

## **The New Prime theorems (291) - (340)**

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### **Abstract**

Using Jiang function  $J_2(\omega)$  we prove that the new prime theorems (291)- (340) contain infinitely many prime solutions and no prime solutions.

# The New Prime theorem (291)

$$P, jP^{502} + k - j (j = 1, \dots, k-1)$$

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## Abstract

Using Jiang function we prove that  $jP^{502} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{502} + k - j (j = 1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{502} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{502} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(502)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3,503$ . From (2) and(3) we have

$$J_2(\omega) = 0 \quad (7)$$

we prove that for  $k = 3, 503$ , (1) contain no prime solutions

**Example 2.** Let  $k \neq 3, 503$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 7$  (1) contain infinitely many prime solutions

## The New Prime theorem (292)

$$P, jP^{504} + k - j (j = 1, \dots, k-1)$$

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### Abstract

Using Jiang function we prove that  $jP^{504} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{504} + k - j (j = 1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{504} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{504} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(504)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3, 5, 7, 13, 19, 29, 43, 127$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 7, 13, 19, 29, 43, 127$  (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 7, 13, 19, 29, 43, 127$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 7, 13, 19, 29, 43, 127$ , (1) contain infinitely many prime solutions

## The New Prime theorem (293)

$$P, jP^{506} + k - j (j = 1, \dots, k-1)$$

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### Abstract

Using Jiang function we prove that  $jP^{506} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{506} + k - j (j = 1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{506} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{506} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(506)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P - 1)$ .

Example 1. Let  $k = 3, 23, 47$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 23, 47$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 23, 47$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 23, 47$  (1) contain infinitely many prime solutions

## The New Prime theorem (294)

$$P, jP^{508} + k - j (j = 1, \dots, k - 1)$$

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### Abstract

Using Jiang function we prove that  $jP^{508} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{508} + k - j (j = 1, \dots, k - 1). \quad (1)$$

contain infinitely many prime solutions or no prime solutions.

**Proof.** We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P - 1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{508} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \{P \leq N : jP^{508} + k - j = \text{prime}\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(508)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_P (P-1)$ .

Example 1. Let  $k = 3, 5, 509$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 509$  (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 509$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 509$  (1) contain infinitely many prime solutions

## The New Prime theorem (295)

$$P, jP^{510} + k - j (j = 1, \dots, k-1)$$

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### Abstract

Using Jiang function we prove that  $jP^{510} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{510} + k - j (j = 1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{510} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{510} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(510)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3, 7, 11, 31, 103$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 7, 11, 31, 103$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 7, 11, 31, 103$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 7, 11, 31, 103$ , (1) contain infinitely many prime solutions

## The New Prime theorem (296)

$$P, jP^{512} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang

Abstract

Using Jiang function we prove that  $jP^{512} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{512} + k - j (j=1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{512} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{512} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(512)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3, 5, 17, 257$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 17, 257$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 17, 257$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 17, 257$ , (1) contain infinitely many prime solutions

# The New Prime theorem (297)

$$P, jP^{514} + k - j (j=1, \dots, k-1)$$

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## Abstract

Using Jiang function we prove that  $jP^{514} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{514} + k - j (j=1, \dots, k-1), \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{514} + k - j] \equiv 0 \pmod{P}, q=1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{514} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(514)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_P (P-1)$ .

Example 1. Let  $k = 3$ . From (2) and(3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3$ , (1) contain no prime solutions.

Example 2. Let  $k > 3$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k > 3$ , (1) contain infinitely many prime solutions

## The New Prime theorem (298)

$$P, jP^{516} + k - j (j = 1, \dots, k-1)$$

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### Abstract

Using Jiang function we prove that  $jP^{516} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{516} + k - j (j = 1, \dots, k-1), \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P - 1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{516} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{516} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(516)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3, 5, 7, 13, 173$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 7, 13, 173$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 7, 13, 173$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 7, 13, 173$ , (1) contain infinitely many prime solutions

## The New Prime theorem (299)

$$P, jP^{518} + k - j (j = 1, \dots, k-1)$$

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### Abstract

Using Jiang function we prove that  $jP^{518} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{518} + k - j (j = 1, \dots, k-1), \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{518} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{518} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(518)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_P (P-1)$ .

Example 1. Let  $k = 3$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3$ , (1) contain no prime solutions.

Example 2. Let  $k > 3$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k > 3$ , (1) contain infinitely many prime solutions

## The New Prime theorem (300)

$$P, jP^{520} + k - j (j = 1, \dots, k-1)$$

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### Abstract

Using Jiang function we prove that  $jP^{520} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{520} + k - j (j = 1, \dots, k-1), \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{520} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{520} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(520)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_P (P-1)$ .

Example 1. Let  $k = 3, 5, 11, 41, 53, 131, 521$ . From (2) and(3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 11, 41, 53, 131, 521$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 11, 41, 53, 131, 521$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 11, 41, 53, 131, 521$ , (1) contain infinitely many prime solutions

## The New Prime theorem (301)

$$P, jP^{522} + k - j(j=1, \dots, k-1)$$

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Abstract

Using Jiang function we prove that  $jP^{522} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{522} + k - j(j=1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{522} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{522} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(522)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

**Example 1.** Let  $k = 3, 7, 19, 59, 523$ . From (2) and(3) we have

$$J_2(\omega) = 0 \quad (7)$$

we prove that for  $k = 3, 7, 19, 59, 523$ , (1) contain no prime solutions

**Example 2.** Let  $k \neq 3, 7, 19, 59, 523$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 7, 19, 59, 523$  (1) contain infinitely many prime solutions

## The New Prime theorem (302)

$$P, jP^{524} + k - j(j = 1, \dots, k-1)$$

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### Abstract

Using Jiang function we prove that  $jP^{524} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{524} + k - j(j = 1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{524} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \{P \leq N : jP^{524} + k - j = \text{prime}\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(524)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_P (P-1)$ .

Example 1. Let  $k = 3, 5, 263$ . From (2) and(3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 263$  (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 263$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 263$ , (1) contain infinitely many prime solutions

## The New Prime theorem (303)

$$P, jP^{526} + k - j (j = 1, \dots, k-1)$$

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Abstract

Using Jiang function we prove that  $jP^{526} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{526} + k - j (j=1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{526} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{526} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(526)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3$ . From (2) and(3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3$ , (1) contain no prime solutions.

Example 2. Let  $k > 3$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k > 3$  (1) contain infinitely many prime solutions

## The New Prime theorem (304)

$$P, jP^{528} + k - j (j=1, \dots, k-1)$$

Chun-Xuan Jiang

Abstract

Using Jiang function we prove that  $jP^{528} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{528} + k - j (j=1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions or no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{528} + k - j] \equiv 0 \pmod{P}, q=1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{528} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(528)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3, 5, 7, 13, 17, 23, 67, 89$ . From (2) and(3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 7, 13, 17, 23, 67, 89$  (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 7, 13, 17, 23, 67, 89$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 7, 13, 17, 23, 67, 89$  (1) contain infinitely many prime solutions

# The New Prime theorem (305)

$$P, jP^{530} + k - j (j=1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

## Abstract

Using Jiang function we prove that  $jP^{530} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{530} + k - j (j=1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{530} + k - j] \equiv 0 \pmod{P}, q=1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{530} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(530)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_P (P-1)$ .

Example 1. Let  $k = 3, 11, 107$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 11, 107$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 11, 107$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 11, 107$ , (1) contain infinitely many prime solutions

## The New Prime theorem (306)

$$P, jP^{532} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

### Abstract

Using Jiang function we prove that  $jP^{532} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{532} + k - j (j = 1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{532} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{532} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(532)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3, 5, 29$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 29$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 29$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 29$ , (1) contain infinitely many prime solutions

## The New Prime theorem (307)

$$P, jP^{534} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

### Abstract

Using Jiang function we prove that  $jP^{534} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{534} + k - j (j = 1, \dots, k-1), \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{534} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{534} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(534)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3, 7, 179$ . From (2) and(3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 7, 179$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 7, 179$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 7, 179$ , (1) contain infinitely many prime solutions

## The New Prime theorem (308)

$$P, jP^{536} + k - j(j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

### Abstract

Using Jiang function we prove that  $jP^{536} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{536} + k - j(j = 1, \dots, k-1), \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{536} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{536} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(536)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3, 5, 269$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 269$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 269$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 269$ , (1) contain infinitely many prime solutions

## The New Prime theorem (309)

$$P, jP^{538} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

### Abstract

Using Jiang function we prove that  $jP^{538} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{538} + k - j (j = 1, \dots, k-1), \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{538} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{538} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(538)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3$ . From (2) and(3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3$ , (1) contain no prime solutions.

Example 2. Let  $k > 3$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k > 3$ , (1) contain infinitely many prime solutions

## The New Prime theorem (310)

$$P, jP^{540} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

### Abstract

Using Jiang function we prove that  $jP^{540} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{540} + k - j (j = 1, \dots, k-1), \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{540} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{540} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(540)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_P (P-1)$ .

Example 1. Let  $k = 3, 5, 7, 11, 13, 19, 31, 37, 61, 109, 181, 271, 541$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 7, 11, 13, 19, 31, 37, 61, 109, 181, 271, 541$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 7, 11, 13, 19, 31, 37, 61, 109, 181, 271, 541$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 7, 11, 13, 19, 31, 37, 61, 109, 181, 271, 541$ , (1) contain infinitely many prime solutions

## The New Prime theorem (311)

$$P, jP^{542} + k - j (j = 1, \dots, k-1)$$

Abstract

Using Jiang function we prove that  $jP^{542} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{542} + k - j (j=1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{542} + k - j] \equiv 0 \pmod{P}, q=1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{542} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(542)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_P (P-1)$ .

Example 1. Let  $k = 3$ . From (2) and(3) we have

$$J_2(\omega) = 0 \quad (7)$$

we prove that for  $k = 3$ , (1) contain no prime solutions

**Example 2.** Let  $k > 3$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k > 3$  (1) contain infinitely many prime solutions

# The New Prime theorem (312)

$$P, jP^{544} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

## Abstract

Using Jiang function we prove that  $jP^{544} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{544} + k - j (j = 1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{544} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{544} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(544)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3, 5, 17, 137$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 17, 137$  (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 17, 137$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 17, 137$ , (1) contain infinitely many prime solutions

## The New Prime theorem (313)

$$P, jP^{546} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
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### Abstract

Using Jiang function we prove that  $jP^{546} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{546} + k - j (j = 1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{546} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{546} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(546)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3, 7, 43, 79, 547$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 7, 43, 79, 547$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 7, 43, 79, 547$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 7, 43, 79, 547$  (1) contain infinitely many prime solutions

## The New Prime theorem (314)

$$P, jP^{548} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

### Abstract

Using Jiang function we prove that  $jP^{548} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{548} + k - j (j = 1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions or no prime solutions.

**Proof.** We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{548} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{548} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(548)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3, 5$ . From (2) and(3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5$  (1) contain no prime solutions.

Example 2. Let  $k > 5$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k > 5$  (1) contain infinitely many prime solutions

## The New Prime theorem (315)

$$P, jP^{550} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

### Abstract

Using Jiang function we prove that  $jP^{550} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{550} + k - j (j = 1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{550} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{550} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(550)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_P (P-1)$ .

Example 1. Let  $k = 3, 11, 23$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 11, 23$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 11, 23$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 11, 23$ , (1) contain infinitely many prime solutions

## The New Prime theorem (316)

$$P, jP^{552} + k - j (j = 1, \dots, k-1)$$

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### Abstract

Using Jiang function we prove that  $jP^{552} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{552} + k - j (j = 1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{552} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{552} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(552)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_P (P-1)$ .

Example 1. Let  $k = 3, 5, 7, 13, 47, 139, 277$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 7, 13, 47, 139, 277$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 7, 13, 47, 139, 277$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 7, 13, 47, 139, 277$ , (1) contain infinitely many prime solutions

## The New Prime theorem (317)

$$P, jP^{554} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{554} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{554} + k - j (j = 1, \dots, k - 1), \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P - 1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{554} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{554} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(554)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P - 1)$ .

Example 1. Let  $k = 3$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3$ , (1) contain no prime solutions.

Example 2. Let  $k > 3$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k > 3$ , (1) contain infinitely many prime solutions

## The New Prime theorem (318)

$$P, jP^{556} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang

Abstract

Using Jiang function we prove that  $jP^{556} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{556} + k - j (j=1, \dots, k-1), \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P - 1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{556} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{556} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(556)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3, 5, 557$ . From (2) and(3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 557$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 557$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 557$ , (1) contain infinitely many prime solutions

# The New Prime theorem (319)

$$P, jP^{558} + k - j (j=1, \dots, k-1)$$

Chun-Xuan Jiang  
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## Abstract

Using Jiang function we prove that  $jP^{558} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{558} + k - j (j=1, \dots, k-1), \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{558} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \tag{3}$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \{P \leq N : jP^{558} + k - j = \text{prime}\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(558)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P-1)$ .

Example 1. Let  $k = 3, 7, 19$ . From (2) and (3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for  $k = 3, 7, 19$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 7, 19$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 7, 19$ , (1) contain infinitely many prime solutions

## The New Prime theorem (320)

$$P, jP^{560} + k - j (j = 1, \dots, k-1)$$

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### Abstract

Using Jiang function we prove that  $jP^{560} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{560} + k - j (j = 1, \dots, k-1), \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{560} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{560} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(560)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_P (P-1)$ .

Example 1. Let  $k = 3, 5, 11, 17, 29, 41, 71, 113, 281$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 11, 17, 29, 41, 71, 113, 281$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 11, 17, 29, 41, 71, 113, 281$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 11, 17, 29, 41, 71, 113, 281$ , (1) contain infinitely many prime solutions

## The New Prime theorem (321)

$$P, jP^{562} + k - j (j = 1, \dots, k-1)$$

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### Abstract

Using Jiang function we prove that  $jP^{562} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{562} + k - j (j = 1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{562} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{562} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(562)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_P (P-1)$ .

**Example 1.** Let  $k = 3,563$ . From (2) and(3) we have

$$J_2(\omega) = 0 \quad (7)$$

we prove that for  $k = 3,563$ , (1) contain no prime solutions

**Example 2.** Let  $k \neq 3,563$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3,563$  (1) contain infinitely many prime solutions

## The New Prime theorem (322)

$$P, jP^{564} + k - j(j = 1, \dots, k-1)$$

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### Abstract

Using Jiang function we prove that  $jP^{564} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{564} + k - j(j = 1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{564} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{564} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(564)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_P (P-1)$ .

Example 1. Let  $k = 3, 5, 7, 283$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 7, 283$  (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 7, 283$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 7, 283$ , (1) contain infinitely many prime solutions

## The New Prime theorem (323)

$$P, jP^{566} + k - j (j = 1, \dots, k-1)$$

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Abstract

Using Jiang function we prove that  $jP^{566} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{566} + k - j (j=1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{566} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{566} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(566)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3$ . From (2) and(3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3$ , (1) contain no prime solutions.

Example 2. Let  $k > 3$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k > 3$  (1) contain infinitely many prime solutions

## The New Prime theorem (324)

$$P, jP^{568} + k - j (j=1, \dots, k-1)$$

Chun-Xuan Jiang

Abstract

Using Jiang function we prove that  $jP^{568} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{568} + k - j (j=1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions or no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{568} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{568} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(568)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3, 5, 569$ . From (2) and(3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 569$  (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 569$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 569$  (1) contain infinitely many prime solutions

# The New Prime theorem (325)

$$P, jP^{570} + k - j (j=1, \dots, k-1)$$

Chun-Xuan Jiang  
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## Abstract

Using Jiang function we prove that  $jP^{570} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{570} + k - j (j=1, \dots, k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{570} + k - j] \equiv 0 \pmod{P}, q=1, \dots, P-1 \tag{3}$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{570} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(570)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where  $\phi(\omega) = \prod_P (P-1)$ .

Example 1. Let  $k = 3, 7, 11, 31, 191, 571$ . From (2) and(3) we have

$$J_2(\omega) = 0 \tag{7}$$

We prove that for  $k = 3, 7, 11, 31, 191, 571$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 7, 11, 31, 191, 571$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 7, 11, 31, 191, 571$ , (1) contain infinitely many prime solutions

## The New Prime theorem (326)

$$P, jP^{572} + k - j (j=1, \dots, k-1)$$

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### Abstract

Using Jiang function we prove that  $jP^{572} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{572} + k - j (j=1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{572} + k - j] \equiv 0 \pmod{P}, q=1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{572} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(572)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3, 5, 23, 53$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 23, 53$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 23, 53$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 23, 53$ , (1) contain infinitely many prime solutions

## The New Prime theorem (327)

$$P, jP^{574} + k - j (j = 1, \dots, k-1)$$

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### Abstract

Using Jiang function we prove that  $jP^{574} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{574} + k - j (j = 1, \dots, k-1), \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{574} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{574} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(574)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3, 83$ . From (2) and(3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 83$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 83$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 83$ , (1) contain infinitely many prime solutions

## The New Prime theorem (328)

$$P, jP^{576} + k - j(j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
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### Abstract

Using Jiang function we prove that  $jP^{576} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{576} + k - j(j = 1, \dots, k-1), \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{576} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{576} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(576)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3, 5, 7, 13, 17, 19, 37, 73, 97, 193, 577$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 7, 13, 17, 19, 37, 73, 97, 193, 577$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 7, 13, 17, 19, 37, 73, 97, 193, 577$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 7, 13, 17, 19, 37, 73, 97, 193, 577$ , (1) contain infinitely many prime solutions

## The New Prime theorem (329)

$$P, jP^{578} + k - j (j=1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

### Abstract

Using Jiang function we prove that  $jP^{578} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{578} + k - j (j = 1, \dots, k - 1), \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P - 1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{578} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P - 1 \quad (3)$$

If  $\chi(P) \leq P - 2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P - 1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{578} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(578)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P - 1)$ .

Example 1. Let  $k = 3$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3$ , (1) contain no prime solutions.

Example 2. Let  $k > 3$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k > 3$ , (1) contain infinitely many prime solutions

## The New Prime theorem (330)

$$P, jP^{580} + k - j (j = 1, \dots, k - 1)$$

Chun-Xuan Jiang

Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that  $jP^{580} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{580} + k - j (j=1, \dots, k-1), \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P - 1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{580} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{580} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(580)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3, 5, 11, 59$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 11, 59$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 11, 59$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 11, 59$ , (1) contain infinitely many prime solutions

# The New Prime theorem (331)

$$P, jP^{582} + k - j (j=1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

## Abstract

Using Jiang function we prove that  $jP^{582} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{582} + k - j (j=1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{582} + k - j] \equiv 0 \pmod{P}, q=1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{582} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(582)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_P (P-1)$ .

Example 1. Let  $k=3, 7$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

we prove that for  $k=3, 7$ , (1) contain no prime solutions

**Example 2.** Let  $k \neq 3, 7$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 7$  (1) contain infinitely many prime solutions

## The New Prime theorem (332)

$$P, jP^{584} + k - j (j=1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

### Abstract

Using Jiang function we prove that  $jP^{584} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{584} + k - j (j=1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{584} + k - j] \equiv 0 \pmod{P}, q=1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{584} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(584)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3, 5, 293$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 293$  (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 293$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 293$ , (1) contain infinitely many prime solutions

## The New Prime theorem (333)

$$P, jP^{586} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

### Abstract

Using Jiang function we prove that  $jP^{586} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{586} + k - j (j = 1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{586} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{586} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(586)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3,587$ . From (2) and(3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3,587$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3,587$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3,587$  (1) contain infinitely many prime solutions

## The New Prime theorem (334)

$$P, jP^{588} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

### Abstract

Using Jiang function we prove that  $jP^{588} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{588} + k - j (j = 1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions or no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{588} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{588} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(588)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3, 5, 7, 13, 43, 197$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 7, 13, 43, 197$  (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 7, 13, 43, 197$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 7, 13, 43, 197$  (1) contain infinitely many prime solutions

## The New Prime theorem (335)

$$P, jP^{590} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
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### Abstract

Using Jiang function we prove that  $jP^{590} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{590} + k - j (j = 1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{590} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \{P \leq N : jP^{590} + k - j = \text{prime}\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(590)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_P (P-1)$ .

Example 1. Let  $k = 3, 11$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 11$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 11$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k = 3, 11$ , (1) contain infinitely many prime solutions

## The New Prime theorem (336)

$$P, jP^{592} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
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Abstract

Using Jiang function we prove that  $jP^{592} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{592} + k - j (j=1, \dots, k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{592} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{592} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(592)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3, 5, 17, 149, 593$ . From (2) and(3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 17, 149, 593$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 17, 149, 593$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 17, 149, 593$ , (1) contain infinitely many prime solutions

# The New Prime theorem (337)

$$P, jP^{594} + k - j (j=1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

## Abstract

Using Jiang function we prove that  $jP^{594} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{594} + k - j (j=1, \dots, k-1), \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{594} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{594} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(594)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_P (P-1)$ .

Example 1. Let  $k = 3, 7, 19, 23, 67, 199$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 7, 19, 23, 67, 199$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 7, 19, 23, 67, 199$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 7, 19, 23, 67, 199$ , (1) contain infinitely many prime solutions

## The New Prime theorem (338)

$$P, jP^{596} + k - j (j=1, \dots, k-1)$$

Chun-Xuan Jiang  
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### Abstract

Using Jiang function we prove that  $jP^{596} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{596} + k - j (j=1, \dots, k-1), \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{596} + k - j] \equiv 0 \pmod{P}, q=1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{596} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(596)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

Example 1. Let  $k = 3, 5$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5$ , (1) contain no prime solutions.

Example 2. Let  $k > 5$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k > 5$ , (1) contain infinitely many prime solutions

## The New Prime theorem (339)

$$P, jP^{598} + k - j (j = 1, \dots, k-1)$$

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

### Abstract

Using Jiang function we prove that  $jP^{598} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{598} + k - j (j = 1, \dots, k-1), \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_p P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{598} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{598} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(598)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_P (P-1)$ .

Example 1. Let  $k = 3, 47, 599$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 47, 599$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 47, 599$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 47, 599$ , (1) contain infinitely many prime solutions

## The New Prime theorem (340)

$$P, jP^{600} + k - j (j = 1, \dots, k-1)$$

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### Abstract

Using Jiang function we prove that  $jP^{500} + k - j$  contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let  $k$  be a given odd prime.

$$P, jP^{500} + k - j (j = 1, \dots, k-1), \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)] \quad (2)$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jq^{500} + k - j] \equiv 0 \pmod{P}, q = 1, \dots, P-1 \quad (3)$$

If  $\chi(P) \leq P-2$  then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If  $\chi(P) = P-1$  then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If  $J_2(\omega) \neq 0$  then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{500} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(500)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \quad (6)$$

where  $\phi(\omega) = \prod_P (P-1)$ .

Example 1. Let  $k = 3, 5, 7, 11, 13, 31, 41, 61, 101, 151, 601$ . From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

We prove that for  $k = 3, 5, 7, 11, 13, 31, 41, 61, 101, 151, 601$ , (1) contain no prime solutions.

Example 2. Let  $k \neq 3, 5, 7, 11, 13, 31, 41, 61, 101, 151, 601$ . From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for  $k \neq 3, 5, 7, 11, 13, 31, 41, 61, 101, 151, 601$ , (1) contain infinitely many prime solutions

**Remark.** The prime number theory is basically to count the Jiang function  $J_{n+1}(\omega)$  and Jiang

prime  $k$ -tuple singular series  $\sigma(J) = \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} = \prod_P \left( 1 - \frac{1 + \chi(P)}{P} \right) \left( 1 - \frac{1}{P} \right)^{-k}$  [1,2], which can count

the number of prime numbers. The prime distribution is not random. But Hardy-Littlewood prime  $k$ -tuple

singular series  $\sigma(H) = \prod_P \left( 1 - \frac{\nu(P)}{P} \right) \left( 1 - \frac{1}{P} \right)^{-k}$  is false [3-17], which cannot count the number of prime

numbers[3].

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Szemerédi’s theorem does not directly to the primes, because it cannot count the number of primes. Cramér’s random model cannot prove any prime problems. The probability of  $1/\log N$  of being prime is false. Assuming that the events “ $P$  is prime”, “ $P+2$  is prime” and “ $P+4$  is prime” are independent, we conclude that  $P$ ,  $P+2$ ,  $P+4$  are simultaneously prime with probability about  $1/\log^3 N$ . There are about  $N/\log^3 N$  primes less than  $N$ . Letting  $N \rightarrow \infty$  we obtain the prime conjecture, which is false. The tool of additive prime number theory is basically the Hardy-Littlewood prime tuples conjecture, but cannot prove and count any prime problems[6].

*Mathematicians have tried in vain to discover some order in the sequence of prime numbers but we have every reason to believe that there are some mysteries which the human mind will never penetrate.*

Leonhard Euler(1707-1783)

*It will be another million years, at least, before we understand the primes.*

Paul Erdos(1913-1996)

Of course, the primes are a deterministic set of integers, not a random one, so the predictions given by random models are not rigorous (Terence Tao, Structure and randomness in the prime numbers, preprint). Erdos and Turán(1936) contributed to probabilistic number theory, where the primes are treated as if they were random, which generates Szemerédi’s theorem (1975) and Green-Tao theorem(2004). But they cannot actually prove and count any simplest prime examples: twin primes and Goldbach’s conjecture. They don’t know what prime theory means, only conjectures.

1991年10月25日蒋春暄用他发明新数学证明费马大定理。设指数  $n = 3P$ ，其中  $P > 3$  是素数，有三个费马方程

$$S_1^{3P} + S_2^{3P} = 1 \tag{1}$$

$$S_1^3 + S_2^3 = \left[ \exp\left(\sum_{j=1}^{P-1} t_{3j}\right) \right]^3 \tag{2}$$

$$S_1^P + S_2^P = \left[ \exp(t_P + t_{2P}) \right]^P \tag{3}$$

欧拉证明  $n = 3$ 。(1) 和 (2) 无有理数解，因此，蒋春暄证明 (3) 无有理数解，对于  $P > 3$ ，这样就全部证明费马大定理，证明  $n = 3$  或  $n = 4$  就全部证明费马大定理。1637年费马证明  $n = 4$ ，因此，1637年费马证明他的最后定理。

1994年2月23日中国著名数论家乐茂华给蒋春暄来信“……Wiles 承认失败情况实际上对您是有利的。”当时中国仍在宣传 Wiles，无人理睬蒋春暄的工作。2009年蒋春暄因首先证明费马大定理获国外金奖，中国不承认这个金奖。