v^2 / c^2)$ (7a) "

This is saying that, because the "snapshot" is taken at $t^1 = 0$, it is justified to consider the equation of

$x^1 = a\,(1 - v^2 / c^2)\,x$

as if it was viewed from from K^1 instead of K.

In other words, for one equation (Einstein7) he lays a unit rod in K^1 and perceives it from K. In the other equation (Einstein7a) he lays a unit rod in K and perceives it from K^1.

Thus manoeuvre enables this value

$a\,(1 - v^2 / c^2)$

to be converted to its inverse. There is no logical basis whatsoever for the assumption that the view can be taken from K^1.

He started his reasoning by stating that he had values of K and needed to find values in K^1 and the equations were derived on this basis. So it is absurd to assume that in one isolated equation he can now perceive from K^1.

One has to conclude that it is not surprising no-one could understand the "logic".

Verbatim Quote from Einstein’s Book

Part of APPENDIX 1 from RELATIVITY. The Special and the General Theory

A Popular Exposition by ALBERT EINSTEIN

Authorised Translation by Robert W. Lawson

Methuen & Co Ltd, 1920

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SIMPLE DERIVATION OF THE LORENTZ TRANSFORMATION (SUPPLEMENTARY TO SECTION XI)

For the relative orientation of the co-ordinate systems indicated in Fig.2, the *x*-axes of both systems permanently coincide. In the present case we can divide the problem into parts by considering first only events which are localized on the *x*-axis. Any such event is represented with respect to the co-ordinate system *K* by the abscissa *x* and the time *t*, and with respect to the system K^1 by the abscissa x^1 and the time t^1. We require to find x^1 and t^1 when *x* and *t* are given.

A light-signal, which is proceeding along the positive axis of *x*, is transmitted according to the equation

$x = ct$

or

$x-ct = 0$ (1)

Since the same light-signal has to be transmitted relative to K^1 with the velocity c, the propagation relative to the system K^1 will be represented by the analogous formula

$x^1-ct^1 = 0$ (2)

Those space-time points (events) which satisfy (1) must also satisfy (2). Obviously this will be the case when the relation

$(x^1-ct^1) = \lambda(x-ct)$ (3)

is fulfilled in general, where λ indicates a constant; for, according to (3), the disappearance of $(x-ct)$ involves the disappearance of (x^1-ct^1).

If we apply quite similar considerations to light rays which are being transmitted along the negative *x*-axis, we obtain the condition

$(x^1+ct^1) = \mu(x+ct)$ (4)

By adding (or subtracting) equations (3) and (4), and introducing for convenience the constants *a* and *b* in place of the constants λ and μ where

$a = ½ (\lambda + \mu)$

and

$b = ½ (\lambda - \mu)$

we obtain the equations

$x^1 = ax-bct$

$ct^1 = act-bx$ (5)

We should thus have the solution of our problem, if the constants *a* and *b* were known. These result from the following discussion.

For the origin of K^1 we have permanently $x^1 = 0$, and hence according to the first of the equations (5)

$x = (bc/a)t$.

If we call *v* the velocity with which the origin of K^1 is moving relative to *K*, we then have

$v = (bc/a)$ (6)

The same value *v* can be obtained from equations (5), if we calculate the velocity of another point of K^1 relative to *K*, or the velocity (directed towards the negative *x*-axis) of a point of *K* with respect to K^1. In short, we can designate *v* as the relative velocity of the two systems.

Furthermore, the principle of relativity teaches us that, as judged from *K*, the length of a unit measuring-rod which is at rest with reference to K^1 must be exactly the same as the length, as judged from K^1, of a unit measuring-rod which is at rest relative to *K*. In order to see how the points of the x^1-axis appear as viewed from *K*, we only require to take a "snapshot" of K^1 from *K*; this means that we have to insert a particular value of *t* (time of *K*), e.*g.* t=0. For this value of *t* we then obtain from the first of the questions (5).

$x^1 = ax$.

Two points of the x^1-axis which are separated by the distance $\Delta x^1 = 1$ when measured in the K^1 system are thus separated in our instantaneous photograph by the distance

$\Delta x = (1/a)$ (7)

But if the snapshot be taken from K^1 (t^1=*o*), and if we eliminate *t* from the equations (5), taking into account the expression (6), we obtain

$x^1 = a(1-[v^2/c^2]\ )x$.

From this we conclude that two points on the x-axis separated by the distance 1 (relative to *K*) will be represented on our snapshot by the distance

$\Delta x^1 = a(1-[v^2/c^2]\ )$ (7a)

But from what has been said, the two snapshots must be identical; hence Δx in (7) must be equal to Δx^1 in (7a), so that we obtain

$a^2 = 1/(1-[v^2/c^2]\ )$ (7b)

The equations (6) and (7b) determine the constants a and b. By inserting the values of these constants in (5), we obtain the first and the fourth of the equations given in Section XI.

$x^1 = (x-vt) / (1-[v^2/ c^2]\ )^{½}$

$t^1 = (t-[vx/c^2]\ ) / (1-[v^2/c^2]\ )^{½}$ (8)

Thus we have obtained the Lorentz transformation for events on the x-axis. It satisfies the condition

$(x^1)^2 - c^2(t^1)^2 = x^2 - c^2t^2$ (8a)