On The Consecutive Integers $n+i-1=(i+1)P_i$

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Abstract

By using the Jiang's function $J_2(\omega)$ we prove that there exist infinitely many integers n such that $n=2P_1$, $n+1=3P_2,\cdots$, $n+k-1=(k+1)P_k$ are all composites for arbitrarily long k, where $P_1, P_2,\cdots,P_k$ are all primes. This result has no prior occurrence in the history of number theory.

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Theorem 1. There exist infinitely many integers n such that the consecutive inegers $n=2P_1$, $n+1=3P_2,\cdots,$ $n+k-1=(k+1)P_k$ are all composites for arbitrarily long k, where $P_1, P_2,\cdots,P_k$ are all primes.

Proof. Suppose that $m=\prod_{i=1}^{k}(i+1)$. We define the prime equations

$$P_i=\frac{m}{i+1}x+1, \tag{1}$$

Where $i=1,2,\cdots,k.$

The Jiang's function [1] is

$$J_2(\omega)=\prod_{3\le P}(P-k-1-\chi(P))\neq 0 \tag{2}$$

where $\chi(P)=-k$ if $P^2|m$; $\chi(P)=-k+1$ if $P|m$; $\chi(P)=0$ otherwise, $\omega=\prod_{2\le P}P$.

Since $J_2(\omega)\to\infty$ as $\omega\to\infty$, there exist infinitely many integers x such that $P_1, P_2,\cdots,P_k$ are all primes.

We have the asymptotic formula of the number of integers $x\le N$ [1]

$$\pi_{k+1}(N,2)\sim\frac{J_2(\omega)\omega^k}{\phi^{k+1}(\omega)}\frac{N}{\log^{k+1}N}, \tag{3}$$

where $\varphi(\omega)=\prod_{2\le P}(P-1)$.

From (1) we have $n=mx+2=2\left(\frac{mx}{2}+1\right)=2P_1$, $n+1=mx+3=3\left(\frac{m}{3}x+1\right)$

$=3P_2,\cdots,$ $n+k-1=mx+k+1=(k+1)\left(\frac{m}{k+1}x+1\right)=(k+1)P_k$.

Example 1. Let $k=5$, we have $n=2\times 53281$, $n+1=3\times 35521$, $n+2=4\times 26641$, $n+3=5\times 21313$, $n+4=6\times 17761.$

Theorem 2. There exist infinitely many integers n such that the consecutive inegers

$n=(1+2^b)P_1$, $n+1=(2+2^b)P_2,\cdots,$ $n+k-1=(k+2^b)P_k$ are all composites for arbitrarily long k, where $P_1, P_2,\cdots,P_k$ are all primes.

Proof. Suppose that $m=\prod_{i=1}^{k}(i+2^b)$. We define the prime equations

$$P_i=\frac{m}{i+2^b}x+1, \qquad (4)$$

Where $i=1,2,\cdots,k.$

The Jiang's function [1] is

$$J_2(\omega)=\prod_{3\le P}(P-k-1-\chi(P))\neq 0 \qquad (5)$$

where $\chi(P)=-k$ if $P^2|m$; $\chi(P)=-k+1$ if $P|m$; $\chi(P)=0$ otherwise.

Since $J_2(\omega)\to\infty$ as $\omega\to\infty$, there exist infinitely many integers x such that $P_1, P_2,\cdots,P_k$ are all primes.

We have the asymptotic formula of the number of integers $x\le N$ [1]

$$\pi_{k+1}(N,2)\sim\frac{J_2(\omega)\omega^k}{\phi^{k+1}(\omega)}\frac{N}{\log^{k+1}N}, \qquad (6)$$

From (4) we have $n=mx+1+2^b=(1+2^b)\left(\frac{m}{1+2^b}x+1\right)=(1+2^b)P_1$, $n+1=$

$mx+2+2^b=(2+2^b)\left(\frac{m}{2+2^b}x+1\right)=(2+2^b)P_2,\cdots,$ $n+k-1=mx+k+2^b=$

$(k+2^b)\left(\frac{m}{k+2^b}x+1\right)=(k+2^b)P_k$.

Example 2. Let $b=1$ and $k=4$, we have $n=3\times 27361$, $n+1=4\times 20521$, $n+2=5\times 16417$, $n+3=6\times 13681$.

Theorem 3. There exist infinitely many integers n such that the consecutive inegers $n=3P_1,\ n+2=5P_2,\cdots,n+2(k-1)=(2k+1)P_k$ are all composites for arbitrarily long k, where $P_1, P_2,\cdots,P_k$ are all primes.

Proof. Suppose that $m = \prod_{i=1}^{k}(2i+1)$. We define the prime equations

$$P_i = \frac{m}{2i+1}x + 1, \qquad (7)$$

Where $i = 1,2,\cdots,k$.

The Jiang's function [1] is

$$J_2(\omega) = \prod_{3 \le P}(P - k - 1 - \chi(P)) \ne 0 \qquad (8)$$

where $\chi(P) = -k$ if $P^2 \mid m$; $\chi(P) = -k+1$ if $P \mid m$; $\chi(P) = 0$ otherwise.

Since $J_2(\omega) \to \infty$ as $\omega \to \infty$, there exist infinitely many integers x such that $P_1, P_2, \cdots, P_k$ are all primes.
We have the asymptotic formula of the number of integers $x \le N$ [1]

$$\pi_{k+1}(N,2) \sim \frac{J_2(\omega)\omega^k}{\phi^{k+1}(\omega)} \frac{N}{\log^{k+1} N}, \qquad (9)$$

From (7) we have $n = mx + 3 = 3(\frac{m}{3}x + 1) = 3P_1$, $n + 2 = mx + 5 = 5(\frac{m}{5}x + 1) =$

$5P_2, \cdots, n + 2(k-1) = mx + 2k + 1 = (2k+1)\left(\frac{m}{2k+1}x + 1\right) = (2k+1)P_k$.

Example 3. Let $k = 4$, we have $n = 3 \times 631$, $n + 2 = 5 \times 379$, $n + 4 = 7 \times 271$, $n + 6 = 9 \times 211$.

Theorem 4. There exist infinitely many integers n such that the consecutive inegers $n = P_1$, $n + 2 = 3P_2, \cdots, n + 2(k-1) = (2k-1)P_k$ are all composites for arbitrarily long k, where $P_1, P_2, \cdots, P_k$ are all primes.

Proof. Suppose that $m = \prod_{i=1}^{k}(2i-1)$. We define the prime equations

$$P_i = \frac{m}{2i-1}x + 1 \qquad (10)$$

where $i = 1,2,\cdots,k$.

The Jiang's function [1] is

$$J_2(\omega) = \prod_{3 \le P} (P - k - 1 - \chi(P)) \ne 0 \qquad (11)$$

where $\chi(P) = -k$ if $P^2 | m$; $\chi(P) = -k + 1$ if $P | m$; $\chi(P) = 0$ otherwise.

Since $J_2(\omega) \to \infty$ as $\omega \to \infty$, there exist infinitely many integers x such that $P_1, P_2, \cdots, P_k$ are all primes.

We have the asymptotic formula of the number of integers $x \le N$ [1]

$$\pi_{k+1}(N,2) \sim \frac{J_2(\omega)\omega^k}{\phi^{k+1}(\omega)} \frac{N}{\log^{k+1} N}, \qquad (12)$$

From (10) we have $n = P_1 = mx + 1$, $n + 2 = mx + 3 = 3(\frac{m}{3}x + 1) = 3P_2, \cdots$,

$$n + 2(k-1) = mx + 2(k-1) = (2k-1)\left(\frac{m}{2k-1}x + 1\right) = (2k-1)P_k.$$

Example 4. Let $k = 4$, we have $n = 9661$, $n + 2 = 3 \times 3221$, $n + 4 = 5 \times 1933$, $n + 6 = 7 \times 1381$.

Theorem 5. There exist infinitely many integers n such that the consecutive inegers $n = 3P_1, n + 4 = 7P_2, \cdots, n + 4(k-1) = (4k-1)P_k$ are all composites for arbitrarily long k, where $P_1, P_2, \cdots, P_k$ are all primes.

Example 5. Let $k = 4$, we have $n = 3 \times 2311$, $n + 4 = 7 \times 991$, $n + 8 = 11 \times 631$, $n + 12 = 15 \times 463$.

Theorem 6. There exist infinitely many integers n such that the consecutive inegers $n = 5P_1, n + 4 = 9P_2, \cdots, n + 4(k-1) = (4k+1)P_k$ are all composites for arbitrarily long k, where $P_1, P_2, \cdots, P_k$ are all primes.

Reference

[1] Chun-Xuan Jiang. Foundations of Santilli's isonumber theory with applications to new cryptograms, Fermat's theorem and Goldbach's conjecture. International Academic Press, 2002 MR 2004c: 11001. http: // www.i-b-r.org/docs/jiang/pdf.