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The New Prime theorem（1）

$P_2 = aP_1 + b$

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Abstract

Using the Jiang function we prove that there exist infinitely many primes P_1 such that $aP_1 + b$ is prime.

Theorem

$$P_2 = aP_1 + b. (a,b) = 1, \quad 2|ab, \qquad (1)$$

There exist infinitely many primes P_1 such that P_2 is prime.

Proof. We have the Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)], \qquad (2)$$

where

$$\omega = \prod_P P,$$

$\chi(P)$ is the number of solutions of congruence

$$aq + b \equiv 0 \pmod{P}, \qquad (3)$$

$q = 1, \cdots, P - 1$.

From (3) we have if $P|ab$ then $\chi(P) = 0$, $\chi(P) = 1$ otherwise.

From (3) and (2) we have

$$J_2(\omega) = \prod_{3 \le P} (P - 2) \prod_{P|ab} \frac{P - 1}{P - 2} \neq 0. \qquad (4)$$

We prove that there exist infinitely many primes P_1 such that P_2 is prime.

We have the best asymptotic formula [1, 2]

$$\pi_2(N,2)=\left|\{P_1\le N: aP_1+b=prime\}\right|\sim\frac{J_2(\omega)\omega}{\phi^2(\omega)}\frac{N}{\log^2 N}$$

$$=2\prod_{3\le P}\left(1-\frac{1}{(P-1)^2}\right)\prod_{P|ab}\frac{P-1}{P-2}\frac{N}{\log^2 N}. \qquad (5)$$

where $\phi(\omega)=\prod_P(P-1)$.

Twin primes theorem [1]. Let $a=1$ and $b=2$. From (1) we have

$$P_2=P_1+2 \qquad (6)$$

From (4) we have

$$J_2(\omega)=\prod_P(P-2)\ne 0 \qquad (7)$$

We prove that there exist infinitely many primes P_1 such that P_1+2 is prime.

From (5) we have

$$\pi_2(N,2)=\left|\{P_1\le N: P_1+2=prime\}\right|\sim 2\prod_{3\le P}\left(1-\frac{1}{(P-1)^2}\right)\frac{N}{\log^2 N}. \qquad (8)$$

Goldbach theorem [1]. Let $a=-1$ and $b=N$. From (1) we have

$$N=P_1+P_2 \qquad (9)$$

From (4) we have

$$J_2(\omega)=\prod_{3\le P}(P-2)\prod_{P|N}\frac{P-1}{P-2}\ne 0 \qquad (10)$$

We prove that every even number $N\ge 6$ is the sum of two primes.

From (5) we have

$$\pi_2(N,2)=\left|\{P_1\le N: N-P=prime\}\right|\sim 2\prod_{3\le P}\left(1-\frac{1}{(P-1)^2}\right)\prod_{P|N}\frac{P-1}{P-2}\frac{N}{\log^2 N} \qquad (11)$$

Reference

[1] Chun-Xuan Jiang, On the Yu-Goldbach prime theorem (Chinese), Guangxi Science, 3 (1996) 9-12.

[2] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. (http://www.wbabin.net/math/xuan2.pdf) (http://vixra.org/pdf/0812.0004v2.pdf)