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# New prime K-tuple theorem (3)

$$P, jP+j+1 (j=1,\cdots,k)$$

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**Abstract**

Using the Jiang function, we prove that for every positive integer $k$ there exist infinitely many primes $P$ such that each of $jP+j+1$ is prime.

**Theorem**

$$P, jP+j+1 (j=1,\cdots,k). \qquad (1)$$

For every psitive integer $k$ there exist infinitely many primes $P$ such that each of $jP+j+1$ is prime.

**Proof**. We have Jiang function [1, 2]

$$J_2(\omega)=\prod_P (P-1-\chi(P)), \qquad (2)$$

where $\omega=\prod_P P$,

$\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k}(jq+j+1)\equiv 0 \pmod P, \qquad (3)$$

where $q=1,\cdots,P-1$.

From (3) we have

If $P\le k+1$ then $\chi(P)=P-2$, if $k+1<P$ then $\chi(P)=k$.

From (3) and (2) we have

$$J_2(\omega)=\prod_{k+1<P}(P-k-1)\neq 0. \qquad (4)$$

We prove that for every positive integer $k$ there exist infinitely many primes $P$ such that each of $jP+j+1$ is prime.

We have the best asymptotic formula [1, 2]

$$\pi_{k+1}(N,2) = \left|\{ P \le N : jP + j + 1 = prime\}\right| \sim \frac{J_2(\omega)\omega^k}{\phi^{k+1}(\omega)} \frac{N}{\log^{k+1} N}. \qquad (5)$$

The author took a day to write this paper.

**References**


[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution.

(http://www.wbabin.net/math/xuan2.pdf) (http://vixra.org/pdf/0812.0004v2.pdf)

[2] Chun-xuan Jiang, The Hardy-Littlewood prime $k$-tuple conjecture is false. http:// www. wbabin.net/math/xuan77.pdf