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The "Last Riddle" of Pierre de Fermat, II

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Some time ago, I published a work entitled, "The Last Riddle" of Pierre de Fermat " in which I had written a proof of the theorem. (See (http://wbabin.net/mitkovsky/mitkovsky6.pdf, in English, and also, http://wbabin.net/mitkovsky/mitkovsky6r.pdf for the Russian version.) The proof was written as mathematical formulas, but the principle of the proof, it seems, was not clear to the mathematicians, who studied the work. Therefore in the present paper, I shall state the principles involved in the theorem in more detail, so that it is more easily understood.

Probably the reason that mathematics was unable to prove Fermat's Last Theorem over an extended period of time, is that it was assumed to be very complicated. Actually, the task is so simple that the time spent on a description of the proof is much longer than the proof of the theorem itself.

To understand this, it is necessary to follow Pierre de Fermat's train of thought, not to prove the theorem, but to deduce it. To any man having a certain mathematical turn of mind and a spatial imagination, it is possible to understand the reasoning which leads to the conclusions. For this purpose we shall temporarily overlook the fact that there is such a problem and concentrate on the Pythagorean Theorem.

We know that a square can be established as the sum of two squares on a right-angled triangle.
Let's accept, that the one side equals z, and the other sides are equal to x and y_1, so that

$$x^2+y_1^2=z^2 \tag{1}$$

Formula (1) means that the area of the square z is equal to the sum of the areas of two squares, one of which has the side x, and the other, side y_1. We shall not consider all the variants of the decomposition of squares; we shall consider only one of them in which z and x are whole positive numbers.

It is known that, if z and x are whole positive numbers, y_1 can be whole or an irrational number. This was known centuries before B.C., and we may assume it was also known by Pierre de Fermat. Let's assume further that Pierre de Fermat, reading "Arithmetica", in which Diophantus of Alexandria described the tasks of decomposition of squares, had asked himself a question - "What will happen if in the transformation we add one more measurement – height?".
Let's answer this question.

Let's add to a square, the property z for height, creating a cube with a volume of z^3. If to the squares on which z^2, is displayed, that is x^2 and y_1^2 we add the same height, we shall see, that the cube with volume z^3 consists of two geometrical figures – a direct parallelepiped, on which lays a square with side x and having height z, and a direct parallelepiped, with side y_1 having height z. Further we shall assume the parallelepiped has straight lines, since all parallelepiped's, used below, have straight lines. Let's write down the formula:

$$x^2 z+y_1^2 z=z^3 \qquad (2)$$

Formula (2) follows from the Pythagorean Theorem and is not subject to doubt.

Let's consider the composition of formula (2)
Here we have accepted that z and x, are whole positive numbers and it follows that the volume of a cube with the height z is a whole positive number, and the volume of the parallelepiped x^2z is also whole positive number.

If y_1 is a whole positive number, the volume of the parallelepiped with the base y_1^2 and height z will be natural number.
If y_1 is an irrational number, the volume of the parallelepiped with the base y_1^2 and height z will be an irrational number. It is clear that an irrational number squared and multiplied by an integer, cannot be a natural number.

It is obvious, that this reasoning on the decomposition of one volumetric figure to two other volumetric figures, will be valid for any height that is set, provided that this height is an integer. Proceeding from this, instead of height z, we can substitute height z^{n-2}. Let's write down the formula:

$$x^2 z^{n-2}+y_1^2 z^{n-2}=z^n \qquad (3)$$

Where n is a natural number.

Formula (3) shows that having changed the height of z to z^{n-2}, instead of cube z^3, with z height, we have a parallelepiped with height z^{n-2} and a volume equal to z^n. The parallelepiped with volume z^n, in formula (3) may been spread over two parallelepipeds.

The base of the first parallelepiped with side x and height z^{n-2}, gives a volume equal to x^2z^{n-2} and this volume expressed numerically, is a natural number.
The base of the second parallelepiped with side y_1 and height z^{n-2}, its volume is equal to $y_1{}^2z^{n-2}$.
With z^{n-2} as a natural number, it is obvious, that if y_1 is an irrational number, the volume $y_1^2z^{n-2}$ is also irrational.

However, for us, the parallelepiped x^2z^{n-2} is uninteresting, as those are more important which will accept x in transformations. Therefore, we can allocate from parallelepiped x^2z^{n-2}, another parallelepiped, in which the square with side x has a height x^{n-2}. The volume of such a parallelepiped will be equal to x^n.

Thus, we have divided parallelepiped x^2z^{n-2} into two parallelepipeds, one with the base x^2 and height x^{n-2}, whose volume is equal to x^n, and a second parallelepiped with base x^2 and height z^{n-2}-x^{n-2} whose volume is equal to $x^2 (z^{n-2}-x^{n-2})$. The total of parallelepiped z^n will be equal to

$$z^n = x^n + x^2 (z^{n-2} - x^{n-2}) + y_1^2 z^{n-2} \qquad (4)$$

Formula (4) shows, that the parallelepiped with the base z^2, and height z^{n-2} and volume z^n consists of three parallelepipeds x^n, $x^2 (z^{n-2} -x^{n-2})$ and $y_1^2z^{n-2}$.

The above-mentioned derivations are obvious and directly follow from the Pythagorean Theorem. We have reached that part of our reasoning in which we can already make assumptions. For example, we can assume that there is an expression:

$$z^n = x^n + y^n \tag{5}$$

Let's consider a situation which can accept y^n incorporating the above conditions, i.e., when z and x are natural numbers. For this purpose we apply formula (4). In this formula we already have deduced the meanings of z^n and x^n. If we compare formulas (4) and (5), it turns out that y^n should be equal to the subtraction of x^n from z^n. Let's write down the formula:

$$y^n = z^n - x^n = x^2\,(z^{n-2} - x^{n-2}) + y_1^2 z^{n-2} \tag{6}$$

Formula (6) shows, that the meaning y^n should consist of the sum of the volumes of two parallelepiped, $x^2(z^{n-2}-x^{n-2})$ and $y_1^2z^{n-2}$. Earlier we have established that the volume of parallelepiped $x^2(z^{n-2}-x^{n-2})$ is always a natural number, and the volume of parallelepiped $y_1^2z^{n-2}$ can be a natural number if y_1 is a natural number, or irrational if y_1 is an irrational number. It is easy to understand that if one is an irrational number, and the second is a natural number, the sum of these will be an irrational number. This implies, that y can be a natural number only in the event that y_1 is a natural number. It is obvious, that x, y_1 and z simultaneously will be integers only in the event that x, y_1 and z are a Pythagorean triple. Let's consider, in what cases this condition can be carried out.

1. *Lets assume that z is an even number.*

We know that it is impossible to spread out a square whose property is an even natural number on two whole squares.

This implies that at any even meaning z, the meaning y_1 will always be an irrational number, provided that z and x are natural numbers. Hence, the volume parallelepiped $y_1^2zn^{-2}$ can not be expressed by a natural number, meaning from formula (6), y^n can not be a natural number. It is thus obvious, that y can not be natural number.

2. *Lets assume, that z is a simple odd number.*

It is known, that if the simple number looks like $4t+1$, this will be a Pythagorean number. It is obvious, that in this case, y_1 will be natural, and the volume of the parallelepiped $y_1^2z^{n-2}$ will be expressed as a natural number. This implies that the meaning y^n (formula 6) will be a natural number and y can be a natural number. Let's accept, that y_1 is a natural number. The meaning of y^n in this case will be a natural number, which follows from formula (6). This means y^n can present a parallelepiped, in which the base is the square with side y, having height y^{n-2} .

Square y^2 can be entered in the base z^2 of parallelepiped z^n. In this case, from the Pythagorean theorem, there should be a meaning for x_1, which will satisfy the condition

$$x_1^2+y^2=z^2 \tag{7}$$

Using formula (7), the parallelepiped z^n can be presented as the sum of two parallelepipeds, one with base x_1^2 and height z^{n-2}, and the second with base y^2 and height z^{n-2}.

$$x_1^2\,z^{n-2}+y^2z^{n-2}=z^n \tag{8}$$

From the parallelepiped y^2z^{n-2} we shall derive a parallelepiped with base y^2 and height y^{n-2}. We shall determine that parallelepiped y^2z^{n-2} consists of two parallelepipeds, one with base y^2, height y^{n-2}, volume y^n, and one with base y^2, height z^{n-2}-y^{n-2}, and volume $y^2\,(z^{n-2}-y^{n-2})$.

In total, parallelepiped z^n will make:

$$z^n = y^n + y^2\,(z^{n-2} - y^{n-2}) + x_1^2z^{n-2} \tag{9}$$

From formulas (5) and (9), it follows,

$$x^n = z^n - y^n = y^2 (z^{n-2} - y^{n-2}) + x_1^2 z^{n-2} \tag{10}$$

From formula (10) follows that if x_1 is an irrational number, x^n can not be a natural number. That contradicts one of the conditions.

The necessary conditions follow from above, that x and y will be natural:

- *That side* y *is natural, then it is necessary that* y_1 *is a natural number.*
- *That side* x is *natural, then it is necessary, that* x_1 *is a natural number.*

These also are necessary conditions.

So it is satisfied as a condition of integrity that for x and y for one z there should be two Pythagorean triples, based on z.
We know, that simple odd numbers of the kind, $4t+1$ are represented as the sum of squares in a unique way. This is easy to prove.

Any one primitive Pythagorean triple (x, y, z) is unequivocally represented as

$m^2-n^2;\ 2mn;\ m^2+n^2$
Where $z=m^2+n^2$, m and n ***are*** natural and mutually simple numbers.

We have accepted, that z is a simple odd natural number. From the theorem of Fermat–Euler, it is known that the simple odd numbers of the kind $4t+1$ are represented as the sum of squares in a unique way, hence, for the equality $z=m^2+n^2$ there is only one pair of numbers, m and n. This implies, that we can have only one meaning of $x=m^2-n^2$ and $y = 2mn$. Hence, we have no other variants in the performance of necessary conditions of natural numbers, except as in the case $x=x_1\ y=y_1$.

Did Pierre de Fermat know about this dependence? Undoubtedly! It is known that it is possible to present the statement that a simple number is the sum of two whole squares only in the event that this number is odd. More precisely, it shows that division by 4 gives 1 in the remainder. This had been formulated by Fermat in 1640 and then, after a hundred years, this statement was proven by Euler. It is known to us as the Fermat–Euler theorem

.
Thus, we came to the conclusion that x and y in equation (5) were natural numbers, and the condition $x=x_1\ y=y_1$ is necessary.

In this case equation (4) will accept

$$z^n = x^n + x^2 (z^{n-2} - x^{n-2}) + y^2 z^{n-2}$$
$$y^n < y^2 z^{n-2}$$

This implies

$$x^n + y^n \neq x^n + x^2 (z^{n-2} - x^{n-2}) + y^2 z^{n-2}$$

In this case, equation (4) will accept
$$z^n = y^n + y^2 (z^{n-2} - y^{n-2}) + x^2 z^{n-2}$$

$$x^n < x^2 z^{n-2}$$

This implies

$$x^n + y^n \neq y^n + y^2 (z^{n-2} - y^{n-2}) + x^2 z^{n-2}$$

Thus, we came to the conclusion that any simple odd natural number z of the kind $4t+1$ does not exist in the natural values x and y, in which there is a resolution of the equation

$$z^n = x^n + y^n.$$

It is obvious that if an odd z does not look like $4t+1$, this number cannot be spread out to a Pythagorean triple.

3. We shall assume, that z is a compound number, and designate it as $z_1 = zk$, where z is a simple odd natural number, k is a natural number. Let's increase this number to degree n, to get the number z_1^n, which can be presented as a parallelepiped in which base, the square has the line z_1 and the height z_1^{n-2} . We admit, we need to spread z_1^n over two numbers, x^n and y^n under the formula

$$z_1^n = x^n + y^n \tag{5}$$

Let's accept that x^n is a natural number, which can be presented vectorally as a parallelepiped with the base x^2 and height x^{n-2}. Parallelepiped x^n is a part of parallelepiped z_1^n, and the base x^2 is a component of the base z_1^2.

However, parallelepiped z_1^n is also possible to be presented as

$$z_1^n = (zk)^n = z^n k^n = z^2 z^{n-2} k^n \tag{11}$$

Formula (11) shows that numbers of the kind z_1^n can be presented vectorially as a parallelepiped, with base z^2 and height $z^{n-2}k^n$, with the side of the square having z as a simple odd natural number. Obviously, as in this case, x^n will be a part of parallelepiped z_1^n, but the base of parallelepiped x^n will be another and this base will be a part of the base z^2. As we have accepted, that - x^n is a natural number, we can accept the base of parallelepiped x^n as a square, whose side is natural whole positive number, which we shall designate x_1. In this case, the volume of parallelepiped x^n is possible to be presented as $x_1^2 a$, where a is the height of parallelepiped $x^n = x_1^2 a$. It is necessary to note, that a can be both a whole and fractional number. However in any case, a is rational number. The meaning of x_1 we can set arbitrarily, guided only by the fact that this number should be whole and less than z.

From the Pythagorean Theorem there should be parity:

$$x_1^2 + y_1^2 = z^2 \tag{12}$$

we can spread parallelepiped z_1^n over two parallelepipeds - one parallelepiped, in which the base is the square of side x_1 and having a height of $z^{n-2}k^n$, and a second parallelepiped, in which the base is a square with the side y_1 having the height $z^{n-2}k^n$, provided that $x_1^2 + y_1^2 = z^2$.

Thus, the volume parallelepiped z_1^n will consist of volumes of two parallelepiped so that it is possible to write down the formula:

$$z_1^n = z^2 z^{n-2} k^n = x_1^2 z^{n-2} k^n + y_1^2 z^{n-2} k^n \tag{13}$$

It is unsettling to notice, that the right and left part of the formula are easily deduced from k^n, and as a result, we have the equation.

$$z^n = x_1^2 z^{n-2} + y_1^2 z^{n-2} \tag{14}$$

In which z is a simple odd natural number. Or,

$$z_1^n / k^n = z^n = x^n / k^n + y^n / k^n \tag{15}$$

It is obvious that a reduction of formula (13) is proportional to k^n. The volume of parallelepiped $x^n / k^n = x_1^2 a / k^n$ was also reduced in such a manner that the height would decrease in k^n and would be equal to height b, where $b = a / k^n$. Thus for b, there can be both a whole, and fractional number, but in any case, b is a rational number.

Parallelepiped $x_1^2 z^{n-2}$ can spread over two parallelepipeds - $x_1^2 b$ and $x_1^2 (z^{n-2} - b)$
Where $x_1^2 b = x^n / k^n$.
The volume parallelepiped z^n will be equal to the sum of volumes of three parallelepiped $x_1^2 b$, $x_1^2 (z^{n-2} - b)$ and $y_1^2 z^{n-2}$

$$z^n = x_1^2 b + x_1^2 (z^{n-2} - b)^2 + y_1^2 z^{n-2} \tag{16}$$

Recognizing that x_1^2 is a natural number, b a rational number, it follows, that $x_1^2 b$ is a rational number. It is possible to conclude from this, that $x_1^2 (z^{n-2} - b)^2$ are rational numbers.
If $x_1^2 b = x^n / k^n$, from formula (15), it follows, that

$$y^n / k^n = x_1^2 (z^{n-2} - b)^2 + y_1^2 z^{n-2} \tag{17}$$

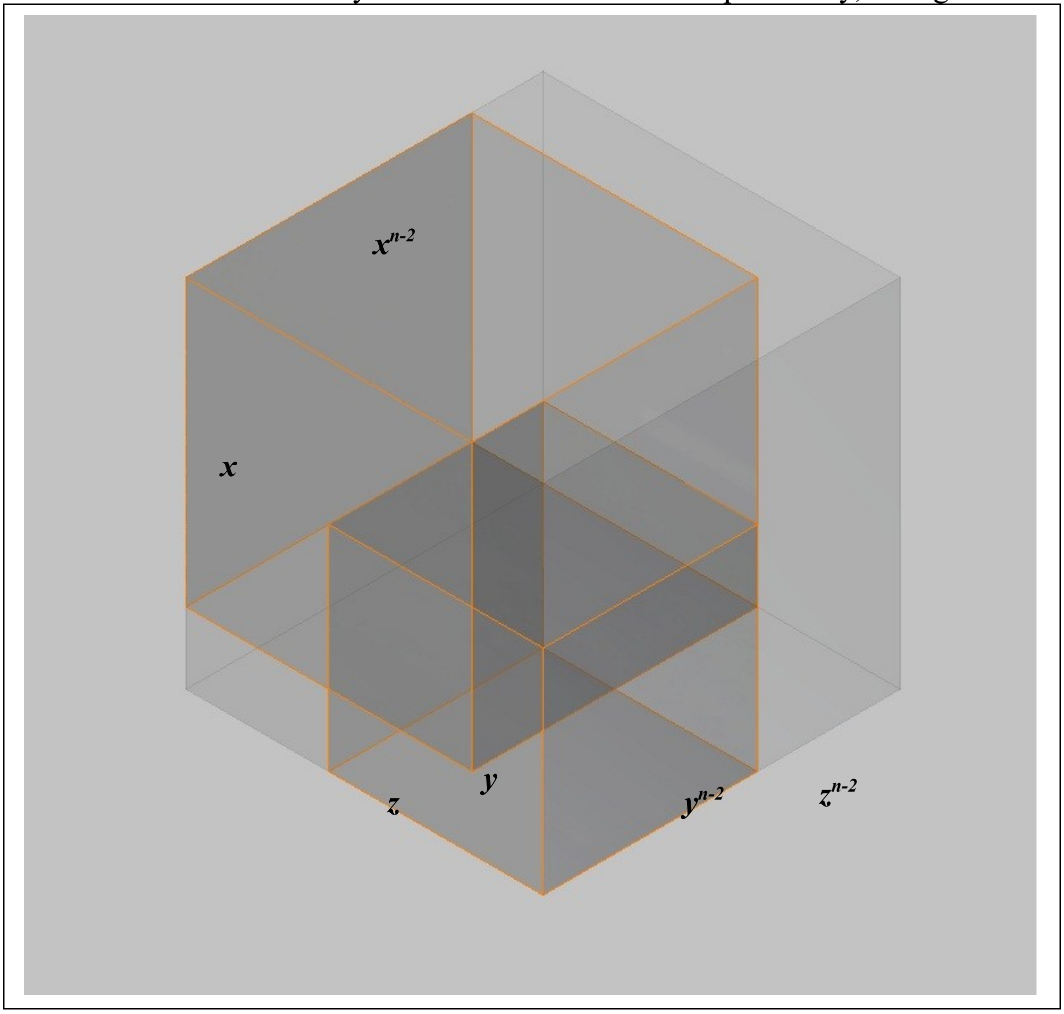
From formula (17) it is obvious that if y_1 is a natural number, y^n / k^n can be a rational number. If y_1 is an irrational number, y^n / k^n will be an irrational number. from what follows, if y_1 is irrational, the number y^n can not be a rational number.

Conditions, in which y_1 is a natural number for the odd whole positive number z of the kind 4t+1 that we have considered above, we have established that for any simple odd natural z of the kind $4t+1$ there does not exist a natural x and y, for which there is a solution of the equation
$z^n = x^n + y^n$.

We have considered all possible variants.

See diagram below:

The above evaluation may be easier to understand pictorially, using the following figure:



Since z can not accept other natural meanings, we came to the conclusion that the equation $z^n = x^n + y^n$ has no solutions for whole positive numbers of $n > 2$.

As we see from the reasoning, the meaning of n does not influence the proof of the theorem, as indicated by Pierre de Fermat. And this really surprising proof, is completely based on the knowledge that was available to Pierre de Fermat. I hope after due study, there will be no doubts that Pierre de Fermat knew the proof of the theorem, which was subsequently named the Last Theorem of Fermat.

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