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DEDUCTIVE ARGUMENTS IN NATURAL SCIENCES

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Argument, inference, conditional

In natural sciences the approach referred to as theoretical can be reduced, without much loss, to the recurrent application of a specific mental procedure called argument or inference. In accordance with this procedure, one initially affirms a few premises and then extracts a conclusion from them. The so-called Carnot theorem can be regarded as a classical example:

Premise 1: Heat is conserved (cannot be converted into work in the heat engine).

Premise 2: Perpetuum mobile of the first kind is impossible.

Conclusion: All reversible heat engines working between the same two temperatures T_1 and T_2 ($T_1 > T_2$) have the same efficiency.

Of course, one could claim that the theoretical approach cannot so easily be reduced to repetitive application of arguments. In other words, the theory could be richer than a sequence of arguments. The admirers of Einstein would insist on the importance of esthetical regulation, divine intuition or whatever else they see in the works of their idol. Yet, even if admirers were right, the discussion of this particular problem would be irrelevant here. We are just going to define and explain two kinds of deductive arguments which, in our view, play an important role in natural sciences. If those deductive arguments are really important, this paper could be valuable. If not, not.

The argument often takes the form of a conditional (if-clause):

If “antecedent”, then “consequent”.

A simple juxtaposition would show that the conditional’s consequent coincides with the conclusion of the argument. As for the conditional’s antecedent, it coincides with the conjunction of the premises of the argument:

Antecedent = Premise 1 & Premise 2 & Premise 3 &….

We have just established an equivalence between argument, inference and conditional. This will allow us to change the linguistic form or the respective term without any shift in meaning. For instance, one often affirms a single premise and suppresses, temporarily or permanently, other premises. In this case the conditional proves particularly suitable. So the Carnot theorem can be reduced to:

If heat is conserved (cannot be converted into work in the heat engine), then all heat engines working between the same two temperatures T_1 and T_2 ($T_1 > T_2$) have the same efficiency.

Needless to say, the suppressed premise (“Perpetuum mobile of the first kind is impossible”) is by no means less important than the explicit one (“Heat is conserved”).

Below are a few examples of conditionals (some quite weird) which, in a certain context, would be accepted as legitimate:

(1) If it is a crow, then it is black.

(2) If it rains, then the soil is wet.

(3) If the temperature of water surpasses 50°C, then the fish dies.

(4) If it both rains and does not rain, then crocodiles can fly.

(5) If it rains, then either crocodiles can fly or it rains.

(6) If $(x'-ct')=\lambda(x-ct)$, then $(x'-ct')=0$ entails $(x-ct)=0$ and $(x-ct)=0$ entails $(x'-ct')=0$.

(7) If heat is conserved (cannot be converted into work in the heat engine), then all reversible heat engines working between the same two temperatures T_1 and T_2 ($T_1 > T_2$) have the same efficiency.

(8) If the speed of light does not depend on the speed of the light source, then I observe your clock running slower than mine and you observe mine running slower than yours. Both observations are correct.

The last three arguments, (6), (7) and (8), are particularly important because their conclusions involve, explicitly or implicitly, *mathematical equations*. This will prove crucial for our definition of *mathematical deductive arguments*.

Absolute truths

In his *Tractatus* Wittgenstein defines propositions that owe their truth or falsehood to the fact that they picture or mirror states of nature. Of course, not all propositions receive their truth from nature but obviously some do. As one claims, for instance, that the speed of light does *not* depend on the speed of the light source (Einstein's light postulate advanced in 1905), one implicitly assumes that nature has already said "True!" or "False!" once and for all, although scientists might be ignorant of or mistaken about nature's verdict.

Insofar as they are independent of our subjectivity, nature's verdicts and true propositions that reflect them can be defined as *absolute truths*. Natural sciences have always been looking for such truths. If Einstein's 1905 light postulate is not an absolute truth, then its antithesis proclaiming that the speed of light does depend on the speed of the light source surely is.

Axioms are allowed to be false but…

Philosophical concepts usually have several births. Wittgenstein's idea that the truth of propositions should be referred to states of nature has obviously something to do with the realist construal of theories involving the assumption that scientific propositions are true or false in virtue of how the world is independently of ourselves. But what if in some particular case, at least initially, we have no idea of how the world is independently of ourselves? Then theoreticians are allowed to advance some initial assumption, an axiom, and draw conclusions from it, although this axiom might prove false in the end. That is, theoreticians are allowed to advance the conditional:

If “axiom that might prove false in the end”, then “conclusion”.

The step seems advantageous: theoreticians believe it is both interesting and useful to obtain new conclusions, even if their origin is uncertain. Moreover, in accordance with the dominant philosophy of Karl Popper, if no contradiction between experiments and the *final* conclusions of the theory has been found, then there is nothing to worry about. That is, no contradiction between experiments and final conclusions and both theoreticians and philosophers of science sleep well. No nightmares where false axioms or invalid arguments mysteriously devastate theoretical science in an irreversible way.

Unfortunately Popper’s philosophy is misleading. Even if the scientific community knows of no experimental evidence contradicting the final conclusions of the theory, a false axiom could still have caused, quite imperceptibly, horrible devastation. Einstein has left texts suggesting that, unlike Popper, he was quite sensitive to the problem. For instance, by the end of his life, he made the following confession:

Einstein 1954 : “I consider it entirely possible that physics cannot be based upon the field concept, that is on continuous structures. Then nothing will remain of my whole castle in the air, including the theory of gravitation, but also nothing of the rest of contemporary physics.”

By 1954 everybody believes that Einstein’s theory has already been confirmed by countless experiments. No falsification at all registered, at least officially. In this atmosphere of triumph and glory Einstein’s apocalyptic scenario looks absolutely mysterious. No falsification at all and yet it is “entirely possible” that “field” and “continuous structures” can be wrong initial concepts. No falsification at all and yet Einstein hints at bidding farewell not only to his own theory but also to “the rest of contemporary physics”. What really is going on in the divine mind in 1954?

The antithesis of “field concept” and “continuous structures”, insofar as the nature of light is concerned, is Newton’s corpuscular theory according to which light consists of (discontinuous) tiny particles nowadays called photons. The important point is that the speed of those particles *does* depend on the speed of the light source, in contradiction to Einstein’s 1905 light postulate. Therefore it is reasonable to suspect that, in his confession, Einstein hints at some link between the “entirely possible” collapse of contemporary physics and the falsehood of his 1905 light postulate. A second confession of Einstein’s gives more support to our suspicion:

Albert Einstein: “If the speed of light is the least bit affected by the speed of the light source, then my whole theory of relativity and theory of gravity is false.”

Perhaps it would have been much fairer for Einstein to combine his two confessions in the following way:

“I consider it entirely possible that the speed of light does depend on the speed of the light source. Then nothing will remain of my whole castle in the air, including the theory of gravitation, but also nothing of the rest of contemporary physics.”

False premise, true conclusion?

It is evident that the truth/falsehood of the conclusion somehow depends on the truth/falsehoods of the premises. This dependence is quite definite sometimes. For instance, logicians would say that, if the premises of a deductive argument are true, then the conclusion cannot be false. For that reason deductive arguments are called *truth-preserving*. On the other hand, it seems that if some of the premises are false, the conclusion could still prove true:

If it rains, then the soil is wet.

It is easy to imagine a situation where it does not rain, that is, the premise is false, and yet the soil is wet (the gardener has watered it), that is, the conclusion is true. Now the crucial question:

Does *any* argument in natural sciences tolerate the combination “false premise, true conclusion”? In particular, does the Carnot theorem tolerate this combination?

To our knowledge, this question has never been asked, at least not in this form. Yet in 1850 Clausius implicitly answered “yes” as he dealt with the Carnot argument:

Carnot argument: If heat is conserved (cannot be converted into work in the heat engine), then all heat engines working between the same two temperatures T_1 and T_2 ($T_1 > T_2$) have the same efficiency.

The premise is false but the conclusion had turned out to be manna for Clausius and Kelvin, an inexhaustible source of miraculous new results. So Clausius found a way to save the Carnot conclusion, although, as we are going to show, the Carnot argument does not tolerate the combination “false premise, true conclusion”.

Kant: Physical propositions are as certain as mathematical propositions

Hume regarded the propositions of physics as protagonists in inductive scenarios, that is, as ones whose certainty is insufficient. In order to counteract Hume’s skepticism, Kant decided to show that physical propositions are just as certain as mathematical propositions. However the former were *synthetic* whereas mathematical propositions were regarded as *analytic*. Kant tried to solve the problem by developing a theory according to which mathematical propositions were also synthetic.

Synthetic and analytic propositions are not supposed to be discussed here but since we have already mentioned them, some minimum information should be given. Consider the proposition:

A statement picturing a state of nature, e.g. “The speed of light is independent of the speed of the light source”, cannot be both true and false.

“A statement picturing a state of nature” is the subject; the property of being unable to be both true and false is the predicate. Clearly, the concept of the predicate is logically included in the concept of the subject; that is, the latter concept is possible just because, in referring to a statement picturing one of its own states, nature can say either “true” or “false” but never both. According to Leibniz, this defines the proposition as *analytic*. In contrast, the proposition:

The crow is black.

is *synthetic*. The concept of the predicate, “black”, is by no means included in the concept of the subject, “crow”: surely somewhere natural selection has produced differently colored plumage. The synthesis of the two concepts just narrows the possibilities, that is, makes the subject more definite.

Yet propositions like “The crow is black”, although synthetic, could not be referred to as “physical” if “physical” is to conform to Kant’s assertion:

Kant: *Physical propositions are as certain as mathematical propositions.*

We shall see that, in contrast, the Carnot argument does conform to Kant's assertion.

Inductive arguments

Among birds, crows and swans play an essential role in the explanation of what is called *induction*. Assume one makes observations and the initial result consists of exclamations of the sort: "Look, a black crow!", "Ah, another black crow!", etc. After making a sufficient number of observations one has the courage to declare, in referring to a bird that has not been identified yet:

If it is a crow, then it is black.

This type of reasoning is called induction. Even if the premises of the inductive argument are true, there is no guarantee that the conclusion is also true; rather, it is reasonable to say it is *probably* true. This can even be converted into a definition of inductive arguments: these are arguments where the premises support the conclusion (render it probable) without guaranteeing its truth.

Induction involves, unavoidably, what is called "the fallacy of affirming the consequent". A generalized inductive argument can be presented in the following way:

If something characterizes any object in the subset, then it characterizes any object in the set.

This argument is obviously invalid. However if we change the places of the antecedent and the consequent, the argument becomes valid:

If something characterizes any object in the set, then it characterizes any object in the subset.

In a purely logical system independent of the empirical world only the latter (valid) conditional would exist. That is, in such a logical system, the antecedent and the consequent of the latter conditional would be, respectively, "the true antecedent" and "the true consequent". Judging from this purely logical system, in the former (invalid) conditional one has fallaciously affirmed "the true consequent" and obtained, as a conclusion, "the true antecedent". Hence "the fallacy of affirming the consequent".

The fallacy of affirming the consequent could be insignificant (that is, the inductive argument could be almost valid), as in the following inductive derivation of the proposition "The sun will rise tomorrow":

If the sun has already risen $n-1$ times, then tomorrow there will be the n^{th} sunrise.

However the fallacy of affirming the consequent could also be extremely dangerous, as in the following argument implicitly but systematically used in Clausius's works:

If something characterizes any object in the subset composed of ideal gas systems (systems where interaction between particles is absent by definition), then it characterizes any object in the set composed of all material systems.

The fallacy of affirming the consequent could also appear in purely deductive contexts. So in Appendix 1 of his 1920 "Relativity" Einstein essentially advances the argument:

If $(x'-ct')=0$ entails $(x-ct)=0$ and $(x-ct)=0$ entails $(x'-ct')=0$, then $(x'-ct')=\lambda(x-ct)$.

Clearly the argument is invalid. Einstein affirms "the true consequent":

$(x'-ct')=0$ entails $(x-ct)=0$ and $(x-ct)=0$ entails $(x'-ct')=0$

and then obtains “the true antecedent”:

$(x'-ct')=\lambda(x-ct)$.

Needless to say, the valid argument is:

If $(x'-ct')=\lambda(x-ct)$, then $(x'-ct')=0$ entails $(x-ct)=0$ and $(x-ct)=0$ entails $(x'-ct')=0$.

Non-mathematical deductive arguments

We have already defined deductive arguments as truth-preserving: if the premises of a deductive argument are true, then the conclusion cannot be false. Two examples of deductive argument:

If it rains and nothing prevents drops from reaching the soil, then the soil is wet.

If a system initially in equilibrium suddenly starts changing, then there are changes in the surroundings. (This argument is clearly deductive: the system in equilibrium cannot start changing unless some jog comes from, that is, unless there are changes in, the surroundings.)

The following features are noteworthy; they show that we are dealing with *non-mathematical deductive arguments*:

1. The conclusion is a *qualitative* proposition, that is, it does not involve, either explicitly or implicitly, mathematical equations.

2. The combination “false premise, true conclusion” is *possible*.

Mathematical deductive arguments

Let us consider the following deductive arguments:

(1) If $(x'-ct')=\lambda(x-ct)$, then $(x'-ct')=0$ entails $(x-ct)=0$ and $(x-ct)=0$ entails $(x'-ct')=0$.

(2) If heat is conserved (cannot be converted into work in the heat engine), then all reversible heat engines working between the same two temperatures T_1 and T_2 ($T_1 > T_2$) have the same efficiency.

(3) If the speed of light does not depend on the speed of the light source, then I observe your clock running slower than mine and you observe mine running slower than yours. Both observations are correct.

The conclusions of all three arguments involve mathematical equations. This is not directly seen in (2) and (3) but a closer examination of the conclusion (2) would reveal the equations $E1=E2=\ldots$, where E1, E2 etc. are efficiencies of reversible heat engines. Similarly, the conclusion (3) implicitly involves equations of the type $t'=t/\gamma$, where t' and t are readings of clocks as seen by an observer and the factor γ is defined in Einstein’s special theory of relativity. (We need no more information about these equations).

Needless to say, deductive arguments whose conclusions involve, explicitly or implicitly, mathematical equations, can be referred to as *mathematical deductive arguments*. Unlike both inductive arguments and non-mathematical deductive arguments, mathematical deductive arguments do not tolerate the combination "false premise, true conclusion". Consider again the argument:

If $(x'-ct')=\lambda(x-ct)$, then $(x'-ct')=0$ entails $(x-ct)=0$ and $(x-ct)=0$ entails $(x'-ct')=0$.

(We never forget that this argument belongs to natural sciences so the variables x', t', x and t are *physical* quantities). Let us assume that the premise, $(x'-ct')=\lambda(x-ct)$, is false. In the absence of additional *ad hoc* assumptions this means either that there is no relation whatsoever between the variables x', t', x and t, or that *any* relation except the equation $(x'-ct')=\lambda(x-ct)$ is admissible. Given this indefiniteness of the premise, can we expect that $(x-ct)=0$ would miraculously continue to entail $(x'-ct')=0$? Of course we cannot. It is obvious that, when the premise is false, the probability that the conclusion can remain true is virtually zero. The generalized version of this reasoning amounts to the following theorem:

Theorem: If any premise of the *mathematical deductive argument* proves false, then the conclusion is false as well. If in some particular case the premise of an argument is false but the conclusion remains true, then the argument is not "mathematical deductive".

Our theorem could be useful in various cases but the arguments (2) and (3) above hide the two most important implications. The argument (2), the Carnot argument, was once fundamental for thermodynamics but then the premise ("Heat is conserved") proved false. In accordance with our theorem, the conclusion is false as well. The argument (3) is fundamental for Einstein's theory of relativity but the conclusion looks absurd. Again in accordance with our theorem, if the premise is false, the conclusion is also false and then "looks absurd" is not the right expression; rather, the conclusion *is* absurd.

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