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THE STEFAN-BOLTZMANN LAW
a simplified derivation

by Miles Mathis

The Stefan-Boltzmann Law is an equation that relates the temperature of a black body to its total radiation:

$J = \sigma T^4$

But if you consult a textbook or the internet or a mainstream physicist, you are never told why we have the fourth power here, though that is the central mechanical question. It is passed off as non-mechanical, a coincidence, or unexplainable. But it is a simple outcome of the E/M field.

The problem is that all the textbook and historical derivations have tried to derive the equation from the flat surface of a black body. The current equation is written to apply to radiation per unit surface area, and the surface area is flat. In deriving this equation, the current derivation starts by looking at a small flat surface radiating out into a half sphere. Not only does this make the math much more difficult than it needs to be—since we have to integrate using spherical coordinates, going from flat to curved—but it hides the mechanics of the field. As you see, we are emitting from a flat surface into a spherical field. That is upside down. We should let our definitional black body be a sphere to start with, since that is how the E/M field is emitted in the real world, at the quantum level as well as at the cosmic or planetary level.

In several papers http://milesmathis.com/moon.html, I have already shown that the E/M field must decrease to the fourth power as it is emitted from spherical objects of any size. It must decrease to the square simply because of the surface area equation. As it is emitted from a spherical body, it moves out into larger shells, and those shells are found by the surface area equation—which has an r^2 in the denominator. But the E/M field is decreasing to a second square because it is expanding inside the gravitational field. Every real sphere creates its own gravitational field, so the E/M field is always being emitted into the gravitational field. Because it simultaneously decreases for both reasons, it must decrease to the fourth power.

The Stefan-Boltzmann Law is therefore just the inverse of this fundamental law of the E/M field. If the E/M field decreases to the fourth power as it is emitted into larger volumes, it must increase to the fourth power with larger temperatures. This is because temperature and volume are inverse measurements. An increase in temperature is operationally exactly the same as a decrease in volume, since temperature increases particle velocity, allowing the particle to cover more distance. If you cover more distance in the same time, you have lowered the effective volume.

We can see this from Charles' Gas Law, which shows the relationship of temperature and volume:

$V \propto T$

The Stefan-Boltzmann Law is just a direct outcome of Charles' Law, and Charles' Law is a tautology—a deduction from the definitions of length and velocity.

You will say, "Wait, Charles' Law is a direct proportion, and you are claiming an inverse proportion! What happened?" What happened is that you are misreading the equation. Yes, with a balloon that is free to expand, an increase in temperature will cause an increase in volume. But if you decrease the volume, the temperature will also increase. In that case, Charles' Law could be written like this

$V \propto 1/T$

It is the second case that is experimentally analogous to our E/M field, not the first case. I only wrote Charles' Law as a direct proportion because that is how it is normally presented. If I had written it as an inverse proportion to start with, you would not have complained, you would have written me off as a bumbler. This way, I could show you the logic of my argument.

You will then say, "With the E/M field we cannot increase the velocity, since the velocity is already c." True, but with electromagnetic radiation, we increase the energy instead of the velocity. Energy also acts in an inverse way to volume, since energy always has a mass equivalence. As you increase mass, you decrease volume, because mass must take up volume. As I have shown http://milesmathis.com/ug.html, mass is operationally or mathematically the same as $\text{length}^3/\text{time}^2$, so any increase in mass must increase length, just as with velocity. The result in the field is the same either way.

You will say, "But the photon has no mass. Giving the photon more energy cannot increase its mass, since it has none." I have shown http://milesmathis.com/photon.html that this interpretation of the photon is used by the standard model only to keep their gauge math working, but they have no proof of it. In fact, by the most famous equation of the 20th century, $E=mc^2$, all energy *must* have a mass equivalence, including the energy of the photon. Even if the photon has no rest mass, according to this equation—which the standard model still accepts—the energy of the photon must act in the equations and in the field as if it does, so that the energy of the photon takes up real space. We can think of this as *moving* mass equivalence, rather than rest mass, since the photon is always moving. We can say that the photon takes up space in the field due to energy or mass, it doesn't really matter to the math or the field. The same result pertains either way: the effective volume of the field is decreased, proving my point.

Because the Stefan-Boltzmann equation has always been derived from a flat surface, we get the strange constant:

$\sigma = 2\pi^5 k^4/15c^2h^3 = 5.67 \times 10^{-8}\ Js^{-1}m^{-2}K^{-4}$

But, of course, if we wrote the equation for a spherical black body, the constant would be different. As you can see, part of that constant is just taking us from degrees Kelvin to watts per square meter. Since the size of a degree Kelvin has nothing to do with a watt, we must get some number transform. But a much more useful and correct equation would give us joules or watts per radius, and in this case the constant might even tell us something interesting and fundamental.

There are two ways to do this. One way is to rerun the derivation from the beginning. But the simplest way is to assume the mainstream derivation is correct, and then convert their number from square meters to a radius. To do this, we just ask what radius would give us one square meter in surface area.

$4\pi r^2 = 1 = m^2$
$r = \sqrt{(1/4\pi)} \approx .282m$

Then we just substitute into the given constant, like this:

$\sigma = 5.67 \times 10^{-8}\ Js^{-1}(4\pi r^2)\text{-}1K^{-4}$
$\sigma = (4 \times 10^{-9} / r^2)\ (Js^{-1}m^{-2}K^{-4})$

Since r is measured in meters, that will give you an answer in watts per square meter, but your surface is now curved, as in the surface area equation. And you have a variable for radius, so you can immediately find the radius of a particle that would emit that amount of radiation at that temperature. All you have to do is let the r variable have no dimensions. In this equation r is just a number. It's dimensions are separated from it, appearing as m^{-2}.

But let us continue unwinding this equation.

$\sigma = (4 \times 10^{-9} / r^2) Kgm^2\ s^{-2}\ s^{-1}\ m^{-2}\ K^{-4}$
$\sigma \approx (1.2/cr^2)(s/m)Kgs^{-3}\ K^{-4}$
$\approx (1.2/cr^2)Kgs^{-2}\ m^{-1}K^{-4}$
$\approx (1.2/cr^2)(m^3\ s^{-2})s^{-2}\ m^{-1}K^{-4}$
$\approx (1.2/cr^2)m^2\ s^{-4}\ K^{-4}$
$\approx (1.2a^2 / cr^2)\ K^{-4}$

What did I just do? First, I noticed that 4×10^{-9} is almost 1/c. Then I reduced the other parameters into acceleration instead. But what does that acceleration variable stand for? It must stand for the gravitational acceleration on or of the surface of our radiating black body, which means it stands for the gravitational field of that body. So we could rewrite the last equation as

$\sigma \approx (1.2g^2 / cr^2)\ K^{-4}$

If we let $g = 9.8$, then the equation becomes

$\sigma \approx (.122g^2 / cr^2)\ K^{-4}$

You ask, "Why let $g = 9.8$? This is any black body we are talking about here, not the earth." Yes, but I have shown that relative to the earth, all spherical bodies have a gravitational acceleration at their surface of 9.8, including the photon. If they didn't, they would not stay the same size relative to the earth. So if we let *g* be the relative acceleration of the surface of the body, we can use 9.8 for any given body.

If we use the numbers we already have, we can continue to simplify:

$T^4(K^{-4}) = 2.56 \times 10^7 r^2 J$

With that equation, we can relate temperature to energy with just the radius of our black body. J will still be found in watts per square meter, since if we re-expand we would still find the same parameters as before. The transform is no longer a constant, it is true, now that it contains r. Nonetheless, it will be much more useful in this form. We can also use this equation:

$r = 2 \times 10^{-4} T^2/\sqrt{J}$

As I said, these new equations will be much more useful than the old ones, since real bodies—especially natural ones—tend to be spheres. The new equations are also much more transparent than the old ones.

Whenever you want to discover the mechanics underneath any equation, first ditch all the modern constants. These constants are there to act as misdirection. Look again at the original constant:

$\sigma = 2\pi^5 k^4/15c^2h^3$

A constant expressed by four other constants! And k is not the Coulomb constant, it is the Boltzmann constant, which is two constants, the gas constant and the Avogadro constant:

$k = R/N_A$

So the Stefan-Boltzmann constant is actually five constants stacked. They really must want to misdirect us, with that many blind alleys. In other papers I have shown that h, Plank's constant, is hiding the photon mass http://milesmathis.com/fine.html; and even π is false in kinematic situations. That is why I skipped that expression of σ and went right into the dimensions. As you have seen, it was much easier to solve by unlocking the dimensions than it would have been trying to unlock all the constants.

Some will say, "This equation can't be right, because it implies that all objects of the same radius act the same as radiators of E/M emission. But density must come into play."

No, all this equation tells us is that a perfect black body—our definitional object—must have a certain density at a given radius. It must have this density to be a perfect black body. If it had a greater or lesser density than this optimal density, it would not act like our definitional black body. This is more information that was hidden underneath the old equations. Physicists had thought that perfect black body absorption and emission was due to molecular makeup or some other factor, but this new equation implies it has to do mainly with density. A variety of materials may be able to create this density in various ways, but it is the density that mathematically determines the black body.

And we have one final new discovery, unlocked by these equations. We found that, other than the radius, the transform was composed of g^2/c. I have simplified the derivation specifically to make the mechanics transparent, but what mechanics have we seen here? Why g^2/c? That transform gives us both the gravitational field and the E/M field, the two fields that determine this equation. We simply rewrite that term as $(g)(g/c)$. The first term is gravity, obviously, and the second is the E/M field. The E/M field travels c through the gravity field, so we have to relate one to the other. Yes, g^2/c is the simplest of the simple unified field transforms. I found it only because I was looking for it.