

New Prime K -tuple Theorem(1)

$$P_1, P_2, P_1 + jP_2 + j (j = 1, \dots, k)$$

and

$$P_1, P_2, P_1 + jP_2 - j (j = 1, \dots, k)$$

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Abstract

Using the Jiang function, we prove that there exist infinitely many primes P_1 and P_2 such that each of $P_1 + jP_2 + j$ is prime and there exist infinitely many primes P_1 and P_2 such that each of $P_1 + jP_2 - j$ is prime.

Theorem 1.

$$P_1, P_2, P_1 + jP_2 + j (j = 1, \dots, k) \quad (1)$$

There exist infinitely many primes P_1 and P_2 such that each of $P_1 + jP_2 + j$ is prime.

Proof. We have the Jiang function [1, 2]

$$J_3(\omega) = \prod_p [(P-1)^2 - \chi(P)] \quad (2)$$

where $\omega = \prod_p P$,

$\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^k (q_1 + jq_2 + j) \equiv 0 \pmod{P}, \quad q_i = 1, \dots, P-1, i = 1, 2. \quad (3)$$

From (3) we have

If $k < P$ then $\chi(P) = k(P-2)$. If $P \leq k$ then $\chi(P) = (P-1)(P-2)$.

From (3) and (2) we have

$$J_3(\omega) = \prod_{P=3}^{P \leq k} (P-1) \prod_{k < P} [(P-1)^2 - k(P-2)] \neq 0 \quad (4)$$

For any positive integer k there exist infinitely many primes P_1 and P_2 such that each

of $P_1 + jP_2 + j$ is prime.

We have the asymptotic formula [1, 2]

$$\pi_{k+1}(N, 3) = \left| \{ P_1, P_2 \leq N : P_1 + jP_2 + j = \text{prime} \} \right| \sim \frac{J_3(\omega) \omega^k}{\phi^{k+2}(\omega)} \frac{N^2}{\log^{k+2} N}, \quad (5)$$

where $\phi(\omega) = \prod_p (P-1)$.

Example 1.

$$P_1, P_2, P_1 + P_2 + 1. \quad (6)$$

From (4) we have

$$J_3(\omega) = \prod_{3 \leq P} [(P-1)^2 - (P-2)] \neq 0 \quad (7)$$

From (5) we have

$$\pi_2(N, 3) \sim 2 \prod_{3 \leq P} \left(1 + \frac{1}{(P-1)^3} \right) \frac{N^2}{\log^3 N}. \quad (8)$$

Example 2.

$$P_1, P_2, P_1 + P_2 + 1, P_1 + 2P_2 + 2. \quad (9)$$

From (4) we have

$$J_3(\omega) = \prod_{3 \leq P} [(P-1)^2 - 2(P-2)] \neq 0 \quad (10)$$

From (5) we have

$$\pi_3(N, 3) \sim \frac{J_3(\omega) \omega^2}{\phi^4(\omega)} \frac{N^2}{\log^4 N}. \quad (11)$$

Example 3.

$$P_1, P_2, P_1 + jP_2 + j (j = 1, \dots, 8). \quad (12)$$

From (4) we have

$$J_3(\omega) = 48 \prod_{11 \leq P} [(P-1)^2 - 8(P-2)] \neq 0. \quad (13)$$

From (5) we have

$$\pi_9(N, 3) \sim \frac{J_3(\omega) \omega^8}{\phi^{10}(\omega)} \frac{N^2}{\log^{10} N}. \quad (14)$$

Theorem 2.

$$P_1, P_2, P_1 + jP_2 - j (j = 1, \dots, k), \quad (15)$$

we have Jiang function

$$J_3(\omega) = \prod_{P=3}^{P \leq k} (P-1) \prod_{k < P} [(P-1)^2 - k(P-2)] \neq 0. \quad (16)$$

For any positive integer k there exist infinitely many primes P_1 and P_2 such that each of $P_1 + jP_2 - j$ is prime.

we have the asymptotic formula

$$\pi_{k+1}(N, 3) = \left| \left\{ P_1, P_2 \leq N : P_1 + jP_2 - j = \text{prime} \right\} \right| \sim \frac{J_3(\omega) \omega^k}{\phi^{k+2}(\omega)} \frac{N^2}{\log^{k+2} N} . \quad (17)$$

References

- [1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution.
<http://www.wbabin.net/math/xuan2.pdf> (<http://vixra.org/pdf/0812.0004v2.pdf>)
- [2] Chun-Xuan Jiang, The Hardy-Littlewood prime k -tuple conjecture is false.
<http://www.wbabin.net/math/xuan77.pdf>