

## New prime K-tuple theorem (5)

$$P_1, P_2, jP_1 + (j+1)P_2 (j = 1, \dots, k)$$

Chun-Xuan Jiang

P. O. Box 3924, Beijing 100854, P. R. China

[Jiangchunxuan@vip.sohu.com](mailto:Jiangchunxuan@vip.sohu.com)

### Abstract

Using the Jiang function, we prove that for every positive integer  $k$  there exist infinitely many primes  $P_1$  and  $P_2$  such that each of  $jP_1 + (j+1)P_2$  is prime.

### Theorem

$$P_1, P_2, jP_1 + (j+1)P_2 (j = 1, \dots, k). \quad (1)$$

For every positive integer  $k$  there exist infinitely many primes  $P_1$  and  $P_2$  such that each of  $jP_1 + (j+1)P_2$  is prime.

**Proof.** We have the Jiang function [1, 2]

$$J_3(\omega) = \prod_p [(P-1)^2 - \chi(P)], \quad (2)$$

where  $\omega = \prod_p P$ ,

$\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^k [jq_1 + (j+1)q_2] \equiv 0 \pmod{P}, \quad (3)$$

$q_i = 1, \dots, P-1, i = 1, 2, \dots$

From (3) we have

If  $P \leq k+1$  then  $\chi(P) = (P-1)(P-2)$ , if  $k+1 < P$  then  $\chi(P) = k(P-1)$ .

From (3) and (2) we have

$$J_3(\omega) = \prod_{P=3}^{P \leq k+1} (P-1) \prod_{k+1 < P} [(P-1)^2 - k(P-1)] \neq 0. \quad (4)$$

We prove that for every positive integer  $k$  there exist infinitely many primes  $P_1$  and

$P_2$  such that each of  $jP_1 + (j+1)P_2$  is prime.

We have the best asymptotic formula [1, 2]

$$\pi_{k+1}(N, 3) = \left| \left\{ P_1, P_2 \leq N : jP_1 + (j+1)P_2 = \text{prime} \right\} \right| \sim \frac{J_3(\omega) \omega^k}{\phi^{k+2}(\omega)} \frac{N^2}{\log^{k+2}}, \quad (5)$$

where  $\phi(\omega) = \prod_p (P-1)$ .

### References

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