

New prime K -tuple theorem (4)

$$P, P + (2j)^2 (j = 1, \dots, k)$$

Chun-Xuan Jiang

P. O. Box 3924, Beijing 100854, P. R. China

Jiangchunxuan@vip.sohu.com

Abstract

Using the Jiang function, we prove that for every positive integer k there exist infinitely many primes P such that each of $P + (2j)^2$ is prime.

Theorem

$$P, P + (2j)^2 (j = 1, \dots, k). \quad (1)$$

For every positive integer k there exist infinitely many primes P such that each of $P + (2j)^2$ is prime.

Proof. We have the Jiang function [1]

$$J_2(\omega) = \prod_p (P - 1 - \chi(P)), \quad (2)$$

where $\omega = \prod_p P$,

$\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^k [q + (2j)^2] \equiv 0 \pmod{P}, \quad (3)$$

where $q = 1, \dots, P - 1$.

From (3) we have

If $P < 2k$ then $\chi(P) = (P - 1)/2$, if $2k < P$ then $\chi(P) = k$.

From (3) and (2) we have

$$J_2(\omega) = \prod_{P=3}^{P < 2k} \frac{P-1}{2} \prod_{2k < P} (P-1-k) \neq 0. \quad (4)$$

We prove that for every positive integer k there exist infinitely many primes P such that each of $P + (2j)^2$ is prime.

We have the best asymptotic formula [1]

$$\pi_{k+1}(N, 2) = \left| \left\{ P \leq N : P + (2j)^2 = \text{prime} \right\} \right| \sim \frac{J_2(\omega) \omega^k}{\phi^{k+1}(\omega)} \frac{N}{\log^{k+1} N}. \quad (5)$$

The author took a day to write this paper.

References

- [1] Chun-Xuan Jiang, Foundations of Santilli's isonumber theory with applications to new cryptograms, Fermat's theorem and Goldbach's conjecture. Inter. Acad. Press, 2002, MR2004c:110011, (<http://www.i-b-r.org/docs/jiang.pdf>) (<http://www.wbabin.net/math/xuan13.pdf>)