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The New Prime theorem（8）

$x^6 + 1091$ has no prime solutions

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Abstract

Using the Jiang function we prove that $x^6 + 1091$ has no prime solutions.

Shanks conjectured[1,2]:

Table 52.

$f(x)$	$f(m)$ is composite for all m up to
$x^6 + 1091$	3905
$x^6 + 82991$	7979
$x^{12} + 4094$	170624
$x^{12} + 488669$	616979

The smallest prime value of the last polynomial has no less than 70 digits.

Theorem 1.

$$(P+1)^6 + 1091 \qquad (1)$$

has no prime solutions

Proof. We have the Jiang function[3]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)], \qquad (2)$$

where $\omega = \prod_P P$,

$\chi(P)$ is the number of solutions of congruence

$$(q+1)^6 + 1091 \equiv 0 \pmod P \qquad (3)$$

$q = 1, \cdots, P-1$.

From (3) we have $\chi(2)=0$, $\chi(3)=2$, $\chi(5)=2$, $\chi(7)=6$.

Substituting it into (2) we have

$$J_2(3)=0,\ J_2(7)=0.$$

We have prove that (1) has no prime solutions.

In the same way we prove that $x^6+82991$ has no prime solutions.

Theorem 2.

$$P^{12}+4094 \qquad (4)$$

has no prime solutions

Proof. We have the Jiang function [3]

$$J_2(\omega)=\prod_P[P-1-\chi(P)], \qquad (5)$$

$\chi(P)$ is the number of solutions of congruence

$$q^{12}+4094\equiv 0 \pmod{P}, \qquad (6)$$

$q=1,\cdots,P-1$.

From (6) we have

$$\chi(5)=4,\quad \chi(13)=12 \qquad (7)$$

Substituting it into (5) we have $J_2(5)=0, J_2(13)=0$. We prove (4) has no prime solutions.

In the same way we are able to prove $x^{12}+488669$ has no prime solutions

Reference

[1] D. Shanks, A low density of primes. J. Recr. Math., 4(1971)272-275.

[2] P. Ribenboim, The New book of prime Number Records, 3rd edition Springer-Verlag, New York, NY, 1995, pp401.

[3] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution.

http://www. wbabin.net/math /xuan2. pdf.