

The New Prime Theorem (11)

$$2 \times a^2 \pm 1$$

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Abstract

Using the Jiang function we prove that $2 \times a^2 \pm 1$ has infinitely many prime solutions

Theorem. We define the prime equation

$$P_1 = 2 \times (P - 1)^2 - 1 \quad (1)$$

There exist infinitely many primes P such that P_2 is a prime.

Proof. We have the Jiang function[1]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)], \quad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$2 \times (q - 1)^2 - 1 \equiv 0 \pmod{P}, \quad q = 1, \dots, P - 1 \quad (3)$$

From (3) we have $\left(\frac{2}{P}\right) = (-1)^{\frac{P^2-1}{8}}$, if $\left(\frac{2}{P}\right) = 1$ then $\chi(P) = 2$, if $\left(\frac{2}{P}\right) = -1$ then $\chi(P) = 0$.

Substituting it into (2) we have

$$J_2(\omega) = \prod_{3 \leq P} \left[P - 2 - (-1)^{\frac{P^2-1}{8}} \right] \neq 0 \quad (4)$$

We prove there exist infinitely many primes P such that P_2 is a prime.

We have the asymptotic formula [1]

$$\pi_2(N, 2) = \left| \left\{ P \leq N : 2 \times (P - 1)^2 - 1 = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega}{2\phi^2(\omega)} \frac{N}{\log^2 N} \quad (5)$$

where $\phi(\omega) = \prod_p (P - 1)$.

In the same way we are able to prove that $2 \times a^2 + 1$ has infinitely many prime solutions.

Reference

[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution.

<http://www.wbabin.net/math/xuan2.pdf>.