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# *Thermoelastic Plane Waves by a Body under Force F: Formulation of the Basic Equation*

Dr_math2010@hotmail.com
Ahmed.S.Arife
Tel: +20102958263

***Abstract :***

In this paper we give the basic equation for this problem and try to derive the equation governor. We consider the force between two homogeneous isotropic thermoelastic solids and give the basic equation.

***Introduction:***

We can write the field equation for the energy of a thermoelastic solid with an elastic force and heat source. A constitutive relation can be written

$$(\lambda+\mu)\nabla(\nabla\cdot u)+\mu\nabla^2 u-\nu\nabla T+F=\rho\frac{\partial^2 u}{\partial t^2} \quad (1)$$

The force is given by

$$F=\mu_e H_0^2\nabla(\nabla\cdot u) \quad (2)$$

We can write the equation as

$$(\lambda+\mu+\mu_e H_0^2)\nabla(\nabla\cdot u)+\mu\nabla^2 u-\nu\nabla T=\rho\frac{\partial^2 u}{\partial t^2} \quad (3)$$

$$K\nabla^2 T=\rho C\frac{\partial^2 T}{\partial t^2}+\nu T_0\frac{\partial^2}{\partial t^2}\nabla\cdot u \quad (4)$$

For a two dimensional problem , the displacement vector **u** is given by

$$u=(u,0,\omega) \quad (5)$$

Where the displacement components u and $\omega$ are related by the potential function

$$u = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z}$$

$$\omega = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x} \qquad (6)$$

Now we will drive the method of finding the basic equation and the constant

$$\nabla \cdot u = \left( \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} + \right) \cdot u$$

$$\nabla \cdot u = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \nabla^2 \phi$$

$$\nabla^2 u = \nabla \cdot \nabla u = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \cdot \left( \frac{\partial u}{\partial x}, 0, \frac{\partial \omega}{\partial z} \right) = \left( \frac{\partial^2 u}{\partial x^2}, 0, \frac{\partial^2 \omega}{\partial z^2} \right)$$

$$\nabla^2 u = \left( \frac{\partial}{\partial x} \frac{\partial^2 \phi}{\partial x^2} - \frac{\partial}{\partial z} \frac{\partial^2 \psi}{\partial x^2}, 0, \frac{\partial}{\partial z} \frac{\partial^2 \phi}{\partial z^2} + \frac{\partial}{\partial x} \frac{\partial^2 \psi}{\partial z^2} \right)$$

$$= \left( \frac{\partial}{\partial x} \frac{\partial^2 \phi}{\partial x^2}, 0, \frac{\partial}{\partial z} \frac{\partial^2 \phi}{\partial z^2} \right) + \left( -\frac{\partial}{\partial z} \frac{\partial^2 \psi}{\partial x^2}, 0, \frac{\partial}{\partial x} \frac{\partial^2 \psi}{\partial z^2} \right)$$

$$\nabla^2 u = \nabla(\nabla^2 \phi) + \Delta(\nabla^2 \psi) \qquad (7)$$

Where

$$\nabla = \left( \frac{\partial}{\partial x}, 0, \frac{\partial}{\partial z} \right)$$

$$\Delta = \left( -\frac{\partial}{\partial z}, 0, \frac{\partial}{\partial x} \right)$$

$$\frac{\partial^2 u}{\partial t^2} = \left( \frac{\partial^2 u}{\partial t^2}, 0, \frac{\partial^2 \omega}{\partial t^2} \right) = \left( \frac{\partial}{\partial x} \frac{\partial^2 \phi}{\partial t^2} - \frac{\partial}{\partial z} \frac{\partial^2 \psi}{\partial t^2}, 0, \frac{\partial}{\partial z} \frac{\partial^2 \phi}{\partial t^2} + \frac{\partial}{\partial x} \frac{\partial^2 \psi}{\partial t^2} \right)$$

$$\frac{\partial^2 u}{\partial t^2} = \left( \frac{\partial}{\partial x} \frac{\partial^2 \phi}{\partial t^2}, 0, \frac{\partial}{\partial z} \frac{\partial^2 \phi}{\partial t^2} \right) + \left( -\frac{\partial}{\partial z} \frac{\partial^2 \psi}{\partial t^2}, 0, \frac{\partial}{\partial x} \frac{\partial^2 \psi}{\partial t^2} \right)$$

$$\frac{\partial^2 u}{\partial t^2} = \nabla \frac{\partial^2 \phi}{\partial t^2} + \Delta \frac{\partial^2 \psi}{\partial t^2} \qquad (8)$$

Now by the first equation we can write that

$$(\lambda + \mu + \mu_e H_0^2)\nabla(\nabla^2 \phi) + \mu\left[\nabla(\nabla^2 \phi) + \Delta(\nabla^2 \psi)\right] - \nu \nabla T = \rho\left[\nabla \frac{\partial^2 \phi}{\partial t^2} + \Delta \frac{\partial^2 \psi}{\partial t^2}\right] \quad (9)$$

$$(\lambda + 2\mu + \mu_e H_0^2)\nabla(\nabla^2 \phi) - \nu T = \rho \frac{\partial^2 \phi}{\partial t^2} \qquad (10)$$

$$\mu \nabla^2 \psi = \rho \frac{\partial^2 \psi}{\partial t^2} \qquad (12)$$

$$K \nabla^2 T = \rho C \frac{\partial^2 T}{\partial t^2} + \nu T_0 \frac{\partial^2}{\partial t^2} \nabla^2 \phi \qquad (13)$$

We can write this equation

$$c_1^2 (\nabla^2 \phi) - \bar{\nu} T = \frac{\partial^2 \phi}{\partial t^2} \qquad (14)$$

$$c_2^2 \nabla^2 \psi = \frac{\partial^2 \psi}{\partial t^2} \qquad (15)$$

$$\bar{K} \nabla^2 T = C \frac{\partial^2 T}{\partial t^2} + \bar{\nu} T_0 \frac{\partial^2}{\partial t^2} \nabla^2 \phi \qquad (16)$$

The constant gives

$$c_1^2 = \frac{\lambda + 2\mu + \mu_e H_0^2}{\rho}$$

$$c_2^2 = \frac{\mu}{\rho}$$

$$\bar{\nu} = \frac{\nu}{\rho} \qquad \bar{K} = \frac{K}{\rho}$$

Multiply 14 by $\nabla^2$. This gives

$$\nabla^4\phi-\nabla^2\frac{\bar{\nu}}{c_1^2}T=\frac{1}{c_1^2}\frac{\partial^2}{\partial t^2}\nabla^2\phi \qquad (17)$$

And by eq 16

$$\nabla^4\phi-\frac{\bar{\nu}}{c_1^2}\left[\frac{C}{\bar{K}}\frac{\partial^2 T}{\partial t^2}+\frac{\bar{\nu}T_0}{\bar{K}}\frac{\partial^2}{\partial t^2}\nabla^2\phi\right]=\frac{1}{c_1^2}\frac{\partial^2}{\partial t^2}\nabla^2\phi \qquad (18)$$

Simplify 18 to give

$$\nabla^4\phi-\nabla^2\left[\frac{1}{c_1^2}\frac{\partial^2\phi}{\partial t^2}+\frac{\bar{\nu}^2T_0}{c_1^2\bar{K}}\frac{\partial^2\phi}{\partial t^2}\right]=\frac{\bar{\nu}C}{c_1^2\bar{K}}\frac{\partial^2 T}{\partial t^2} \qquad (19)$$

By equation 14

$$c_1^2\frac{\partial^2}{\partial t^2}\nabla^2\phi-\bar{\nu}\frac{\partial^2 T}{\partial t^2}=\frac{\partial^4\phi}{\partial t^4}$$

This gives

$$\bar{\nu}\frac{\partial^2 T}{\partial t^2}=c_1^2\frac{\partial^2}{\partial t^2}\nabla^2\phi-\frac{\partial^4\phi}{\partial t^4}$$

and multiply by $C/c_1^2K$ to give

$$\frac{\bar{\nu}C}{c_1^2\bar{K}}\frac{\partial^2 T}{\partial t^2}=\frac{C}{\bar{K}}\frac{\partial^2}{\partial t^2}\nabla^2\phi-\frac{C}{c_1^2\bar{K}}\frac{\partial^4\phi}{\partial t^4}$$

Now by eq 19

$$\nabla^4\phi-\nabla^2\left[\frac{1}{c_1^2}\frac{\partial^2\phi}{\partial t^2}+\frac{\bar{\nu}^2T_0}{c_1^2\bar{K}}\frac{\partial^2\phi}{\partial t^2}\right]=\frac{C}{\bar{K}}\frac{\partial^2}{\partial t^2}\nabla^2\phi-\frac{C}{c_1^2\bar{K}}\frac{\partial^4\phi}{\partial t^4} \qquad (20)$$

We can but $\varepsilon=\bar{\nu}^2T_0/c_1^2C$ in eq 20 gives

$$\nabla^4\phi - \nabla^2\left[\frac{1}{c_1^2} + (1+\varepsilon)\frac{C}{K}\right]\frac{\partial^2\phi}{\partial t^2} + \frac{C}{c_1^2\overline{K}}\frac{\partial^4\phi}{\partial t^4} = 0 \qquad (21)$$

We assume now the solution

$$\phi \,\square\, \phi e^{-i\,\omega t}, T \,\square\, T e^{-i\,\omega t}, \psi \,\square\, \psi e^{-i\,\omega t} \qquad (22)$$

we can define the constant

$$A = \frac{1}{c_1^2} + (1+\varepsilon)\frac{C}{K}$$

$$B = \frac{C}{c_1^2\overline{K}} \qquad (23)$$

And

$$\frac{\partial^2\phi}{\partial t^2} = -\omega^2\phi e^{i\,\omega t}$$

$$\frac{\partial^4\phi}{\partial t^4} = \omega^4\phi e^{i\,\omega t} \qquad (24)$$

By the  constant and equation 24 write 21

$$\nabla^4\phi + A\,\omega^2\nabla^2\phi + B\,\omega^4\phi = 0 \qquad (25)$$

Solution 25 gives

$$\delta_{1,2}^2 = \omega^2\lambda_{1,2}^2$$

Where

$$\lambda_{1,2}^2 = \frac{-A \pm\sqrt{A^2 - 4B}}{2}$$

***Reference :***


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