

Einstein's Derivation of $E=mc^2$ and the Photoelectric Effect

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Abstract

This article will explain the derivation of Einstein's SR theory and the photoelectric effect. Einstein succeeded in explaining the photoelectric effect using the basic quantum concept. The photoelectric effect generally refers to the emission or ejection of electrons from the surface of a metal in response to incident light. The energy of the photon can be written as the frequency ν times Planck's constant, which equals $h\nu$. When incident on a metallic surface, part of the energy $h\nu_0$ is used to free the electron from the metal, and the other part appears as the kinetic energy of the photoelectrons, $mv^2/2$. This paper will derive mc^2 by Newton's law through a very nice formal application

Introduction

Newton's law gives mass m and the velocity v as the force F . When changing the mass, the derivative can be given by this equation:

$$F = \frac{d}{dt}(m v)$$

This function lead us to the derivative to find the work function

$$F = v \frac{dm}{dt} + m \frac{dv}{dt}$$

The work is $dw = E$ and we can find E

$$dw = \int F . dx = \int (v dm + m dv) \cdot \frac{dx}{dt} = \int v^2 dm + m v dv$$

The last equation gives E using v , To change the mass in the SR derivative, Einstein take it as,

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

We can replace the last one and the derivative gives

$$m^2 c^2 - m^2 v^2 = m_o^2 c^2 \xrightarrow{D} c^2 dm - v^2 dm - v m dv = 0$$

The integration gives,

$$E_t = \int_{m_o}^m c^2 dm = c^2 (m - m_o)$$

From last equation, the total E gives

$$E = m c^2$$

The Klein-Gordon Equation:

We can derive the Klein Gordon equation by dE

$$E = h\nu \xrightarrow{1} p = \frac{h}{\lambda} \xrightarrow{2} E = pc$$

Newtonian mechanics gives $p = mv$ and the energy is given by

$$\int dE = \int F dx = \int \frac{dp}{dt} dx = \int v dp$$

We can find v by E

$$v = \frac{c^2 p}{E}$$

Klein Gordon

$$\int E dE = \int c^2 p dp$$

The integration gives :

$$E^2 = m^2 c^4 + c^2 p^2$$

This results in an operator equation for the wave function. We can see by the relativistic energy relation that we can write the conservation of energy as,

$$h\nu = h\nu_o + \frac{1}{2}m\nu^2$$

Where $h\nu_o$ is the Klein Gordon work function and m is rest mass $E=cp$

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$