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The New Prime theorem (10)

There are finite Mersenne primes

and

There are finite repunits primes

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Abstract

Using the Jiang function, we prove the finite Mersenne primes and the finite repunits primes.

Theorem. Suppose the prime equation

$$P_1 = \frac{(P-1)^{P_0}-1}{P-2}. \qquad (1)$$

where P_0 is a given prime.

There exist infinitely many primes P such that P_1 is a prime.

Proof. We have the Jiang function[1]

$$J_2(\omega) = \prod_P [P-1-\chi(P)], \qquad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\frac{(q-1)^{P_0}-1}{q-2} \equiv 0 \pmod P, \quad q = 1, \cdots, P-1. \qquad (3)$$

$\chi(P_0) = 1$, $\chi(P) = P_0 - 1$ if $P \equiv 1 \pmod{P_0}$, $\chi(P) = 0$ otherwise.

Since $J_2(\omega) \neq 0$, there exist infinitely many primes P such that P_1 is a prime.

We have the asymptotic formula [1]

$$\pi_2(N,2) = \left|\{P \le N : P_1 = prime\}\right| \sim \frac{1}{P_0 - 1} \frac{J_2(\omega)\omega}{\phi^2(\omega)} \frac{N}{\log^2 N}. \qquad (4)$$

where $\phi(\omega) = \prod_P (P-1)$.

Let $P = 3$. From (1) we have equation of Mersenne numbers [2]

$$P_1 = 2^{P_0} - 1. \qquad (5)$$

From (4) we have

$$\pi_2(3,2) = \left|\{3 \le N : 2^{P_0} - 1 = prime\}\right| \sim \frac{1}{P_0 - 1} \frac{J_2(\omega)\omega}{\phi^2(\omega)} \frac{3}{\log^2 3} \to 0 \text{ as } P_0 \to \infty \qquad (6)$$

We prove the finite Mersenne primes.

Let $P = 11$. From (1) we have equation of repunits numbers [2]

$$P_1 = \frac{10^{P_0} - 1}{9}. \qquad (7)$$

From (4) we have

$$\pi_{11}(11,2) = \left|\left\{11 \le N : \frac{10^{P_0} - 1}{9} = prime\right\}\right| \sim \frac{1}{P_0 - 1} \frac{J_2(\omega)\omega}{\phi^2(\omega)} \frac{11}{\log^2 11} \to 0 \text{ as } P_0 \to \infty. \qquad (8)$$

We prove the finite repunits primes.

In the same way we are able to prove that $(a^{P_0} - 1)/(a-1)$ with $a = 4,6,10,12,\cdots$, has the finite prime solutions.

Reference

[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution.

http://www. wbabin.net/math /xuan2. pdf.

[2] P. Ribenboim, The new book of prime number records, 3rd edition, spring-Verlag, New York, NY, 1995.