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The "Last Riddle" of Pierre de Fermat

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The Last and Greatest Theorem

For more than three and a half centuries, this theorem of Pierre de Fermat has occupied the minds of hundreds and thousands of mathematicians. It is assumed that it is based on the work of Diophantus of Alexandria and is related to the Pythagorean theorem. Now that the theorem has been proven with the help of modern mathematical methods, the interest in it has gradually died away. However, the proof has not given an answer to two riddles: whether there exists a simple proof, and whether Pierre de Fermat really knew that proof. It would appear that we will never find the answer to these questions, or at least, the second of them. In this work, I shall attempt to answer both.

There is an equation,

$$x^n+y^n = z^n \tag{1}$$

Where n is a natural number.
Let's present z^n as

$$z^n=z^2 * z^{n-2} \tag{2}$$

Formula (2) shows, where on the basis of a hypercube z^n is square, which party is equal z.
For a known number x there should necessarily be a number y_1, which will satisfy the condition

$$x^2+y_1^2=z^2 \tag{3}$$

Let's substitute formula (3) in formula (2)

$$z^n= (x^2+y_1^2)z^{n-2} \tag{4}$$

Meaning it is possible to present z^{n-2} as

$$z^{n-2}=x^{n-2} + (z^{n-2} - x^{n-2})$$
$$z^{n-2}= y_1^{n-2} + (z^{n-2} - y_1^{n-2}) \tag{5}$$

On first sight, it seems that the formulas in (5) are nonsense, but this is not so. The first formula shows that part z^{n-2} of hypercube z^n consists of a piece x^{n-2}, which is a part of hypercube x^n, and also another distance, which equals $z^{n-2} - x^{n-2}$. The second formula, accordingly shows, that the part z^{n-2} of hypercube z^n consists of a piece, y_1^{n-2}, and another distance, which is equal to $z^{n-2} - y_1^{n-2}$.
Let's substitute expressions (5) in formula (4). So that it is clear where they are inserted, the formulas in (5) are allocated as follows:

$$z^n= (x^2+y_1^2)z^{n-2}= x^2 \underline{z^{n-2}}+y_1^2 \underline{z^{n-2}}= x^2 \underline{(x^{n-2} + (z^{n-2} - x^{n-2}))}+ y_1^2 \underline{(y_1^{n-2} + (z^{n-2} - y_1^{n-2}))}=$$
$$=x^n+ x^2 (z^{n-2} - x^{n-2}) + y_1^n + y_1^2 (z^{n-2} - y_1^{n-2}) \tag{6}$$

Let's substitute formula (6) in formula (1) and we shall establish that the meaning is equal to y^n

$y^n = z^n - x^n$

$$y^n = x^n + x^2 (z^{n-2} - x^{n-2}) + y_1^n + y_1^2 (z^{n-2} - y_1^{n-2}) - x^n = x^2 z^{n-2} - x^n + y_1^n + y_1^2 z^{n-2} - y_1^n =$$
$$= x^2 z^{n-2} + y_1^2 z^{n-2} - x^n \qquad (7)$$

Formula (7) shows a dependence of the meaning y^n on the meaning y^1.
For a known number y there should be a number x^1, which will satisfy the condition:

$$x_1^2 + y^2 = z^2 \qquad (8)$$

Let's substitute formula (8) in formula (2)

$$z^n = (x_1^2 + y^2) z^{n-2} \qquad (9)$$

It is possible to present number z^{n-2} as,

$$z^{n-2} = x_1^{n-2} + (z^{n-2} - x_1^{n-2}) \qquad (10)$$
$$z^{n-2} = y^{n-2} + (z^{n-2} - y^{n-2})$$

Let's substitute the expressions of (10) in formula (9)

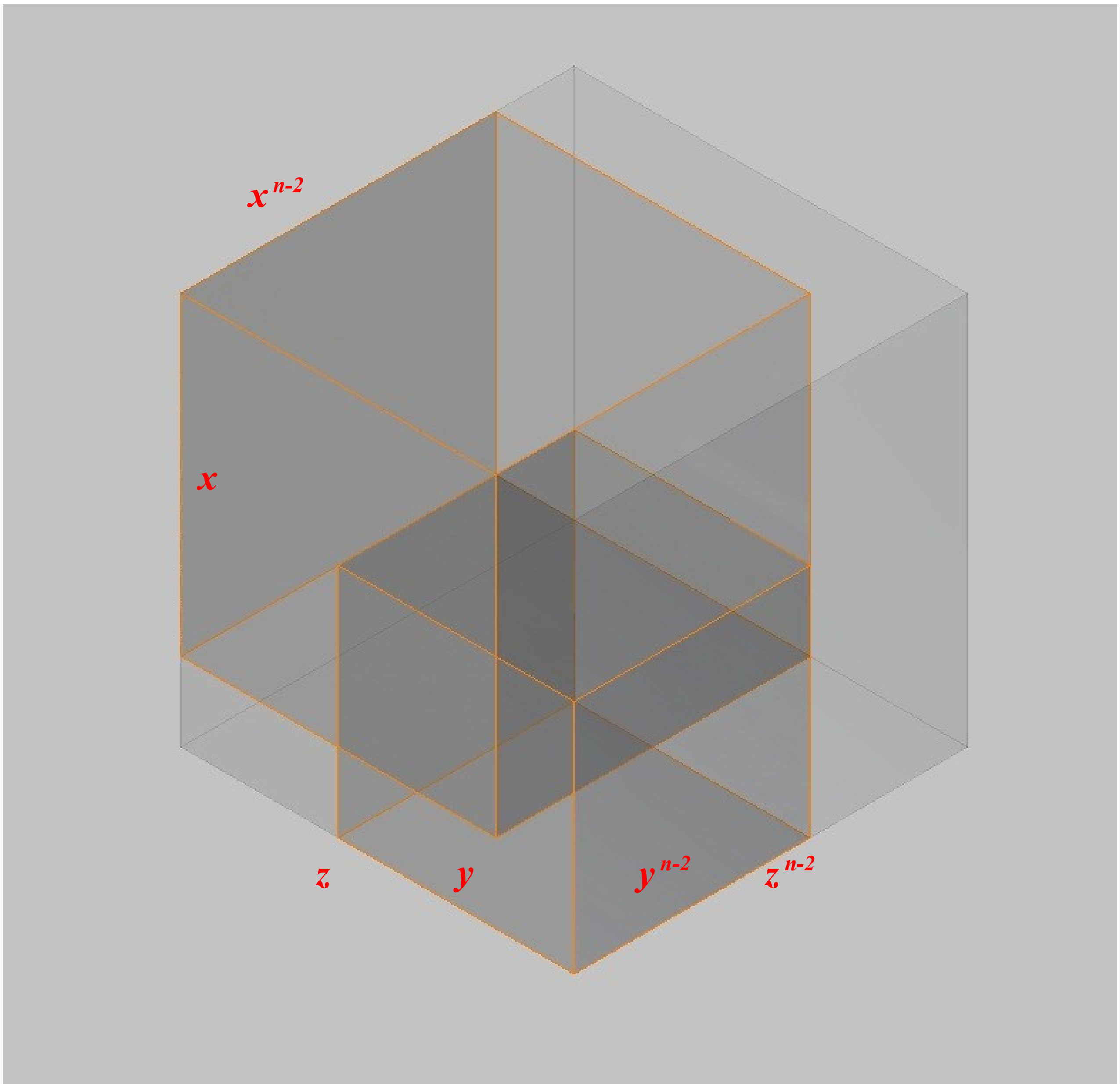
$$z^n = (x_1^2 + y^2) z^{n-2} = x_1^2 z^{n-2} + y^2 z^{n-2} = x_1^2 (x_1^{n-2} + (z^{n-2} - x_1^{n-2})) + y^2 (y^{n-2} + (z^{n-2} - y^{n-2})) = \qquad (11)$$
$$= x_1^n + x_1^2 (z^{n-2} - x_1^{n-2}) + y^n + y^2 (z^{n-2} - y^{n-2})$$

(This is similar to formulas (5) and (6))
Let's substitute formula (11) in formula (1) and we shall establish that it is equal to x^n.

$x^n = z^n - y^n$

$$x^n = x_1^2 z^{n-2} + y^2 z^{n-2} - y^n \qquad (12)$$

Formula (12) shows the dependence of meaning x^n on meaning x_1.
For better understanding, the transformations made can be presented in the following figure



The behavior at higher transformations does not give restrictions on the meanings x, y and z. Further, suppose, that x, y and z are natural numbers.

First. Let's consider formula (7).
Let's accept, that x and z - natural numbers. By the first condition that y - a natural number is that y^n is a natural number. If y^n is not a natural number, y cannot be a natural number. This is obvious.

Formula (7) includes a little composed. Proceeding from the condition, that x and z - natural numbers, in the formula (7) x^2z^{n-2} - natural number, x^n - natural number. Is it necessary to establish, whether y_1 is a natural number?

It is known on the condition that an equation of the kind $x^2+y^2=z^2$, has the correct interpretation, it should carry out the identity

$$(m^2-n^2)^2+4m^2n^2=(m^2+n^2)^2 \qquad (13)$$

We have accepted, that z - natural number, hence for number z, there should be two integers m and n, with which the equality will be carried out

$$z = m^2+n^2$$

Accordingly, for formula (3), the equality should be carried out

$$x=m^2-n^2;\ y_1=2mn;\ where\ \ z=m^2+n^2 \qquad (14)$$

Recognizing that the meaning (importance) of $y_1=2mn$ is attained by the product of natural numbers, it follows that y_1 - a natural number, means, y^n in formula (7) – a natural number. Thus, we have established, that at the observance of the above-considered conditions, y^n in formula (7) is a natural number, which means, ***y* can be natural number.**

Second. Let's consider formula (12).
Let's accept, that y and z - natural numbers. By the first condition that x - a natural number is that x^n is a natural number. If x^n is not a natural number, x cannot be a natural number.

Formula (12) includes a little composed. Proceeding from the condition, that Is it necessary to establish, whether is x_1 natural number? We have accepted, that z - natural number, hence, for number z there should be two integers m_1 and n_1, upon which the equality will be carried out

$$z=m_1^2+n_1^2$$

To satisfy the condition of formulas (13), there should be the equality

$$x_1=m_1^2-n_1^2;\ y=2m_1n_1;\ where\ z=m_1^2+n_1^2 \qquad (15)$$

Recognizing that the meaning (importance) of $x_1=m_1^2-n_1^2$ is identified by the actions of the above integers, it is possible to draw the conclusion, that x_1 - a natural number, means, x^n in formula (12) - natural number.

Thus, we have established, that with the observance of the above-considered conditions, x^n in formula (12) is a natural number, which means, ***x* can be a natural number**.

Third. Let's consider formulas (7) and (12) in aggregate. It is known and is proven, that any simple number cannot be submitted as the sum of squares of two integers in two and more ways. It is essentially different (i. e. Not turning out by rearrangement or composed, one of the other).

It is known and is proven that if the equation $x^2+y^2=z^2$ has even one solution in natural numbers x, y it is necessary and sufficient that the number z has at least one divider of the kind $4t+1$, where t is an integer. That, is number z should be either a simple number, or a simple number multiplied by an integer. This implies that there cannot be two various pair numbers m and n, m_1 and n_1, of which the number z can be made.

This means, $m_1=m,\ n_1=n.$

From this, we judge that the meanings (importance) n and n_1, m and m_1, in parities (ratio) (14) and (15) are equal among themselves, which means $x=x_1\ y=y_1$.

This implies, that equations (3) and (8) can be simultaneously correct in natural numbers, ***only if*** $x=x1\ y=y1$. If these conditions are not carried out, one of the equations, (3) and (8) will not have the correct solution in natural numbers.

In the case where $x=x_1\ y=y_1$ formulas (6) and (11) will accept

$$z^n=x^n+y^n+x^2(z^{n-2}-x^{n-2})+y^2(z^{n-2}-y^{n-2})$$
$$z^n=x^2z^{n-2}+y^2z^{n-2} \qquad (16)$$

It is necessary to compare formulas (16) and (1).

$$x^2 z^{n-2} + y^2 z^{n-2} \neq x^2\, x^{n-2} + y^2\, y^{n-2}$$

It is obvious, that formula (16) cannot be equal to formula (1), except for, as an example, when ***n=2***. In this case, formula (16) will accept,

$$z^2 * z^{n-2} = x^2 z^{n-2} + y^2 z^{n-2}$$
$$z^2 = x^2 + y^2$$

This implies, that the expression $x^n+y^n = z^n$ can not have a solution in natural numbers at $n > 2$.
The theorem is proven.

A Note:

There are Pythagorean triplets available with an equal z result.
For example:

7, 24, 25
15, 20, 25

We see, that the z factors are equal, but x and y are different. On the first sight, it seems that such triplets are not covered in the above-mentioned proof.

Actually, close attention to it reveals that it is a compound of a given Pythagorean triplet

15=3*5, 20=4*5, 25=5*5,

That is, it reduces to the Pythagorean triplet 3, 4, 5.

The equation of the sum of degrees looks like:

$$(3*5)^n + (4*5)^n = (5*5)^n$$

That is, it is derived from the Pythagorean triplet 3, 4, 5. The equation of the sum of degrees appears as:

$$3^n + 4^n = 5^n$$

To this equation we can apply the proofs stated above.

In a general view, such equations based on compound Pythagorean triplets, will look like:

$$(x*i)^n + (y*i)^n = (z*i)^n$$

After a reduction of i n the equation, it will accept

$$x^n+y^n = z^n$$

Where i - whole positive number, z - simple number or number received by multiplication of simple numbers by an integer.

In a general view, formula (13) for such numbers will look like

$$(m^2-n^2)^2 i^2 + 4m^2n^2 i^2 = (m^2+n^2)^2 i^2, \text{ where } i = 1,2,3\ldots i$$

This also does not contradict the above-offered proof.

Conclusion

After I had come to the above conclusion, Fermat's logic when he formulated the famous theorem was completely clear to me.

I think, that, studying the eighth task of Diophantus in which the question of decomposition of a square to two whole squares is considered, Pierre de Fermat had asked himself a simple question: “How would these transformations apply if one more side was added to a given square to create a cube (hypercube)?" From this the question logically follows: “What would be the case, if even one of the numbers thus generated were whole and positive?". The answer to these questions is obvious and is stated above.

It is possible to assume from this conclusion that Pierre de Fermat undoubtedly knew the proof of the Theorem, and due to its simplicity, probably had not found it necessary to write it down .

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