

Gaps Among Products of m Primes

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Abstract

Using Jiang function $J_2(\omega)$ we prove gaps among products of m prime:

$$d(x) = d(x+1) = d(x+5-3) = d(x+7-3) = \cdots = d(x+P_n-3) = m > 1 \text{ infinitely-often,}$$

where P_n denotes the n -th prime.

Theorem 1. Let $P_3 = 5$, gaps among products of two primes

$$d(x) = d(x+1) = d(x+2) = 2 \text{ infinitely-often.} \quad (1)$$

where $d(x)$ represents the number of distinct prime factors of x , $d(x) = \sum_{P|x} 1$, $d(3) = 1$,

$$d(15) = 2, \quad d(105) = 3.$$

Proof (see[1] p.146 theorem 3.1.154). Prime equations are

$$\beta_1 = 10\alpha + 1, \quad \beta_2 = 15\alpha + 2, \quad \beta_3 = 6\alpha + 1 \quad (2)$$

We have Jiang function

$$J_2(\omega) = 3 \prod_{7 < P} (P-4) \neq 0, \quad (3)$$

where $\omega = \prod_{2 \leq P}$

We prove that $J_2(\omega) \neq 0$ there exist infinitely many odd integers α such that β_1 , β_2 and β_3 are primes.

We have asymptotic formula

$$\left| \{ \alpha \leq N : 10\alpha + 1, 15\alpha + 2, 6\alpha + 1 \} \right| > \frac{J_2(\omega)\omega}{\phi^4(\omega)} \frac{N}{\log^4 N}, \quad (4)$$

where $\phi(\omega) = \prod_{2 \leq P} (P-1)$.

From (2) we have

$$\begin{aligned} 3\beta_1 &= 30\alpha + 3, \\ 3\beta_1 + 1 &= 30\alpha + 4 = 2(15\alpha + 2) = 2\beta_2, \\ 3\beta_1 + 2 &= 30\alpha + 5 = 5(6\alpha + 1) = 5\beta_3 \end{aligned} \quad (5)$$

From (5) we prove

$$d(3\beta_1) = d(3\beta_1 + 1) = d(3\beta_1 + 2) = 2 \text{ infinitely-often.} \quad (6)$$

We prove that there exist infinitely many triples of consecutive integers, each being the products of two distinct primes.

Theorem 2. Let $P_3 = 5$, gaps among products of m primes.

$$d(x) = d(x+1) = d(x+2) = m > 1 \text{ infinitely-often} \quad (7)$$

Proof (see [1] p.148, theorem 3.1.158). Suppose that $u, u+1$ and $u+2$ are three consecutive integers, each being the products of $m-1$ distinct primes. Let $M = u(u+1)(u+2)$. We define the three prime equations

$$\beta_1 = \frac{2M}{u}\alpha + 1, \quad \beta_2 = \frac{2M}{u+1}\alpha + 1, \quad \beta_3 = \frac{2M}{u+2}\alpha + 1 \quad (8)$$

Using Jiang function $J_2(\omega)$ we can prove that there exist infinitely many integers α such that β_1, β_2 and β_3 are primes.

From (8) we have

$$u\beta_1 = 2M\alpha + u$$

$$u\beta_1 + 1 = 2M\alpha + u + 1 = (u+1)\left(\frac{2M}{u+1}\alpha + 1\right) = (u+1)\beta_2$$

$$u\beta_1 + 2 = 2M\alpha + u + 2 = (u+2)\left(\frac{2M}{u+2}\alpha + 1\right) = (u+2)\beta_3 \quad (9)$$

We prove

$$d(x) = d(x+1) = d(x+2) = m > 1 \text{ infinitely-often.} \quad (10)$$

Theorem 3. Let $P_4 = 7$, gaps among products of two primes.

$$d(x) = d(x+2) = d(x+4) = 2 \text{ infinitely-often.} \quad (11)$$

Proof [1,2,3]. Prime equations are

$$\beta_1 = 70\alpha + 1, \quad \beta_2 = 42\alpha + 1, \quad \beta_3 = 30\alpha + 1 \quad (12)$$

Using Jiang function $J_2(\omega)$ [1] we can prove that there exist infinitely many integers α such that β_1, β_2 and β_3 are primes.

From (12) we have

$$3\beta_1 = 210\alpha + 3,$$

$$3\beta_1 + 2 = 210\alpha + 5 = 5(42\alpha + 1) = 5\beta_2$$

$$3\beta_1 + 4 = 210\alpha + 7 = 7(30\alpha + 1) = 7\beta_3 \quad (13)$$

We prove

$$d(3\beta_1) = d(3\beta_2 + 2) = d(3\beta_1 + 4) = 2 \text{ infinitely-often.} \quad (14)$$

Theorem 4. Let $P_4 = 7$, gaps among products of m primes.

$$d(x) = d(x + 2) = d(x + 4) = m > 1 \text{ infinitely-often.} \quad (15)$$

Proof [1, 2, 3]. Suppose that $u, u + 2$ and $u + 4$ are three odd integers, each being the products of $m - 1$ distinct primes. Let $M = u(u + 2)(u + 4)$

We define three prime equations

$$\beta_1 = \frac{2M}{u}\alpha + 1, \quad \beta_2 = \frac{2M}{u+2}\alpha + 1, \quad \beta_3 = \frac{2M}{u+4}\alpha + 1 \quad (16)$$

Using Jiang function $J_2(\omega)$ [1] we can prove that there exist infinitely many integers α such that β_1, β_2 and β_3 are primes.

From (16) we have

$$u\beta_1 = 2M\alpha + u,$$

$$u\beta_1 + 2 = 2M\alpha + u + 2 = (u + 2)\left(\frac{2M}{u+2}\alpha + 1\right) = (u + 2)\beta_2,$$

$$u\beta_1 + 4 = 2M\alpha + u + 4 = (u + 4)\left(\frac{2M}{u+4}\alpha + 1\right) = (u + 4)\beta_3. \quad (17)$$

We prove

$$d(x) = d(x + 2) = d(x + 4) = m > 1 \text{ infinitely-often.} \quad (18)$$

Theorem 5. Let $P_4 = 7$, gaps among products of m primes.

$$d(x) = d(x + 1) = d(x + 2) = d(x + 4) = m > 1 \text{ infinitely-often.} \quad (19)$$

Proof. From (12) we have prime equations

$$\beta_1 = 70\alpha + 1, \quad \beta_2 = 105\alpha + 2, \quad \beta_3 = 42\alpha + 1, \quad \beta_4 = 30\alpha + 1 \quad (20)$$

Using Jiang function $J_2(\omega)$ [1] we can prove there exist infinitely many odd integers α such that $\beta_1, \beta_2, \beta_3$ and β_4 are primes

From (20) we have

$$3\beta_1 = 210\alpha + 3$$

$$3\beta_1 + 1 = 210\alpha + 4 = 2(105\alpha + 2) = 2\beta_2$$

$$3\beta_1 + 2 = 210\alpha + 5 = 5(42\alpha + 1) = 5\beta_3$$

$$3\beta_1 + 4 = 210\alpha + 7 = 7(30\alpha + 1) = 7\beta_4. \quad (21)$$

We prove

$$d(3\beta_1) = d(3\beta_1 + 1) = d(3\beta_1 + 2) = d(3\beta_1 + 4) = 2 \text{ infinitely-often.} \quad (22)$$

Using Jiang function we can prove that

$$d(x) = d(x + 1) = d(x + 2) = d(x + 4) = m > 1 \text{ infinitely-often.} \quad (23)$$

Theorem 6. Gaps among products of m primes.

$$d(x) = d(x + 1) = d(x + 5 - 3) = d(x + 7 - 3) = \dots = d(x + P_n - 3) = m > 1 \text{ infinitely-often.}$$

(24)

where P_n denotes the n -th prime.

Proof. Let $\omega_n = \prod_{2 \leq P \leq P_n} P$. We define the prime equations

$$\beta_1 = \frac{\omega_n}{3}\alpha + 1, \beta_2 = \frac{\omega_n}{2}\alpha + 2, \beta_3 = \frac{\omega_n}{5}\alpha + 1, \beta_4 = \frac{\omega_n}{7}\alpha + 1, \dots, \beta_n = \frac{\omega_n}{P_n}\alpha + 1. \quad (25)$$

Using Jiang function $J_2(\omega)$ [1] we can prove that there exist infinitely many odd integers α such that $\beta_1, \beta_2, \dots, \beta_n$ are primes.

From (25) we have

$$\begin{aligned} 3\beta_1 &= \omega_n\alpha + 3, \\ 3\beta_1 + 1 &= \omega_n\alpha + 4 = 2\left(\frac{\omega_n}{2}\alpha + 2\right) = 2\beta_2, \\ 3\beta_1 + 2 &= \omega_n\alpha + 5 = 5\left(\frac{\omega_n}{5}\alpha + 1\right) = 5\beta_3, \\ 3\beta_1 + 4 &= \omega_n\alpha + 7 = 7\left(\frac{\omega_n}{7}\alpha + 1\right) = 7\beta_4, \\ &\dots\dots \\ 3\beta_1 + P_n - 3 &= \omega_n\alpha + P_n = P_n\left(\frac{\omega_n}{P_n}\alpha + 1\right) = P_n\beta_n. \end{aligned} \quad (26)$$

From (26) we have

$$d(3\beta_1) = d(3\beta_1 + 1) = d(3\beta_1 + 2) = d(3\beta_1 + 4) = \dots = d(3\beta_1 + P_n - 3) = 2 \quad \text{infinitely-often.} \quad (27)$$

Using Jiang function $J_2(\omega)$ [1] we can prove that

$$d(x) = d(x+1) = d(x+5-3) = d(x+7-3) = \dots = d(x+P_n-3) = m > 1 \quad \text{infinitely-often.} \quad (28)$$

Theorem 7. Gaps between products of two primes.

$$d(x) = d(x+3) = 2 \quad \text{infinitely-often} \quad (29)$$

Proof. We define prime equations

$$\beta_1 = 30\alpha + 1, \quad \beta_2 = 12\alpha + 1. \quad (30)$$

Using Jiang function we can prove that there exist infinitely many integers α such that β_1 , and β_2 are primes.

From (30) we have

$$\begin{aligned} 2\beta_1 &= 60\alpha + 2, \\ 2\beta_1 + 3 &= 60\alpha + 5 = 5(12\alpha + 1) = 5\beta_2 \end{aligned} \quad (31)$$

From (31) we prove

$$d(2\beta_1) = d(2\beta_1 + 3) = 2 \quad \text{infinitely-often.} \quad (32)$$

Theorem 8. Gaps between products of m primes.

$$d(x) = d(x+3) = m > 1 \text{ infinitely-often} \quad (33)$$

Proof. Suppose that u and $u+3$ are two integers, each being the products of $m-1$ distinct primes.

Let $M = 6u(u+3)$. We define prime equations

$$\beta_1 = \frac{M}{u}\alpha + 1, \quad \beta_2 = \frac{M}{u+3}\alpha + 1 \quad (34)$$

Using Jiang function we can prove that there exist infinitely many integers α such that β_1 , and β_2 are primes.

From (34) we have

$$\begin{aligned} u\beta_1 &= M\alpha + u, \\ u\beta_1 + 3 &= M\alpha + u + 3 = (u+3)\left(\frac{M}{u+3}\alpha + 1\right) = (u+3)\beta_2 \end{aligned} \quad (35)$$

From (35) we prove

$$d(x) = d(x+3) = m > 1 \text{ infinitely-often.} \quad (36)$$

Theorem 9. Gaps between products of two primes.

$$d(x) = d(x+6) = 2 \text{ infinitely-often} \quad (37)$$

Proof. We define prime equations

$$\beta_1 = 66\alpha + 1, \quad \beta_2 = 30\alpha + 1. \quad (38)$$

Using Jiang function we can prove that there exist infinitely many integers α such that β_1 , and β_2 are primes.

From (38) we have

$$\begin{aligned} 5\beta_1 &= 330\alpha + 5, \\ 5\beta_1 + 6 &= 330\alpha + 11 = 11(30\alpha + 1) = 11\beta_2 \end{aligned} \quad (39)$$

From (39) we prove

$$d(5\beta_1) = d(5\beta_1 + 6) = 2 \text{ infinitely-often.} \quad (40)$$

Theorem 10. Gaps between products of m primes.

$$d(x) = d(x+6) = m > 1 \text{ infinitely-often} \quad (41)$$

Proof. Suppose that u and $u+6$ are two integers, each being the products of $m-1$ distinct primes. Let $M = 30u(u+6)$. We define prime equations

$$\beta_1 = \frac{M}{u}\alpha + 1, \quad \beta_2 = \frac{M}{u+6}\alpha + 1 \quad (42)$$

Using Jiang function we can prove that there exist infinitely many integers α such that β_1 , and β_2 are primes.

From (42) we have

$$\begin{aligned} u\beta_1 &= M\alpha + u, \\ u\beta_1 + 6 &= M\alpha + u + 6 = (u+6)\left(\frac{M}{u+6}\alpha + 1\right) = (u+6)\beta_2 \end{aligned} \quad (43)$$

From (43) we prove

$$d(x) = d(x+6) = m > 1 \text{ infinitely-often.} \quad (44)$$

Theorem 11. Gaps between products of two primes.

$$d(x) = d(x+5040) = 2 \text{ infinitely-often} \quad (45)$$

Proof. Suppose $M = 2 \times 3 \times 11 \times 5051$. We define prime equations

$$\beta_1 = \frac{M}{11}\alpha + 1, \quad \beta_2 = \frac{M}{5051}\alpha + 1. \quad (46)$$

Using Jiang function we can prove that there exist infinitely many integers α such that β_1 and β_2 are primes.

From (46) we have

$$\begin{aligned} 11\beta_1 &= M\alpha + 11, \\ 11\beta_1 + 5040 &= M\alpha + 5051 = 5051\left(\frac{M}{5051}\alpha + 1\right) = 5051\beta_2 \end{aligned} \quad (47)$$

From (47) we prove that

$$d(x) = d(x+5040) = 2 \text{ infinitely-often.} \quad (48)$$

Using Jiang function we can prove that

$$d(x) = d(x+5040) = m > 1 \text{ infinitely-often} \quad (49)$$

Theorem 12. Gaps between products of two primes.

We study general solutions of

$$d(x) = d(x \pm 5040) = 2 \text{ infinitely-often} \quad (50)$$

Proof. We define a prime equation

$$P_2 = P_1 \pm 5040. \quad (51)$$

Using Jiang function $J_2(\omega)$ we can prove that there exist infinitely many prime P_1 such that P_2 is a prime.

From (51) suppose $M = 2 \times 3 \times P_1 \times P_2$. We define prime equations

$$\beta_1 = \frac{M}{P_1}\alpha + 1, \quad \beta_2 = \frac{M}{P_2}\alpha + 1 \quad (52)$$

Using Jiang function $J_2(\omega)$ we can prove that there exist infinitely many integers α such that β_1 and β_2 are primes.

From (52) we have

$$\begin{aligned} P_1\beta_1 &= M\alpha + P_1, \\ P_1\beta_1 \pm 5040 &= M\alpha + P_1 \pm 5040 = M\alpha + P_2 = P_2\left(\frac{M}{P_2}\alpha + 1\right) = P_2\beta_2. \end{aligned} \quad (53)$$

We prove that

$$d(x) = d(x \pm 5040) = 2 \text{ infinitely-often.} \quad (54)$$

Theorem 13. Gaps between products of two primes.

$$d(x) = d(x+5) = 2 \text{ infinitely-often} \quad (55)$$

Using $2 \times 43 + 5 = 7 \times 13$ we define prime equations,

$$\beta_1 = 39\alpha + 4, \quad \beta_2 = 6\alpha + 1 \quad (56)$$

From (56) we have

$$\begin{aligned} 2\beta_1 &= 78\alpha + 8, \\ 2\beta_1 + 5 &= 78\alpha + 13 = 13(6\alpha + 1) = 13\beta_2. \end{aligned} \quad (57)$$

We redefine prime equations

$$\beta_1 = 42\alpha + 1, \quad \beta_2 = 12\alpha + 1 \quad (58)$$

From (58) we have

$$\begin{aligned} 2\beta_1 &= 84\alpha + 2, \\ 2\beta_1 + 5 &= 84\alpha + 7 = 7(12\alpha + 1) = 7\beta_2. \end{aligned} \quad (59)$$

Theorem 14. Gaps between products of two primes.

$$d(x) = d(x+7) = 2 \text{ infinitely-often} \quad (60)$$

Using $2 \times 29 + 7 = 5 \times 13$ we define prime equations,

$$\beta_1 = 30\alpha - 1, \quad \beta_2 = 12\alpha + 1 \quad (61)$$

From (61) we have

$$\begin{aligned} 2\beta_1 &= 60\alpha - 2, \\ 2\beta_1 + 7 &= 60\alpha + 5 = 5(12\alpha + 1) = 5\beta_2. \end{aligned} \quad (62)$$

We redefine prime equations

$$\beta_1 = 26\alpha + 3, \quad \beta_2 = 4\alpha + 1 \quad (63)$$

From (63) we have

$$\begin{aligned} 2\beta_1 &= 52\alpha + 6, \\ 2\beta_1 + 7 &= 52\alpha + 13 = 13(4\alpha + 1) = 13\beta_2. \end{aligned} \quad (64)$$

Theorem 15. Gaps between products of two primes.

$$d(x) = d(x+9) = 2 \text{ infinitely-often} \quad (65)$$

Using $2 \times 13 + 9 = 5 \times 7$ we define prime equations,

$$\beta_1 = 15\alpha - 2, \quad \beta_2 = 6\alpha + 1 \quad (66)$$

From (66) we have

$$\begin{aligned} 2\beta_1 &= 30\alpha - 4, \\ 2\beta_1 + 9 &= 30\alpha + 5 = 5(6\alpha + 1) = 5\beta_2. \end{aligned} \quad (67)$$

We redefine prime equations

$$\beta_1 = 42\alpha - 1, \quad \beta_2 = 12\alpha + 1 \quad (68)$$

From (68) we have

$$\begin{aligned} 2\beta_1 &= 84\alpha - 2, \\ 2\beta_1 + 9 &= 84\alpha + 7 = 7(12\alpha + 1) = 7\beta_2. \end{aligned} \quad (69)$$

Theorem 16. Gaps between products of two primes.

$$d(x-11) = d(x) = 2 \text{ infinitely-often} \quad (70)$$

Using $2 \times 23 - 11 = 5 \times 7$ we define prime equations,

$$\beta_1 = 20\alpha + 3, \quad \beta_2 = 8\alpha - 1 \quad (71)$$

From (71) we have

$$\begin{aligned} 2\beta_1 &= 40\alpha + 6, \\ 2\beta_1 - 11 &= 40\alpha - 5 = 5(8\alpha - 1) = 5\beta_2. \end{aligned} \quad (72)$$

We redefine prime equations

$$\beta_1 = 21\alpha + 2, \quad \beta_2 = 6\alpha - 1 \quad (73)$$

From (73) we have

$$\begin{aligned} 2\beta_1 &= 42\alpha + 4, \\ 2\beta_1 - 11 &= 42\alpha - 7 = 7(6\alpha - 1) = 7\beta_2. \end{aligned} \quad (74)$$

Goldston *et. al* proved only

$$d(x) = d(x + n \leq 6) = 2 \text{ infinitely-often [4].} \quad (75)$$

References

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美国匈牙利土耳其数学家正在研究 x 和 $x + n \leq 6$ 两个数。有无限多个 x 使得 x 和 $x \leq 6$ 两个数每个都是两个素数相乘 [4]。例如 $x = 14 = 2 \times 7$, $x + 1 = 15 = 3 \times 5, \dots$, $x = 86 = 2 \times 43$, $x + 6 = 91 = 7 \times 13$, 这就是当代国际数学最高水平, 受到当代数学界的关注。他们并没有证明这个问题, 它比哥德巴赫猜想难一万倍。蒋春暄看到 [4] 以后, 国外就这么点水平吹到天上去, 决定写本文, 在国内外散发。蒋春暄 2002 年结果 [1]。从定理一得出 $x = 93 = 3 \times 31$, $x + 1 = 94 = 2 \times 47$, $x + 2 = 95 = 5 \times 19$, 有无限多个 x 使得 x , $x + 1$, $x + 2$ 三个数每个数都是两个素数相乘。从定理二得出, $x = 1727913 = 3 \times 11 \times 52361$, $x + 1 = 1727914 = 2 \times 17 \times 50821$, $x + 2 = 1727915 = 5 \times 7 \times 49369$, 有无限多个 x 使得每个数都是 m 个素数相乘。定理六, x , $x + 1$, $x + 2$, $x + 4, \dots$, $x + P_n - 3$ 有 n 个数, 有无限多个 x 使得每个数都是 m

个素数相乘，这样成果在过去没有数学家想象过，这是素数分布一个重要规律。将来一定会有广泛的应用。这是数学美！这是人类数学中最伟大成就，在中国被评为最大伪科学，中国不承认蒋春暄成就。母校北航不承认蒋春暄是北航的学生，献给北航母校被拒绝！2009年12月中科院数学院与北航联合创办“华罗庚数学班”，中国全面封杀蒋春暄成果。这样事件只能在中国才存在，全世界任何国家都不会发生“蒋春暄现象”事件！

It was therefore a great surprise for Erdos (and probably for other number theorists as well) when C.Spiro proved in 1981 that $d(x)=d(x + 5040)$ infinitely-often. It is theorem 11. There cannot modern prime theory without Jiang function.