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THE SIMPLE PROOF OF GOLDBACH'S CONJECTURE

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Abstract

Here I solve Goldbach's Conjecture by the simplest method possible. I do this by first calculating probabilites for prime and non-prime meetings. Then I redefine and transform these probability fractions into densities, allowing me to develop a proof without probabilities. These densities allow me to calculate minimum numbers of pair meetings for any given prime density. These minimum pair meetings create a new rule that disallows certain meetings and requires others. One pair meeting that is required is at least one prime pair.

Goldbach's Conjecture is that any even number may be expressed as the sum of two primes. If this conjecture is false, then there must be at least one even number that cannot be expressed as two primes. I will show that this is impossible, thereby confirming Goldbach's Conjecture.

It has always been thought that there may be a simple solution to Goldbach's Conjecture. In this paper I will show that solution.

In addition, I will state my proof in sentences, rather than in equations only. This is the sort of proof I believe that Goldbach wanted for his conjecture. He asked a simple question, hoping for a simple answer. The question is one that can be understood by non-professional mathematicians, and I want my answer to be understood by them, too. The other reason I do this is that even with professional mathematicians, a straightforward, concise proof in equations only has become impossible with Goldbach's Conjecture. The theoretical confusions and misunderstandings as well as the multitude of mathematical errors concerning this problem have built up over the centuries, and it now requires a good deal of exposition and comment to clear up all these misunderstandings. I will not be able to clear up every single error made in regard to Goldbach's Conjecture, of course. That would take a book, or possibly a library. But with a few pages of commentary and equations I believe I can clear away the main errors, both theoretical and mathematical, showing the clear logic of my proof.

If you want to see the proof stated in the baldest possible way, with no commentary or explanation, you can skip to the last paragraph, which simply states the new rule. This is unlikely to satisfy you, however.

The full argument begins by my stating some very basic things. We can list all the possible sums of any even number. The number 20, for example:

20 =
1 + 19
2 + 18
3 + 17
4 + 16
5 + 15
6 + 14

7 + 13
8 + 12
9 + 11
10 + 10

You can immediately see that we have three and only three possible combinations: prime + prime, prime + non-prime, non-prime + non-prime. From now on I will abbreviate prime and non-prime as P and N. Now, if GC is false, then P + P is no longer a possibility, and we are left with the possible sums N + P or N + N.

We are given that some fraction of all integers is prime. If that fraction were > 1/4, then we would need no further proof of GC, since in that case the number of primes would be greater than non-prime odds, overwhelming all tables like the one I just made. A prime would have to meet a prime. At low numbers, this is precisely what happens. But we know that the fraction of primes is nowhere near 1/4. As our even number gets larger the fraction of primes get smaller and smaller. There are complicated proofs that show a limit to this fraction, but I will not need to make use of them. My proof can be completed without a known fraction at a given even number, and this is just as well, since no one has yet provided an equation for developing that fraction for any even number. And even knowing that fraction does nothing to prove or disprove GC.

Let us take some fractions less than 1/4 and see what we find. Let our fraction of primes to all numbers be 1/6. That is the same as saying that 1/6 of all the numbers in our list of sums is P. The first thing I did in my procedure is apply probabilities to this list. I will follow my line of thought so that you can see how I arrived at my proof. I imagined that since we are asking an addition question about numbers found by multiplying or factoring we will have created a list that is nearly random. That is, I assume that whether or not a P meets a N in the list is purely a matter of chance. Then I calculate the probability of a P meeting an N. The way this is usually calculated is like this: if 1/6 of the numbers are P, then the probability of a P meeting a P are 1/36. The probability of a P meeting an N are 2(1/6 x 5/6) = 10/36. (We can have P + N or N + P, so we must multiply by two). N meeting N is 25/36.

But this math is not quite correct. Several errors are made by those who have pursued this line of thought before me, and I will correct those errors as I go, pointing them out to you. I can correct them because I also made them before I saw my mistake. To reach my solution, I had to pass all the blocks that have stopped everyone else. The above probabilities are not exactly correct, since a lot of those N's are evens. The probability that an even will meet an even in our list is 1. It is 100%. Therefore we have to jettison all the evens from the table, since they skew the probabilities. Probabilities only apply to things that can vary. We can find no variance with a meeting of evens. An even *must* meet an even in our list, and so probability cannot apply. We have to look at the probabilities of the odd numbers involved only. This makes our fractions the following: if 1/6 of the numbers are P, then (approximately) 1/3 of the odds are P. This makes the probability of P + P = 1/9. The probability of a P meeting an N are 2(1/3 x 2/3) = 4/9. N + N is also 4/9.

Now let us continue. If GC is false, then all P's must meet N's. We have disallowed P + P, remember. This means that the fraction of sums N + P must be twice the fraction of P's. Each P creates a sum N + P, and we have P's on both sides of the table. This means that if the fraction of P's is 1/3, we need to find 2/3 of our odd sums are N + P. We need to find 2/3 or 6/9 N + P sums, but statistically we can expect only 4/9.

No matter what fraction you choose, you find the same thing. If the fraction of primes is 1/8, then the probability of finding N + P are 6/16, but we require 8/16 to avoid any P + P sums. If the fraction of primes is 1/10, then the probability of finding N + P are 8/25, but we require 10/25 to avoid any P + P sums. The probability is always strongly in favor of a P + P.

The probability *against* covering all primes with non-primes is even stronger. The probability we calculated above applies to each meeting, but if we calculate the probability of **all** meetings, we must first find all combinations. Let us say we have a prime density of 1/8 and we have 2000 primes. That would mean our given number is about 16,000. A high number, but not very high. I have already calculated the probability of finding an N + P sum at that density: 6/16 or 3/8. But we have 2000 draws. That makes our total probability somewhere in the realm of (3/8)^2000. There is only one way to create the correct outcome, which is that every single draw is N + P. The P's can never meet eachother. This means that our probability of finding all P's covered by N's is (3/8)^2000/1. That is already so small a number that my online scientific calculator won't allow it. I am not going to check my math, because even if I have calculated the probability in slightly the wrong way this in this instance, it is clear that the correct answer is a tiny fraction, one that is much much smaller than the probability of a prime meeting a prime. I am not concerned with finding the precise answer here, since it doesn't affect my proof. All I need do is show you that the probability is astronomically large against covering all primes with non-primes, even at the number 16,000. You can imagine how great the probability would be for even larger numbers.

The second mistake that is made is stopping here. Almost everyone who applies probabilities to Goldbach's Conjecture assumes that these probability fractions cannot be transformed into usable fractions of some sort, ones that will provide a real proof. They say that proofs cannot come from probabilities, and they give up. It is true that probabilities cannot provide a proof, since probabilities can vary. We need something that does not vary.

Before we get into that proof, let us look at another list. Let us take a hypothetical even number and try to force it to falsify GC. Take the even number X (it is 60, but we will pretend we don't know that). We will create our own primes to force the list to read the way we want it to.

X =
N + P
N + N
P + N
N + N
N + P
N + N
N + N
N + N
P + N
N + N
N + P
N + N
P + N
N + N
N + N
N + N
N + N
N + N
P + N
N + N
N + P
N + N
N + N
N + N
P + N
N + N
N + P
N + N
N + N
N + N

This list would falsify GC, since no P meets a P. The fraction of P's is 1/6, and the fraction of N + P sums is 1/3. You can see that one fraction is twice the other, and that this must be the case whenever we have no P + P sums.

To transform our probabilities into fractions that we can use for a proof, let us ask why this table cannot actually happen. It has no obvious flaws, since everything looks to be in order. Is there a rule for filling in the table that everyone has missed so far?

To parallel this question, we can ask why the probabilities work like they do. Why do the probabilities tell us that we should very strongly expect to find a P + P sum? Even at very low prime densities, the probabilities still tell us we should strongly expect to find a P + P sum, and the probabilities tell us even more strongly that we should not expect the N's to exactly cover all the P's. With large numbers, the probability of coverage is astronomically small, as I have shown.

Most people who have studied this question have told us that it is easy to intuit that as your prime density drops your probability of finding a prime pair drops. But their intuition does not answer the probabilities, which tell us the opposite. The probabilities tell us that we are much more likely to find a prime pair than to find that our non-primes will have covered every single prime. I would say that the intuition of these mathematicians is faulty, and that we are better to rely on logic than intuition. Logic dictates that what is true for probabilities is also true for non-probabilities. The odds are telling us very strongly to pursue a non-probability solution to Goldbach based on these fractions.

So that is what I did. To turn these fractions from probabilites into usable fractions, I treated them as densities. Then I used these densities to calculate minimum values for meetings. A density is not a probability or statistic. A prime density is a firm number that applies to a real existing relationship in each situation, or with each given even number. This relationship is the primary factor in determining how the meetings of P's and N's occur, as you must see. Speaking of densities allows us to extend our probability arguments and equations into a proof that dispenses with variance and allows for a final solution. With densities, we no longer have to assume that the meetings are random, either. We are concerned with density *and only density*. This density can be created randomly or with any generator you like; it does not affect the density calculations I will do.

We don't even have to know the actual prime density for any given even number. I will show that we can take any possible density and achieve the proof. My proof will work with any given density, down to the tiniest fractions. That is, the density of primes could be 1/1000 or less and my proof would still work. This finding is surprising, I admit, but it is a logical outcome of the math, as you will see.

With that in mind, I can show you exactly what is wrong with the example above, the one I created to force all P's to meet N's. Let us take a closer look at the N + N sums. But we need to look at only the N + N sums that include odd non-primes. I will call these terms NPO's. In my example we have five of these sums.

X =
NPO + P
N + N
P + NPO
N + N
N + NPO
N + N
NPO + NPO
N + N
P + NPO
N + N
NPO + P

N + N
P + NPO
N + N
NPO + NPO
N + N
NPO + NPO
N + N
P + NPO
N + N
NPO + P
N + N
NPO + NPO
N + N
P + NPO
N + N
NPO + P
N + N
NPO + NPO
N + N

So, the fraction of NPO-NPO meetings to odd meetings is 1/3. We have 15 odd pairs and 5 NPO + NPO pairs. The problem occurs when we look at the prime and NPO densities. In this example, the prime density is exactly 10/59. We have 60 terms, but we have only 59 numbers less than 60. That last N + N is 30 + 30, so the second 30 is a repeating term and does not count in the density. This means that we have a density everywhere on the number line for NPO's of 20/59. Notice that this density applies to the whole table above and to each side of the table. A density is not just a fraction with the real number of primes over the number less than X. A density, once calculated, can be applied to all parts of the line and any part of the line. [It can also vary in given parts, but we will touch on that possibility later.]

What should look strange is that we have created in our table a fraction of NPO meetings that is less than our density of NPO's. How did that happen? It happened because the table is not a good representation of the number line. It is a visualization that does not work. It allows you to do things that are illegal mathematically and not be aware of it. There are rules to filling in these slots. You can't just fill in a slot because a term seems to fit there.

If the density of NPO's is an *average* of 20/59 (it does not have to be that density at every point or interval), then we cannot possibly find a fraction of odd sums that is less than that density. To show this, I must create another visualization that shows this where the table does not.

Take two facing walls in a room that are exactly the same size. Paint 10/59th of that wall yellow. That will stand for our primes. Paint 20/59th red. That will be the NPO's. Paint 29/59th blue. That will be the evens. With the blue, you must paint the left side of one wall and the right side of the other, like a mirror image, so that blue always meets blue. If we bring the walls together, blue will cancel itself out, and we can forget about it after this. The question, of course, is how the yellow meets the red. How much orange paint will be created when we move the walls together and have them touch eachother?

We can have red meet red, yellow meet yellow or yellow meet red. Let us seek a minimum amount of red-red meeting. To do this we force as much red to meet yellow as possible. We forbid yellow from meeting yellow and then cover all available yellow with red. The leftover red will have to meet red, and that will be our minimum amount. We have half as much yellow as red, so we must lose half our red in the meeting.

What is our total density of each color now? We have 20/59 orange (red+yellow) and 10/59 red (red+red). But we must remember that $10/59 \neq 5/30$ or 1/6. Therefore, although our wall appears at first to corroborate our table, it does not. The table is a list of actual terms; the wall is a representation of densities.

This is where the final and worst mistake is made, by those that may have made it this far. They have assumed that you could switch mid-problem to a density that expressed NPO's to odd numbers. That is, they assumed that we had a yellow density in the non-blue areas of 1/3 here, but we don't. We can't reduce our density and start talking of a density in the non-blue part of the wall. **The density already applies to sub-parts of the wall** as well as both walls together and each wall separately. So we cannot reduce it and find a sub-density of the odd parts of the two walls. You cannot reduce the density and find a sub-density. Or, you can, but you have to do it in the traditional way, by reducing the fraction in a legal way, such as 10/59 = x/30, and then cross multiplying to find the answer. You can't just say, "We have 30 odds and 10 primes, therefore the density of primes to odds is 1/3." **The density of primes to odds does not generate the meetings, the overall density of primes does.** Those who reduce in this way have made a terrible mathematical error: it is that simple.

We have half as much yellow as red, but we do not have a density of 1/3 yellow in the non-blue section or of 1/6 on each wall or both walls. Again, the density of yellow is 10/59, period, and it applies to the whole number line and every part of the number line. Both walls have that density and each wall has that density.

Those who have made the mistake likely made it because they wrote the density (as I have) as the real number of primes over the real number of integers less than X. This fools them into thinking they can reduce that into sub-areas, but they can't. Once we define that fraction as a density, the numerator no longer stands for the actual number of primes in the table and the denominator no longer stands for the actual number less than X. We would be better off to write it as a density of 20/118 or something, so that we are no longer tempted to think of the numerator as the real number of primes and start reducing it in faulty ways.

Now let us return to the solution. In the graph, we found 1/3 of odds sums were NPO + NPO. But if we reduce our density fraction here (in the correct way) we find that 10/59 = 5.085/30. If we double that to represent a meeting or sum, we find 10.17/30 or 1.017/3. That is our minimum density of meetings or sums of NPO's given the density of primes. But 1/3 is less than that. It is therefore mathematically disallowed. The table we forced to work above fails for a mathematical reason. You cannot generate only five NPO-NPO meetings with 10/59 primes. Your number of NPO-NPO meetings must be greater than half your number of primes. This forces one of your necessary N + P meetings to fail, turning it into a P + P meeting.

I will say that in a different way: If we divide all possible non-blue wall meetings into fifteen meetings (like fifteen equal size patches), at least six must be red-red, since that is the first available number that satisfies the requirement of being at least 1.017/3. If six out of fifteen are red-red, then some yellow will be uncovered by red, and it will have to meet yellow. A yellow-yellow meeting is a prime pair.

Varying the local density does not change this fact, since a lower density of red paint on one given part of the wall must create a higher density at all other points. Less meetings of red-red in one vicinity must create more meetings in all other vicinities, in order to keep the overall density constant.

Some will say that if you have considerably more red paint on one wall than the other wall the outcome must be affected. But this is not true. Even if we allow that the density of NPO's above the halfway mark of a given number must be greater than the density below, we cannot find a variation from my proof. Less red paint on one wall will give you more yellow paint on that wall, but the overall density of yellow paint is also constant for each given number, so more yellow on one wall means less on the other wall. This means that the total amount of red paint that can meet yellow will stay the same. The minimum red-red events therefore cannot change, no matter what you do. This is because to minimize your red-red meetings, you force as much red to meet yellow as possible. You cover all yellow with red. But if your total amount of yellow on both walls does not change, you cannot change your minimum number of red-red meetings. So having a higher density of NPO's at higher numbers does not affect my proof.

Others will say that when I was calculating probabilites, I jettisoned the evens, since they skewed the problem. But now I am doing the opposite. Yes, I am. I am forbidding you from jettisoning the evens, since the density must include them. We aren't concerned with variance now, since we aren't calculating probabilities. Our fraction is not a probability, it is now a density. The density must include all numbers less than X, since that is how it is defined. Notice that you cannot say that the density of evens is 100%, allowing it to be ignored. That doesn't make any sense. Nor can you find a density of primes relative to odds. That doesn't make any sense either, since there isn't any way to subtract out the evens. Every potential spread of primes will create different gaps, and these gaps will be changed in different ways by filtering out evens. So, not only is it impossible to achieve, it is both senseless and counterproductive. It is not how the numbers actually match up. The primes and NPO's match up *around* the evens, therefore the density must express this. The density we have does that, so it would be absurd to propose filtering evens to solve a problem we do not have. Filtering would actually unsolve a problem we have just solved.

Now, if you go back to our table (the number 60) and apply this fact to the list, you find that to create another NPO + NPO sum, you must switch a P with an N. **You cannot do so without creating a P + P sum.**

You can see that my new rule utterly overthrows the tacit assumptions of the example table and every possible table or even number. My minimum can be calculated in the same way for any given even number or prime density, and in every case it forces the appearance of at least one prime pair. We had assumed that we could put an N wherever there was a space for it in a list, but we can't. We have to be scrupulously careful with how many of each sort of sum we create, since many possible combinations will contradict the given prime density. A prime density of 10/59 at the number 60 *requires* that we find at least one prime pair, no matter where the primes turn up in the sequence. Real positions and densities will create other possible numbers of prime pairs, but I have now proven that no possible density or position can create zero prime pairs. Lower and lower densities of primes will create higher and higher densities of NPO's, and this will always force the N + N sums to invade the expected N + P sums, turning at least one of them into a P + P.

One thing this does is match our findings from the probability calculations, and shows that I have simply extended those odds into a non-probability solution. Our probabilities told us that no matter how low we took our density of primes, the odds were very strongly in favor of finding at least one P + P. This was because the odds were even more strongly against covering all primes with non-primes.

At any rate, I turned the probability fractions into usable fractions by looking at densities and minimum values. These minimum values provide a general proof without having to look at actual sums or the real distribution of primes. I could avoid all contact with prime generators, as well as esoteric fields or non-linear maths. Although it is possible there are other more complex solutions, I believe this is the simplest possible solution to Goldbach's Conjecture.

So, the general rule is that your total density of NPO + NPO sums to all sums cannot be less than half your total density of NPO's. It therefore cannot be less than the prime density. But to cover all primes, you must create a fraction of NPO + NPO sums that is less than the prime density. This being disallowed, one of the NPO + P sums must switch to an NPO + NPO sum. Since the total number of NPO's and P's is fixed for each given situation, to create the extra NPO + P sum requires the creation of a P + P sum.