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The New Prime Theorem（13）

$n \times a^n \pm 1$ and $n \times 2^n \pm 1$

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Abstract

Using the Jiang function we prove that $n \times a^n \pm 1$ have infinitely many prime solutions and $n \times 2^n \pm 1$ have finite prime solutions.

Theorem. We define the irreducible prime equation

$$P_1 = n \times (P-1)^n + 1 \qquad (1)$$

For every positive integer n there exist infinitely many primes P such that P_1 is a prime.

Proof. We have the Jiang function[1]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)], \qquad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$n \times (q-1)^n + 1 \equiv 0 \pmod{P}, \quad q = 1, \cdots, P-1. \qquad (3)$$

From (3) we have that if $n = 3b+2$ then $\chi(3) = 1$, $\chi(3) = 0$ otherwise, $\chi(P) < P-1$. We have

$$J_2(\omega) \neq 0. \qquad (4)$$

We prove that there exist inifinitely many primes P such that P_2 is a prime.

We have asymptotic formula [1]

$$\pi_2(N,2) = \left|\{P \le N : n \times (P-1)^n + 1 = prime\}\right| \sim \frac{J_2(\omega)\omega}{n\phi^2(\omega)} \frac{N}{\log^2 N} \qquad (5)$$

where $\phi(\omega) = \prod_P (P-1)$.

Let $P = 3$. From (1) we have Cullen equation

$$P_1 = n \times 2^n + 1 \qquad (6)$$

From (5) we have

$$\pi_2(3,2) \sim \frac{J_2(\omega)}{n\phi^2(\omega)} \frac{3}{\log^2 3} \to 0 \text{ as } n \to \infty \qquad (7)$$

We prove the finite Cullen primes.

In the same way we are able to prove that $n \times a^n - 1$ has infinitely many prime solutions, $n \times 2^n - 1$ has definite prime solutions and $h \times 2^n \pm 1$ have finite prime solutions.

Reference

[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution.

http://www. wbabin.net/math /xuan2. pdf.