

On the Fundamental Theorem in Arithmetic Progression of Primes

(Negation of 2006 a Fields Medal)

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Abstract

Using Jiang function we prove the fundamental theorem in arithmetic progression of primes [1-3]. The primes contain only $k < P_{g+1}$ long arithmetic progressions, but the primes have no $k > P_{g+1}$ long arithmetic progressions. 这是决定素数等差数列长度关键条件, 这是蒋春暄发现的, 过去任何文献都找不到。为了使中国人了解本文内容在重要地方加一点中文说明。Terence Tao is recipient of 2006 Fields medal. Green and Tao proved that the primes contain arbitrarily long arithmetic progressions which is absolutely false[4-9]. They do not understand the arithmetic progression of primes [4-15]. The school of mathematics at institute for advanced study has long been recognized as the leading international center of research and postdoctoral training in pure mathematics. Why do they support and publish (in Ann. Math.) false papers? For example Green, Tao, Kra, Vu, Goldston and other mathematicians. Since they do not understand the prime numbers. 有这样单位支持陶哲轩一定会获得 2006 年菲尔茨奖, 国际很多数学大奖都是他们提名的, 他们成员都获得世界数学大奖, 这就是当代数学水平。

否定 2006 年一个菲尔茨奖, 格林—陶哲轩定理是完全错的

证明素数等差数列要满足两个条件: 一要证明它是否有无限多素数解? 二如果有要找到计算素数个数公式。利用Jiang函数这个问题蒋春暄于 1995 年就彻底解决了, 参看文献 1-3。根据美国 Notices of AMS Oct 2006p.1041 报道陶哲轩获得 2006 年菲尔

茨奖理由:The first highlight is Tao work with Ben Green,a dramatic new result about the fundamental building blocks of mathematics, the prime numbers, 即格林-陶哲轩定理.格林和陶哲轩用概率数论研究素数等差数列。我看过他们所有论文,他们没有得出任何有意义的结果。在他们论文中讨论概率数论,没有接触素数。陶哲轩因解决素数等差数列获得 2006 年菲尔茨奖。在 2006 年国际数学家大会上,他一小时报告中根本没谈素数,只在最后提一下素数,他们根本不懂什么是素数等差数列。因为他们是在以前大数学家猜想基础上完成这项工作。所以全世界数学家都承认他们的工作,过去有关这方面猜想和定理都是错的。2009 年 3 月 22 日我找 1995 年预印本,再研究格林-陶哲轩定理后才提出素数等差数列中一个基本定理。最后写成本文。无人同我讨论,边打印边修改。格林-陶哲轩定理是本文系 1,不是定理。但他们没研究系 1。在 1995 年预印本中,用我获得公式和从素数表所得结果一样。所以我所获得结果是对的。本文已在<http://www.wbabin.net/math/xuan39.pdf>上网,全世界都可看到。

20 世纪最伟大数学家当代 Euler 保罗·厄尔多斯 (Paul Erdos) 说:“至少还再过 100 万年,我们才可能理解素数”。他创建概率数论,研究素数等差数列基本思路是他 1936 年提出来。他的学生 Szemerédi 1975 年开始完成他的想法,而后 Furstenberg 和 Gowers (获得 1998 年菲尔茨奖) 进一步证明 Szemerédi 定理。最后格林和陶哲轩证明 Szemerédi 定理。这些数学家都获得世界数学大奖,这说明什么?说明当代没有一个数学家真正理解素数等差数列。他们认为素数是随机的没有规律的,只能用概率理论来研究。得出结果只能是一个估计上限和下限,不能定量计算。这就当代素数理论是最高水平。因此可以说,当代没有一位数论专家真正了解素数等差数列。它是素数理论中最复杂的问题,他们认为要真正了解它是 100 万年以后的事。

计算素数等差数列专家指出格林—陶哲轩定理对我们没有一点用处。因为素数理论是一个计算问题,不是概率问题。普林斯顿高级研究所 (IAS) 是全世界纯数学领导中心,2007-2008 年举办格林-陶哲轩定理讨论会。2008 年<数学年刊>发表格林和陶哲轩错误论文,无理拒绝发表蒋春暄正确的论文。说明他们也不理解什么是素数等差数列。德国最高数学研究所 Max Planck 数学所今年举办格林-陶哲轩定理研讨会,他们得到全世界广泛的支持,因为他们不理解素数。要否定他们结果谈何容易。我把结果公布就行了!科学问题不是权力不是权威。它的力量是无敌的!真理永存!蒋春暄素数理论否定了 20 世纪几乎所有素数理论结果!

陶哲轩是当代吹起来数学天才,最近他和很多人合作研究其它问题,概率问题,组合问题。素数问题对他来说太难了!从他最近发表论文内容来看.格林-陶哲轩定理也可能是他研究素数问题最后一篇文章。但他仍要宣传格林-陶哲轩定理,例如 Manin 书只把他们文章引用一下,或把结果提一下.他和很多人合作,有他的名字文章马上发表,所以很多人都乐意和他合作。从 2007 年 2008 年他发表论文来看是一般数学,都是和其他人合作的,成果是他的还是别人的也说不清。目前国际数论研究是一潭死水,只有格林和陶哲轩比较活跃。他提出加法组合学吸引很多年轻人,关键内容仍是格林—陶哲轩定理。国外 wikibin 科学网报道,蒋春暄否定黎曼假设,用他发现函数证明哥德巴赫猜想和孪生素数猜想,建立了 iso 数论基础。比怀尔斯先证明了费马大定理。中国不需要费马大定理证明成果。怀尔斯抢夺费马大定理成果。他获得世界所有数学

大奖，包括 2005 年中国式邵逸夫数学大奖。怀尔斯应该感谢中国院士对他的支持。如中国院士支持蒋春暄，怀尔斯什么也得不到。这也算中国数学一大丑闻。

最近美国 Clay 数学所设立 100 万美元黎曼假设奖金委员会给蒋春暄来信：文章发表两年后就即可申请 100 百万美元奖，但他们不对论文进行评定。蒋春暄否定黎曼假设 1998 年 1999 年 2002 年在美国发表，全文 2005 年在美国发表。没有人提出异议。要得奖，需要一批数学家支持宣传，需要中国政府支持宣传，这两个条件都没有。在这种情况下，蒋春暄对 Clay 来信采取不理态度。因为中国至今不承认蒋春暄成果，被评为垃圾和伪科学。桑蒂利宣传蒋春暄否定黎曼假设成果在国内外家喻户晓，谁也不能抢夺这个成果。也没有像怀尔斯这样的人来抢夺蒋春暄成果。蒋春暄可以获得世界所有数学大奖，中国不需要，蒋春暄也不需要。这是中国也是全世界最大数学丑闻。这种事情也只能在中国才会出现。

中科院数学院对格林-陶哲轩定理非常重视，组织一批老中青数学家研究，开了多次讨论会，不知得出什么样结果。听说他们终于了解格林-陶哲轩定理是当代最伟大的成就，应该好好学习。这叫与国际水平接轨。国外得大奖就是最高水平。2002 年 3 月 5 日何祚庥院士在九届五次政协会议上宣布：蒋春暄研究是伪科学。这是人民大会堂丑闻。

高斯和欧拉是大数学家，他们研究内容很广泛，但最重要数论特别是素数论。他们研究是超前没人和他们合作，他们是单干户。蒋春暄也是干户！陶哲轩不是单干户！

Van Vu 来信：请不要寄材料给我，陶哲轩仍在联合 Van Vu, Ben Green, Tamar Ziegler, Tim Austin, Katz, Jean, Kevin Costello and others. 写文章宣传格林-陶哲轩定理。《数学年刊》仍在发表他们的文章，否定 2006 年菲尔茨奖问题很大，连我国国外最好朋友都不来信。这可能是目前国际数学界一件大事。但在中国更无数学家来关心这件事。

陶哲轩写一文：What is good mathematics? 什么是好数学？即宣传格林-陶哲轩定理是好数学。其实是最坏数学。他们没证明素数等差数列任何东西，把一些与素数等差数列无关东西 ergodic theory, harmonic analysis, discrete geometry and additive combinatorics 戴在素数等差数列头上，这样就证明素数等差数列，他得出长度很大不知他们如何猜出来的。像怀尔斯一样把椭圆曲线戴在费马大定理头上，证明费马大定理。这是目前国际研究数论一特点，你必须相信它。因为数论中难题很难只能用这种办法。

科学时报 2006-9-25 报道在 2004 年前还没有人能证明任意长度的素数等差数列存在，这是个一步登天的杰作。2005 年 1 日美国《发现》杂志将格陶成果这项证明列入 2004 年度最重要的 100 项科学发现之一。这才引起蒋春暄注意，研究格林-陶哲轩定理。写出本文，它在蒋的工作只能算一个小芝麻。王元说：我不敢想象天下会有这样伟大的成就。这一吹在中国影响巨大。因为在这篇文章中引用陈氏定理。格林和陶哲轩肯定看过蒋春暄文章和书，他们也会拉关系。这样会引起中国人对他们工作重视，反过来中国更不承认蒋春暄工作，这和中科院数学院所说哥德巴赫猜想还是陈景润工作最高无人超越一致。格林和陶哲轩不承认蒋春暄工作。那只有彻底否定格林-陶哲轩定理。但格林拒绝蒋文，送给他电子邮件都拒收。现在有网络和电子邮件可以把

文章让全世界都知道！从他们证明素数等差数列就可以看出他们在数学上没有重大的发现。

Theorem. The fundamental theorem in arithmetic progression of primes.

这部分蒋春暄 1995 年就完成有预印本作证

We define the arithmetic progression of primes [1-3].

$$P_{i+1} = P_1 + \omega_g i, i = 0, 1, 2, \dots, k - 1, \quad (1)$$

where $\omega_g = \prod_{2 \leq P \leq P_g}$ is called a common difference, P_g is called g -th prime.

We have Jiang function [1-3]

$$J_2(\omega) = \prod_{3 \leq P} (P - 1 - X(P)), \quad (2)$$

$X(P)$ denotes the number of solutions for the following congruence

$$\prod_{i=1}^{k-1} (q + \omega_g i) \equiv 0 \pmod{P}, \quad (3)$$

where $q = 1, 2, \dots, P - 1$.

If $P \mid \omega_g$, then $X(P) = 0$; $X(P) = k - 1$ otherwise. From (3) we have

$$J_2(\omega) = \prod_{3 \leq P \leq P_g} (P-1) \prod_{P_{g+1} \leq P} (P-k). \quad (4)$$

(4) 是证明素数等差数列有解和无解一个关键公式, 这是蒋春暄 1995 年发现的。If $k = P_{g+1}$ then $J_2(P_{g+1}) = 0$, $J_2(\omega) = 0$, there exist finite primes P_1 such that P_2, \dots, P_k are primes. If $k < P_{g+1}$ then $J_2(\omega) \neq 0$, there exist infinitely many primes P_1 such that P_2, \dots, P_k are primes. The primes contain only $k < P_{g+1}$ long arithmetic progressions, but the primes have no $k > P_{g+1}$ long arithmetic progressions. We have the best asymptotic formula [1-3]

$$\begin{aligned} \pi_k(N, 2) &= \left| \left\{ P_1 + \omega_g i = \text{prime}, 0 \leq i \leq k-1, P_1 \leq N \right\} \right| \\ &= \frac{J_2(\omega) \omega^{k-1}}{\phi^k(\omega)} \frac{N}{\log^k N} (1 + o(1)), \end{aligned} \quad (5)$$

where $\omega = \prod_{2 \leq P} P$, $\phi(\omega) = \prod_{2 \leq P} (P-1)$, ω IS called primorial, $\phi(\omega)$ Euler function.

如果有解 (5) 是计算素数个数公式。过去没有任何人提供这个公式, 是蒋春暄于 1995 年发现的。

Suppose $k = P_{g+1} - 1$. From (1) we have

$$P_{i+1} = P_1 + \omega_g i, i = 0, 1, 2, \dots, P_{g+1} - 2. \quad (6)$$

From (4) we have [1-2]

$$J_2(\omega) = \prod_{3 \leq P \leq P_g} (P-1) \prod_{P_{g+1} \leq P} (P - P_{g+1} + 1) \rightarrow \infty \text{ as } \omega \rightarrow \infty \quad (7)$$

We prove that there exist infinitely many primes P_1 such that $P_2, \dots, P_{P_{g+1}-1}$ are primes

for all P_{g+1} .

From (5) we have

$$\begin{aligned} \pi_{P_{g+1}-1}(N, 2) &= \\ \prod_{2 \leq P \leq P_g} \left(\frac{P}{P-1} \right)^{P_{g+1}-2} \prod_{P_{g+1} \leq P} &= \frac{P^{P_{g+1}-2} (P - P_{g+1} + 1)}{(P-1)^{P_{g+1}-1}} \frac{N}{(\log N)^{P_{g+1}-1}} (1 + o(1)). \end{aligned} \quad (8)$$

From (8) we are able to find the smallest solutions $\pi_{P_{g+1}-1}(N, 2) > 1$ for large P_{g+1} .

Theorem is foundations for arithmetic progression of primes。这为素数等差数列计算提供的理论基础。在什么地方存在素数等差数列。以上结果就彻底全面解决素数等差数列问题, 就这么简单。

Example 1. Suppose $P_1 = 2, \omega_1 = 2, P_2 = 3$. From (6) we have the twin primes theorem

$$P_2 = P_1 + 2. \quad (9)$$

From (7) we have

$$J_2(\omega) = \prod_{3 \leq P} (P-2) \rightarrow \infty \text{ as } \omega \rightarrow \infty, \quad (10)$$

We prove that there exist infinitely many primes P_1 such that P_2 are primes. From (8) we have the best asymptotic formula

$$\pi_2(N, 2) = 2 \prod_{3 \leq P} \left(1 - \frac{1}{(P-1)^2} \right) \frac{N}{\log^2 N} (1 + o(1)). \quad (11)$$

Twin prime theorem is the first theorem in arithmetic progression of primes. Green and Tao do not prove the twin prime theorem. Therefore Green – Tao theorem is absolutely false [4-9]. The prime distribution is order rather than randomness. The arithmetic progressions of primes are not directly related to ergodic theory, harmonic analysis, discrete geometry and additive combinatorics. Conjectures and theorems on arithmetic progressions of primes are absolutely false [4-15], because they do not understand the arithmetic progressions of primes. 陶哲轩宣布孪生素数定理是无法证明的，所以他们没有证明素数理论中任何问题。

Example 2. Suppose $P_2 = 3, \omega_2 = 6, P_3 = 5$. From (6) we have

$$P_{i+1} = P_1 + 6i, i = 0, 1, 2, 3. \quad (12)$$

From (7) we have

$$J_2(\omega) = 2 \prod_{5 \leq P} (P-4) \rightarrow \infty \text{ as } \omega \rightarrow \infty, \quad (13)$$

We prove that there exist infinitely many primes P_1 such that P_2, P_3 and P_4 are primes. From (8) we have the best asymptotic formula

$$\pi_4(N, 2) = 27 \prod_{5 \leq P} \frac{P^3(P-4)}{(P-1)^4} \frac{N}{\log^4 N} (1 + o(1)). \quad (14)$$

陶哲轩他更没能力证明比孪生素数定理更难问题。

Example 3. Suppose $P_9 = 23, \omega_9 = 223092870, P_{10} = 29$. From (6) we have

$$P_{i+1} = P_1 + 223092870i, i = 0, 1, 2, \dots, 27. \quad (15)$$

From (7) we have

$$J_2(\omega) = 36495360 \prod_{29 \leq P} (P-28) \rightarrow \infty \text{ as } \omega \rightarrow \infty, \quad (16)$$

We prove that there exist infinitely many primes P_1 such that P_2, \dots, P_{28} are primes.

From (8) we have the best asymptotic formula

$$\pi_{28}(N, 2) = \prod_{2 \leq P \leq 23} \left(\frac{P}{P-1} \right)^{27} \prod_{29 \leq P} \frac{P^{27} (P-28)}{(P-1)^{28}} \frac{N}{\log^{28} N} (1 + o(1)). \quad (17)$$

From (17) we are able to find the smallest solutions $\pi_{28}(N_0, 2) > 1$.

(17) 是目前计算数论专家攻关的对象。这公式对他们有所帮助。

On May 17, 2008, Wroblewski and Raanan Chermoni found the first known case of 25 primes:

$$6171054912832631 + 366384 \times \omega_{23} \times n, \text{ for } n = 0 \text{ to } 24.$$

Theorem can help in finding for 26, 27, 28, ..., primes in arithmetic progressions of primes.

Corollary 1. Arithmetics progression with two prime variables

这是格林-陶哲轩定理主要研究对象但他们没有得出下列正确结果。

Suppose $\omega_g = d$. From (1) we have

$$P_1, P_2 = P_1 + d, P_3 = P_1 + 2d, \dots, P_k = P_1 + (k-1)d, (P_1, d) = 1. \quad (18)$$

From (18) we obtain the arithmetic progression with two prime variables: P_1 and P_2 ,

$$P_3 = 2P_2 - P_1, \quad P_j = (j-1)P_2 - (j-2)P_1, \quad 3 \leq j \leq k < P_{g+1}. \quad (19)$$

We have Jiang function [3]

$$J_3(\omega) = \prod_{3 \leq P} [(P-1)^2 - X(P)], \quad (20)$$

$X(P)$ denotes the number of solutions for the following congruence

$$\prod_{j=3}^k [(j-1)q_2 - (j-2)q_1] \equiv 0 \pmod{P}, \quad (21)$$

where $q_1 = 1, 2, \dots, P-1; q_2 = 1, 2, \dots, P-1$.

From (21) we have

$$J_3(\omega) = \prod_{3 \leq P \leq k} (P-1) \prod_{k < P} (P-1)(P-k+1) \rightarrow \infty \text{ as } \omega \rightarrow \infty. \quad (22)$$

We prove that there exist infinitely many primes P_1 and P_2 such that P_3, \dots, P_k are primes for $3 \leq k < P_{g+1}$. (22) 这才是证明第一个正确结果。

we have the best asymptotic formula

$$\begin{aligned} \pi_{k-1}(N,3) &= \left| \left\{ (j-1)P_2 - (j-2)P_1 = \text{prime}, 3 \leq j \leq k, P_1, P_2 \leq N \right\} \right| \\ &= \frac{J_3(\omega)\omega^{k-2}}{\phi^k(\omega)} \frac{N^2}{\log^k N} (1+o(1)), \end{aligned} \quad (23)$$

From (23) we have the best asymptotic formula

$$\pi_{k-1}(N,3) = \prod_{2 \leq P \leq k} \frac{P^{k-2}}{(P-1)^{k-1}} \prod_{k < P} \frac{P^{k-2}(P-k+1)}{(P-1)^{k-1}} \frac{N^2}{\log^k N} (1+o(1)). \quad (24)$$

(24) 这才是证明第二个正确结果。这两个结果 (22) 和 (24) 彻底全面证明了系 1。

From (24) we are able to find the smallest solution $\pi_{k-1}(N_0,3) > 1$ for large $k < P_{g+1}$.

Example 4. Suppose $k = 3$ and $P_{g+1} > 3$. From (19) we have

$$P_3 = 2P_2 - P_1. \quad (25)$$

From (22) we have

$$J_3(\omega) = \prod_{3 \leq P} (P-1)(P-2) \rightarrow \infty \text{ as } \omega \rightarrow \infty, \quad (26)$$

We prove that there exist infinitely many primes P_1 and P_2 such that P_3 are primes. From (24) we have the best asymptotic formula

$$\pi_2(N,3) = 2 \prod_{3 \leq P} \left(1 - \frac{1}{(P-1)^2} \right) \frac{N^2}{\log^3 N} (1+o(1)) = 1.32032 \frac{N^2}{\log^3 N} (1+o(1)). \quad (27)$$

Example 5. Suppose $k = 4$ and $P_{g+1} > 4$. From (19) we have

$$P_3 = 2P_2 - P_1, \quad P_4 = 3P_2 - 2P_1. \quad (28)$$

From (22) we have

$$J_3(\omega) = 2 \prod_{5 \leq P} (P-1)(P-3) \rightarrow \infty \text{ as } \omega \rightarrow \infty, \quad (29)$$

We prove that there exist infinitely many primes P_1 and P_2 such that P_3 and P_4 are primes. From (24) we have the best asymptotic formula

$$\pi_3(N,3) = \frac{9}{2} \prod_{5 \leq P} \frac{P^2(P-3)}{(P-1)^3} \frac{N^2}{\log^4 N} (1 + o(1)). \quad (30)$$

Example 6. Suppose $k = 5$ and $P_{g+1} > 5$. From (19) we have

$$P_3 = 2P_2 - P_1, \quad P_4 = 3P_2 - 2P_1, \quad P_5 = 4P_2 - 3P_1. \quad (31)$$

From (22) we have

$$J_3(\omega) = 2 \prod_{5 \leq P} (P-1)(P-4) \rightarrow \infty \text{ as } \omega \rightarrow \infty, \quad (32)$$

We prove that there exist infinitely many primes P_1 and P_2 such that P_3 , P_4 and P_5 are primes. From (24) we have the best asymptotic formula

$$\pi_4(N,3) = \frac{27}{2} \prod_{5 \leq P} \frac{P^3(P-4)}{(P-1)^4} \frac{N^2}{\log^5 N} (1 + o(1)). \quad (33)$$

Green and Tao study only **corollary 1**, which is not the theorem [4-9].

Corollary 2. Arithmetic progression with three prime variables

这部分在任何书和文章都找不到是蒋春暄提出的。

From (18) we obtain the arithmetic progression with three prime variables: P_1, P_2 and P_3

$$P_4 = P_3 + P_2 - P_1, \quad P_j = P_3 + (j-3)P_2 - (j-3)P_1, \quad 4 \leq j \leq k < P_{g+1} \quad (34)$$

We have Jiang function

$$J_4(\omega) = \prod_{3 \leq P} ((P-1)^3 - X(P)), \quad (35)$$

$X(P)$ denotes the number of solutions for the following congruence

$$\prod_{j=4}^k (q_3 + (j-3)q_2 - (j-3)q_1) \equiv 0 \pmod{P}, \quad (36)$$

where $q_i = 1, 2, \dots, P-1, i = 1, 2, 3$.

Example 7. Suppose $k = 4$ and $P_{g+1} > 4$. From (34) we have

$$P_4 = P_3 + P_2 - P_1. \quad (37)$$

From (35) and (36) we have

$$J_4(\omega) = \prod_{3 \leq P} (P-1)(P^2 - 3P + 3) \rightarrow \infty \text{ as } \omega \rightarrow \infty, \quad (38)$$

We prove that there exist infinitely many primes P_1 and P_2 and P_3 such that P_4 are primes. we have the best asymptotic formula

$$\pi_2(N,4) = 2 \prod_{3 \leq P} \left(1 + \frac{1}{(P-1)^3} \right) \frac{N^3}{\log^4 N} (1 + o(1)). \quad (39)$$

For $k \geq 5$ from (35) and (36) We have Jiang function

$$\begin{aligned} J_4(\omega) &= \prod_{3 \leq P < (k-1)} (P-1)^2 \\ &\quad \times \prod_{(k-1) \leq P} (P-1)[(P-1)^2 - (P-2)(k-3)] \rightarrow \infty \\ \text{as } \omega &\rightarrow \infty. \end{aligned} \quad (40)$$

We prove that there exist infinitely many primes P_1 and P_2 and P_3 such that P_4, \dots, P_k are primes for $5 \leq k < P_{g+1}$.
we have the best asymptotic formula

$$\begin{aligned} \pi_{k-2}(N,4) &= \left| \{P_3 + (j-3)P_2 - (j-3)P_1 = \text{prime}, 4 \leq j \leq k, P_1, P_2, P_3 \leq N\} \right| \\ &= \frac{J_4(\omega)\omega^{k-3}}{\phi^k(\omega)} \frac{N^3}{\log^k N} (1 + o(1)). \end{aligned} \quad (41)$$

From (41) we have

$$\begin{aligned} \pi_{k-2}(N,4) &= \prod_{2 \leq P < (k-1)} \frac{P^{k-3}}{(P-1)^{k-2}} \prod_{(k-1) \leq P} \frac{P^{k-3}[(P-1)^2 - (P-2)(k-3)]}{(P-1)^{k-1}} \frac{N^3}{\log^k N} (1 + o(1)). \end{aligned} \quad (42)$$

From (42) we are able to find the smallest solution $\pi_{k-2}(N_0,4) > 1$ for large $k < P_{g+1}$.

Corollary 3. Arithmetic progression with four prime variables

这部分是蒋春暄提出的

From (18) we obtain the arithmetic progression with four prime variables: P_1, P_2, P_3 and P_4

$$\begin{aligned} P_5 &= P_4 + 2P_3 - 3P_2 + P_1, & P_j &= P_4 + (j-3)P_3 - (j-2)P_2 + P_1, \\ 5 \leq j &\leq k < P_{g+1} \end{aligned} \quad (43)$$

We have Jiang function

$$J_5(\omega) = \prod_{3 \leq P} [(P-1)^4 - X(P)], \quad (44)$$

$X(P)$ denotes the number of solutions for the following congruence

$$\prod_{j=5}^k [q_4 + (j-3)q_3 - (j-2)q_2 + q_1] \equiv 0 \pmod{P}, \quad (45)$$

where

$$q_i = 1, \dots, P-1, i = 1, 2, 3, 4$$

Example 8. Suppose $k = 5$ and $P_{g+1} > 5$. From (43) we have

$$P_5 = P_4 + 2P_3 - 3P_2 + P_1. \quad (46)$$

From (44) and (45) we have

$$J_5(\omega) = 12 \prod_{5 \leq P} (P-1)(P^3 - 4P^2 + 6P - 4) \rightarrow \infty \quad \text{as } \omega \rightarrow \infty. \quad (47)$$

We prove there exist infinitely many primes P_1, P_2, P_3 and P_4 such that P_5 are primes.

We have the best asymptotic formula

$$\pi_2(N, 5) = \frac{J_5(\omega)\omega}{\phi^5(\omega)} \frac{N^4}{\log^5 N} (1 + o(1)). \quad (48)$$

Example 9. Suppose $k = 6$ and $P_{g+1} > 6$. From (43) we have

$$P_5 = P_4 + 2P_3 - 3P_2 + P_1, \quad P_6 = P_4 + 3P_3 - 4P_2 + P_1. \quad (49)$$

From (44) and (45) we have

$$J_5(\omega) = 10 \prod_{5 \leq P} (P-1)(P^3 - 5P^2 + 10P - 9) \rightarrow \infty \quad \text{as } \omega \rightarrow \infty. \quad (50)$$

We prove there exist infinitely many primes P_1, P_2, P_3 and P_4 such that P_5 and P_6 are primes.

We have the best asymptotic formula

$$\pi_3(N, 5) = \frac{J_5(\omega)\omega^2}{\phi^6(\omega)} \frac{N^4}{\log^6 N} (1 + o(1)). \quad (50)$$

For $k \geq 7$ from (44) and (45) we have Jiang function

$$\begin{aligned} J_5(\omega) &= 6 \prod_{5 \leq P \leq (k-4)} (P-1)(P^2 - 3P + 3) \\ &\times \prod_{(k-4) < P} \left\{ (P-1)^4 - (P-1)^2 [(P-3)(k-4) + 1] - (P-1)(2k-9) \right\} \rightarrow \infty \\ &\text{as } \omega \rightarrow \infty \end{aligned} \quad (51)$$

We prove there exist infinitely many primes P_1, P_2, P_3 and P_4 such that P_5, \dots, P_k are primes.

We have best asymptotic formula

$$\pi_{k-3}(N,5) = \left| \{P_4 + (j-3)P_3 - (j-2)P_2 + P_1 = \text{prime}, 5 \leq j \leq k, P_1, \dots, P_4 \leq N\} \right|$$

$$= \frac{J_5(\omega)\omega^{h-4}}{\phi^k(\omega)} \frac{N^4}{\log^k N} (1 + o(1)).$$

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