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Presentation of my invention of a Flocculator (using the Logarithmic Spiral), including design fundamentals.

Note: proof of the invention can be produced on request

This paper introduces the design criteria of hydraulic flumes of spiral shape, in the attempt to facilitate the preliminary dimensioning of structures having that shape, as they were successfully used in South Africa to condition the raw water running inside them after flocculants were added at their inlet.

Along these hydraulic structures the sections gently expand reducing the velocity of the raw conditioned water and facilitating floc formation through reduction of turbulence.

They output water containing large floc into aptly designed sedimentation tanks and the system spiral flocculator-tank has proved to be a very successful choice in primary treatment of raw water.

The logarithmic spiral is an invention/discovery attributed to the mathematical genius of Jacob Bernoulli, it responds the calculus equation using the notations:

$$U' = U \qquad \text{or} \qquad \frac{d\rho(\vartheta)}{d\vartheta} = \rho(\vartheta)$$

Satisfied by $\rho(\vartheta) = k\, e^{\vartheta}$

Due to its particular property to undergo numeric mutation without change it was from the beginning considered a symbol of immortality and associated to the tenet:

“eadem numero mutata, resurgo”,

The logarithmic spiral is a basic universal shape since the configuration of many galaxies approximates it and along the ages many tried to make practical and ornamental use of it.

As I am aware sections of logarithmic spiral are used in the centrifugal propellers of the devices pumping liquids and gases (water pumps, turbines etc…).

The advent of computing devices provided the tools necessary to overcome the difficulties inherent with the repeated calculation of the exponential functions and therefore made possible to design conditioning flumes based on the use of this mathematical function.

I applied this spiral shape to design the raw water conditioners allowing the build up of chemical floc after flocculants had been introduced at their central mixer, and their success is proven by the fact that they have been built without changes along a time span of more than thirty years, and presently are in full operative function.

The use of the logarithmic spiral then came aptly to justify again its claim to immortality since its application to the cycle of water through a device having its shape and contributing to return water in a pristine life giving condition gives food for reflection.

An application of such importance, (serving 10 to 12 million people everyday) even in the event that will be superseded and made obsolete in the future, will certainly leave a sign of my passage on Earth….

General formula of the spiral:

$$\rho = A + B\, e^{\frac{n\pi}{C}} \quad \text{for} \quad \vartheta = \frac{n\pi}{C}$$

and for $0 < n < N$ the value of N is defining the range of a segment of spiral

Unknowns to be determined:

N = number of half spires
A, B, C (coefficients)

ρ_N maximum radius

In order to obtain the above unknowns for a segment of spiral, (from 0 to N) was necessary to establish the area occupied by its external borders inside which was inserted its shape and within which input and outlet, width of the flumes and height of the water contained within its walls could be mathematically tested.

In order to do the preliminary design was necessary to assume the values of the required Fixed hydraulic Data, the flow F the time of permanence T , and the value of the coefficient of roughness M (the Manning coefficient, chosen in this case, was within the values $M \sim 0.015 \div 0.018$).

The flow at the input and outlet needed to move at an acceptable design velocity of the water and this would have determined the value of the mixing factor G (turbulence factor) which measured the blending at a reasonable time of permanence T of the treated raw water (based on the characters of the floc in formation and therefore of the chemicals used).

Note: the preliminary design will be shown here through an exercise prepared at hoc.

The Geometric Data are obtained through sensible approximate geometric dimensioning of the spiral flume dictated by experience and confirmed through rough initial valuation of the mixing factor G and the time of retention T.

Use of the spiral represents an efficient choice, and its optimization, made through numeric modeling, enables to obtain a good design and further economic improvement.

Note: from these initial data the design proceeds through trial and error until all the physical requirements are satisfied (i.e. a sensible backwater curve whose variation of height of water (from the outlet back to the input) is associated to a sensible average value of G, T, and GT, and acceptable G values at the input and at the outlet, this presentation is preliminary but once given the geometry of the flume, a program was developed, permitting to vary the height of the water in the spiral flume, the values of flow, and the Manning coefficient (within an acceptable set of practical dimensions of the structure), whereas the area of the whole spiral flume is including the area delimiting the central rapid mixer, and therefore giving the radius ρ_0 and the basic geometric data are chosen within dimensions

permitting periodic access inside the flume when maintenance work is required (see below).
Note: the advantage in using the logarithmic spiral shape is in the fact that the origin is not at the center of the coordinates and this allows the insertion of the rapid mixer at the center.

Formulation of spiral configurations

Geometric Data (obtained from hydraulic requirements, through progressive dimensioning):

$$\rho_0 \qquad \rho_N \qquad a=\rho_2-\rho_0 \qquad b=\rho_N-\rho_{N-2}$$

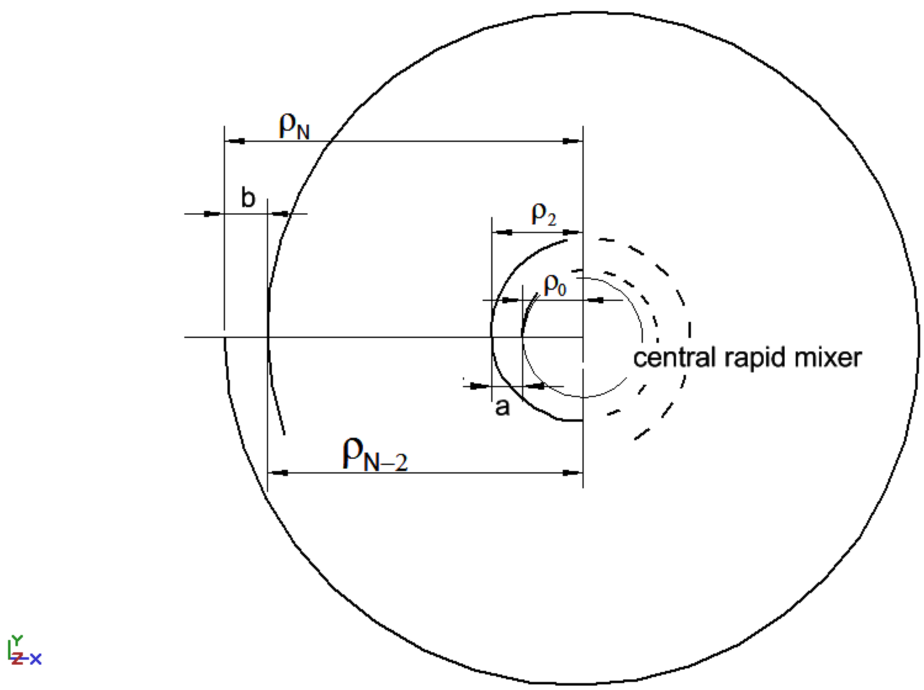


Figure 1

For n =(0,2,N-2,N)

H height of the spiral flume

to calculate approximately the mixing factor admissible at the input and the outlet sections (if we overlook the buildup of the backwater) we need the value of the flow (which is a project datum) and we need to guess the input and output velocities (v_i and v_f).

N is the number of half spires and in the example in fig 2 below is N=16, which is a value of first approximation obtained through the simple following procedure adopted below, in which the initial and final (input and output velocities of the flume described by the spiral are not to exceed: $v_{imax} \cong 0.8$ m/sec and $v_{fmin} \cong 0.6$ m/sec (giving a local mixing factor ~80> G > ~50 which is within an acceptable range of magnitudes):

$$v_{max} > v > v_{min}$$

For geometric initial values of : $\rho_0 = 3$ m we guess for a and b the following initial values:

$$a = \rho_2 - \rho_0 = 1.6 \text{ m}$$

$$b = \rho_N - \rho_{N-2} = 2.2 \text{ m}$$

Note: the dimensioning will use the above criteria whereas the preliminary initial data are used as above trying to guess the initial and final G , as :

$$\frac{G_{initial} + G_{Final}}{2} \cong G_{Average}$$

Since the required characteristic of the flocculator is :

$$G * T \cong 35000 \div 50000 \text{ [--]}$$

An average mixing factor (dissipation function) $G \cong 50 \div 80$ sec-1 will permit us to assume a time of retention $T \cong$ 800 sec including a retention time of $20 \div 30$ sec in the central rapid mixer

Fixing the Height of flocculator $H \cong 1.7$ m
And a Flow $\cong 2.5\ m^3/sec$ or $\cong$ 215 Ml/day

$v_i = F / A_i = F/(H\, a)$ we have $a = F/(H\, v_i)$

and $b = F/(H\, v_f)$

Note: v_i (input) ~ 0.8÷0.9 m/sec and v_f ~ 0.6÷0.7 m/sec (output), depend from the chemical flocculant/s used, and are velocities chosen in such a way that the floc growing inside the flume is not subject to unnecessary turbulence.

The approximate value of the area of our flocculator will be:

$$A_F = \frac{\text{Flow} \cdot T}{H} \cong 1176 \quad m^2$$

Once obtained through estimation the A_F we obtain (also as approximated value) the Larger spiral radius

$$\overline{\rho}_N \cong \sqrt{\frac{A_F}{\pi}} = \sqrt{\frac{1176}{\pi}} \cong 19.5 \quad m$$

Make it $\overline{\rho}_N \cong 19.7\ m$ to take into account the presence of the central rapid mixer.

And for the central Rapid Mixer (assuming a retention Time T_{RM} = 25“) the area A_{RM} will be:

$$A_{RM} = \frac{\text{Flow} \cdot T_{RM}}{H} \cong \frac{2.5 \cdot 25}{1.7} = 36.7 \quad m^2$$

From which we get a Rapid Mixer radius $\overline{\rho}_0$:

$$\overline{\rho}_0 \cong \sqrt{\frac{A_{RM}}{\pi}} = \sqrt{\frac{36.7}{\pi}} \cong 3.4 \quad m$$

A program was developed which elaborated all these data and printed the coefficients A, B, C, and N the number of half spires came out as an integer, it permits first approximation dimensioning of the section of spiral flume.

Note: the second stage of design requires the calculation of the backwater curve and since a better approximation of the velocities in the flume is determined by a value of hydraulic load between output and input, in this second stage, we need to model the flume through repeated adjustments of the dimensions until we are satisfied that the flocculator has reached an acceptable shape.

Since above we calculated a ρ_N of first approximation, the program will give a new value always close to it.

The section of spiral having the above coefficients, and an integer 2N or 2N + 1 of half spires, will have to be tested hydraulically for consistence with the characteristic G, retention time, backwater gain of water level (backwater curve) etc…

A first approximation overlooks the change of height H in the flume due to the backwater build-up and needs an approximate calculation of G for the input and for the outflow sections as given here below:

$$G_i = \sqrt{\frac{9.8 \cdot v_i}{1.148e-6} \left(\frac{v_i M}{\left(\frac{aH}{2H+a} \right)^{\frac{2}{3}}} \right)^2}$$

$$G_f = \sqrt{\frac{9.8 \cdot v_f}{1.148e-6}\left(\frac{v_f M}{\left(\frac{bH}{2H+b}\right)^{\frac{2}{3}}}\right)^2}$$

Note: M, is the Manning coefficient

The mathematical path is briefly outlined below for a single spiral containing a single spiral flume intended for conditioning of the raw water going to the primary sedimentation tanks.

Mathematical path
(see Fig 1)

n=N (1)

$$b = \rho_N - \rho_{N-2}$$

$$b = A + B\, e^{\frac{N\pi}{C}} - \left(A + B\, e^{\frac{(N-2)\pi}{C}}\right)$$

From which:

$$b = B\, e^{\frac{(N-2)\pi}{C}}\left(e^{\frac{2\pi}{C}} - 1\right)$$

(2)

$$a = \rho_2 - \rho_0$$ (See pg 7 above)

$$a = A + B\, e^{\frac{2\pi}{C}} - \left(A + B\, e^{0}\right)$$

From which:

$$a = B\left(e^{\frac{2\pi}{C}} - 1\right)$$

(3)

$$\rho_0 = A + B$$

dividing (1) and (2):

$$\frac{b}{a} = e^{\frac{(N-2)\pi}{C}}$$

(4)

The general formula, supposing that N is an integer, gives ρ_N :

(4*) $$\rho_N = A + B\, e^{\frac{N\pi}{C}}$$

From (1)

$$\rho_N - b = A + B\, e^{\frac{(N-2)\pi}{C}}$$

Substituting the (3)

$$\rho_N - b = \rho_0 - B + B\, e^{\frac{(N-2)\pi}{C}}$$

We have:

$$\rho_N - \rho_0 - b = B\left(e^{\frac{(N-2)\pi}{C}} - 1\right)$$

Which substituting the (4) is Giving a B made up of known terms:

$$B = \frac{\rho_N - \rho_0 - b}{\left(\frac{b}{a} - 1\right)}$$

B is now expressed in function of known values and by consequence A is given as function of B:

$$A = \rho_0 - B$$

Now from the (2) we get C:

$$e^{\frac{2\pi}{C}} = \frac{a}{B} + 1$$

Giving:

$$C = \frac{2\pi}{\ln\left(\frac{a}{B} + 1\right)}$$

From the (4*) we get N which for better geometric positioning preferably has to be an integer

$$N = \text{int}\left(\frac{C}{\pi}\ln\frac{\rho_N - A}{B}\right)$$

Resuming:

Check G_i and G_f for hydraulic consistency

from first approximation DATA

get B from DATA (a, b, ρ_0, ρ_N)
get A from DATA (ρ_0, B)
get C from DATA (a, B)
get N from DATA (ρ_N, A, B, C)

use the N integer to refine the data (through calculation of backwater curve) and build up a better preliminary approximation of the spiral.

The above calculations with $a < b$ produce the conventional spiral in fig 2

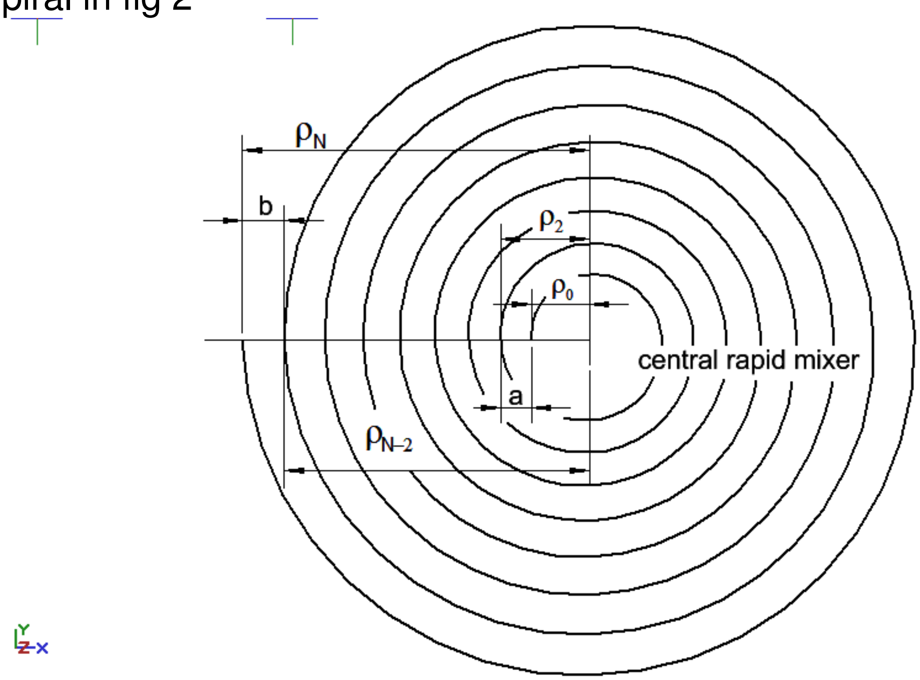
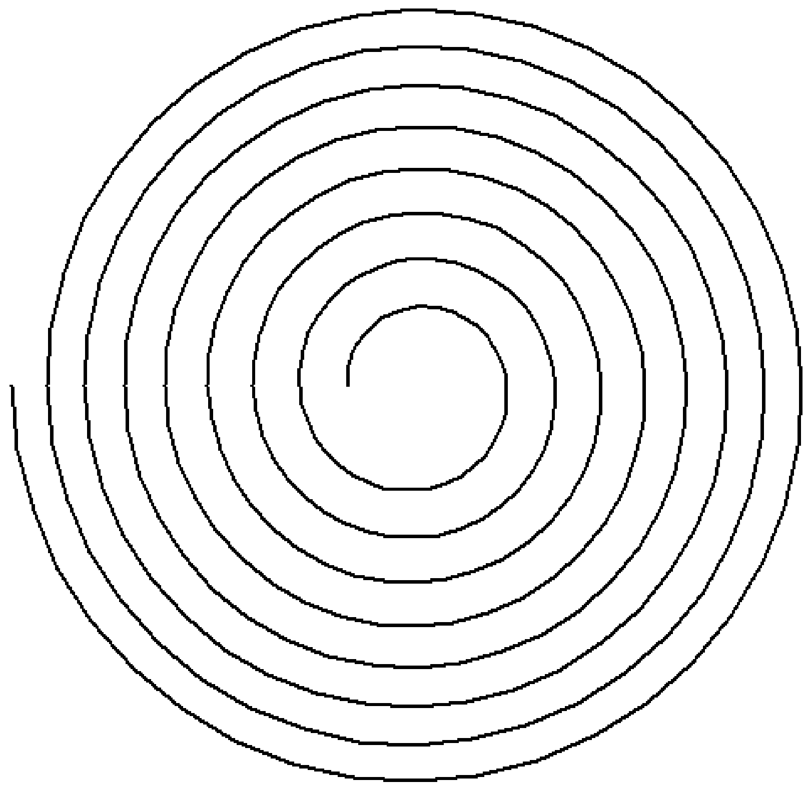


Figure 2

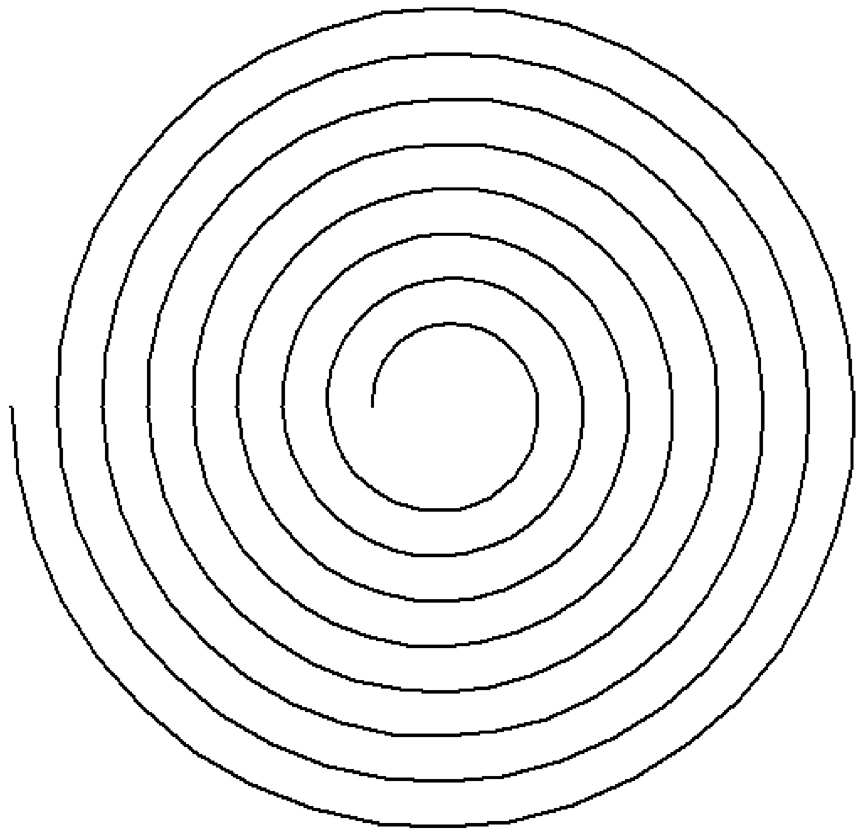
but if we use a > b in the input data an interesting result comes out, as it is shown in fig 3 , it represents a configuration whose possibility of application will be shown in due course.



inverted configuration of the spiral in fig 1

Figure 3

The program developed can also produce a configuration using a constant flume spiral for a=b , this solution also can be the subject of some interesting applications.



constant width spiral flume

Figure 4

The picture below shows an existing spiral conditioner using N=17 half spires:

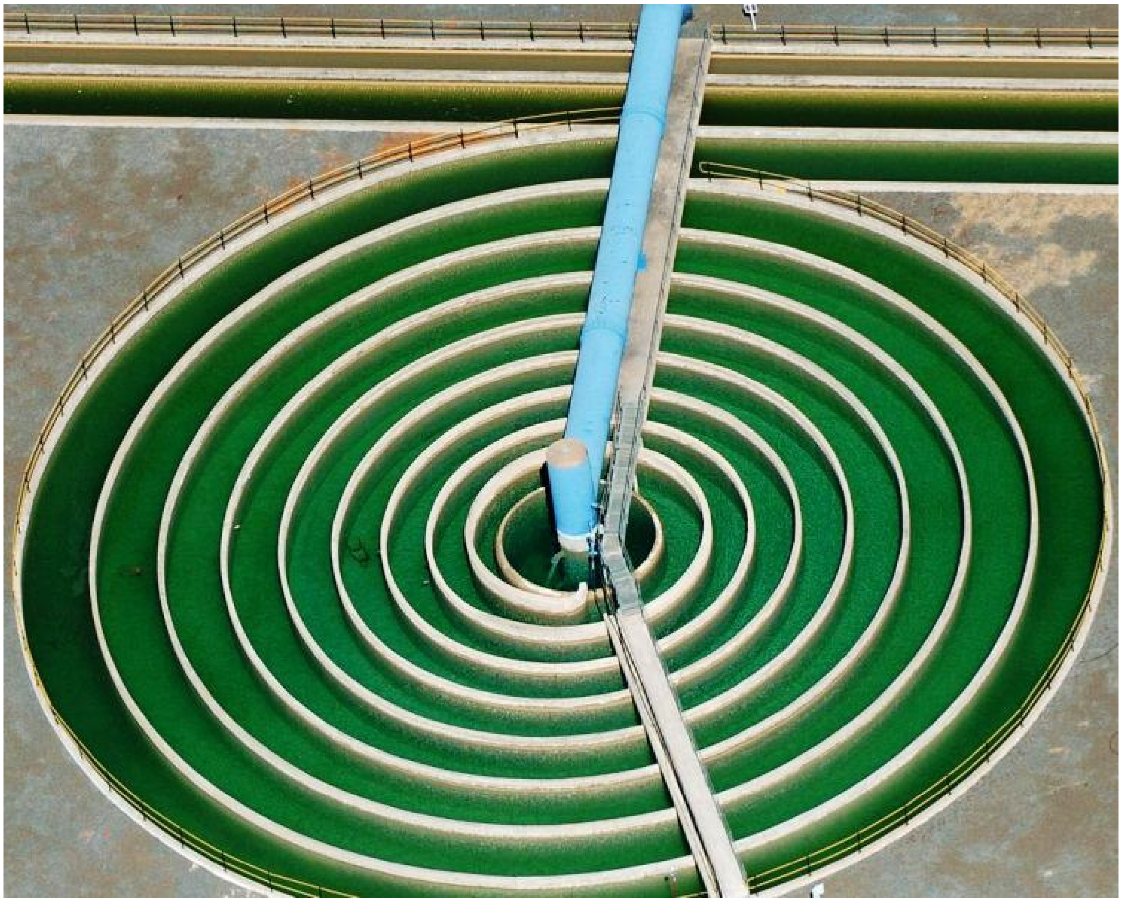


Figure 5

Coordinates of Plants using the Spiral conditioners and sedimentation tanks (see Google Earth)

Zuikerbosch 26° 40' 50.37" S
28° 00' 23,10 E

Vereeniging 26° 41' 10,58" S
27° 55' 04,18" E

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