Riemann Paper (1859) Is False

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Abstract

In 1859 Riemann defined the zeta function $\zeta(s)$. From Gamma function he derived the zeta function with Gamma function $\bar{\zeta}(s)$. $\bar{\zeta}(s)$ and $\zeta(s)$ are the two different functions. It is false that $\bar{\zeta}(s)$ replaces $\zeta(s)$. Therefore Riemann hypothesis (RH) is false. The Jiang function $J_n(\omega)$ can replace RH.

AMS mathematics subject classification: Primary 11M26.

In 1859 Riemann defined the Riemann zeta function (RZF) [1]

$$\zeta(s)=\prod_P(1-P^{-s})^{-1}=\sum_{n=1}^{\infty}\frac{1}{n^s}\ , \qquad (1)$$

where $s=\sigma+ti, i=\sqrt{-1}$, σ and t are real, P ranges over all primes. RZF is the function of the complex variable s with $\sigma\geq 0, t\neq 0$, which is absolutely convergent.

In 1896 J. Hadamard and de la Vallee Poussin proved independently [2]

$$\zeta(1+ti)\neq 0. \qquad (2)$$

In 1998 Jiang proved [3]

$$\zeta(s)\neq 0, \qquad (3)$$

where $0\leq\sigma\leq 1$.

Riemann paper (1859) is false [1]. We define Gamma function [1, 2]

$$\Gamma\left(\frac{s}{2}\right)=\int_0^{\infty} e^{-t}t^{\frac{s}{2}-1}dt\ . \qquad (4)$$

For $\sigma>0$. On setting $t=n^2\pi x$, we observe that

$$\pi^{-\frac{s}{2}}\Gamma\left(\frac{s}{2}\right)n^{-s}=\int_0^{\infty} x^{\frac{s}{2}-1}e^{-n^2\pi x}dx\ . \qquad (5)$$

Hence, with some care on exchanging summation and integration, for $\sigma>1$,

$$\pi^{-\frac{s}{2}}\Gamma\left(\frac{s}{2}\right)\overline{\zeta}(s)=\int_0^{\infty} x^{\frac{s}{2}-1}\left(\sum_{n=1}^{\infty} e^{-n^2\pi x}\right)dx$$

$$=\int_0^{\infty} x^{\frac{s}{2}-1}\left(\frac{\vartheta(x)-1}{2}\right)dx, \qquad (6)$$

where $\overline{\zeta}(s)$ is called Riemann zeta function with gamma function.

$$\vartheta(x):=\sum_{n=-\infty}^{\infty} e^{-n^2\pi x}, \qquad (7)$$

is the Jacobi theta function. The functional equation for $\vartheta(x)$ is

$$x^{\frac{1}{2}}\vartheta(x)=\vartheta(x^{-1}), \qquad (8)$$

and is valid for $x>0$.

Finally, using the functional equation of $\vartheta(x)$, we obtain

$$\bar{\zeta}(s)=\frac{\pi^{\frac{s}{2}}}{\Gamma\left(\frac{s}{2}\right)}\left\{\frac{1}{s(s-1)}+\int_1^{\infty}(x^{\frac{s}{2}-1}+x^{-\frac{s}{2}-\frac{1}{2}})\cdot(\frac{\vartheta(x)-1}{2})dx\right\}. \qquad (9)$$

From (9) we obtain the functional equation

$$\pi^{-\frac{S}{2}}\Gamma\left(\frac{s}{2}\right)\bar{\zeta}(s)=\pi^{-\frac{1-s}{2}}\Gamma\left(\frac{1-s}{2}\right)\bar{\zeta}(1-s). \qquad (10)$$

The function $\bar{\zeta}(s)$ satisfies the following

1. $\bar{\zeta}(s)$ has no zero for $\sigma>1$;
2. The only pole of $\bar{\zeta}(s)$ is at $s=1$, it has residue 1 and is simple;
3. $\bar{\zeta}(s)$ has trivial zeros at $s=-2,-4,...$ but $\zeta(s)$ has no zeros;
4. The nontrivial zeros lie inside the region $0\le\sigma\le1$ and are symmetric about both the vertical line $\sigma=1/2$.

The strip $0\le\sigma\le1$ is called the critical strip and the vertical line $\sigma=1/2$ is called the critical line.

Conjecture (The Riemann Hypothesis). All nontrivial zeros of $\bar{\zeta}(s)$ lie on the critical line $\sigma=1/2$, which is false. [3]

$\bar{\zeta}(s)$ and $\zeta(s)$ are the two different functions. It is false that $\bar{\zeta}(s)$ replaces $\zeta(s)$, Pati proved that is not all complex zeros of $\bar{\zeta}(s)$ lie on the critical line: $\sigma=1/2$ [4].

Schadeck pointed out that the falsity of RH implies the falsity of RH for finite fields [5, 6]. RH is not directly related to prime theory. Using RH mathematicians prove many prime theorems which is false. In 1994 Jiang discovered Jiang function $J_n(\omega)$ which can replace RH, if $J_n(\omega)\ne0$ then the prime equation has infinitely many prime solutions; and if $J_n(\omega)=0$ then the prime equation has finitely many prime solutions. By using $J_n(\omega)$ Jiang proves about 600 prime theorems including the Goldbach's theorem, twin prime theorem and theorems on arithmetic progressions in primes [7, 8].

In the same way we have a general formula involving $\overline{\zeta}(s)$

$$\int_0^\infty x^{s-1}\sum_{n=1}^\infty F(nx)dx=\sum_{n=1}^\infty \int_0^\infty x^{s-1}F(nx)dx$$

$$=\sum_{n=1}^\infty \frac{1}{n^s}\int_0^\infty y^{s-1}F(y)dy=\overline{\zeta}(s)\int_0^\infty y^{s-1}F(y)dy\,, \qquad (11)$$

where $F(y)$ is arbitrary.

From (11) we obtain many zeta functions $\overline{\zeta}(s)$ which are not directly related to the number theory.

The prime distributions are order rather than random. The arithmetic progressions in primes are not directly related to ergodic theory, harmonic analysis, discrete geometry, and combinatories. Using the ergodic theory Green and Tao prove that there exist infinitely many arithmetic progressions of length k consisting only of primes which is false [9, 10, 11]. Fermat's last theorem (FLT) is not directly related to elliptic curves. In 1994 using elliptic curves Wiles proved FLT which is false [12]. There are Pythagorean Theorem and FLT in the complex hyperbolic functions and complex trigonometric functions. In 1991 without using any number theory Jiang proved FLT which is Fermat's marvelous proof [7, 13].

Primes Represented by $P_1^n+mP_2^n$ [14]

（1）Let $n=3$ and $m=2$. We have

$$P_3=P_1^3+2P_2^3\,.$$

We have Jiang function

$$J_3(\omega)=\prod_{3\le P}(P^2-3P+3-\chi(P))\neq 0\,,$$

Where $\chi(P)=2P-1$ if $2^{\frac{P-1}{3}}\equiv 1$ (mod P); $\chi(P)=-P+2$ if $2^{\frac{P-1}{3}}\not\equiv 1$ (mod P); $\chi(P)=1$ otherwise.

Since $J_n(\omega)\neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime.

We have the best asymptotic formula

$$\pi_2(N,3) = \left|\{P_1, P_2 : P_1, P_2 \le N, P_1^3 + 2P_2^3 \text{ prime}\}\right|$$

$$\sim \frac{J_3(\omega)\omega}{6\Phi^3(\omega)} \frac{N^2}{\log^3 N} = \frac{1}{3}\prod_{3\le P} \frac{P(P^2-3P+3-\chi(P))}{(P-1)^3} \frac{N^2}{\log^3 N}.$$

where $\omega = \prod_{2\le P} P$ is called primorial, $\Phi(\omega) = \prod_{2\le P} (P-1)$.

It is the simplest theorem which is called the Heath-Brown problem [15].

（2）Let $n = P_0$ be an odd prime, $2|m$ and $m \ne \pm b^{P_0}$.

we have

$$P_3 = P_1^{P_0} + mP_2^{P_0}$$

We have

$$J_3(\omega) = \prod_{3\le P} (P^2 - 3P + 3 - \chi(P)) \ne 0,$$

where $\chi(P) = -P + 2$ if $P|m$; $\chi(P) = (P_0 - 1)P - P_0 + 2$ if $m^{\frac{P-1}{P_0}} \equiv 1$ (mod P); $\chi(P) = -P + 2$ if $m^{\frac{P-1}{P_0}} \not\equiv 1 (\text{mod } P)$; $\chi(P) = 1$ otherwise.

Since $J_n(\omega) \ne 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime.

We have

$$\pi_2(N,3) \sim \frac{J_3(\omega)\omega}{2P_0\Phi^3(\omega)} \frac{N^2}{\log^3 N}.$$

The Polynomial $P_1^n + (P_2+1)^2$ Captures Its Primes [14]

（1）Let $n = 4$, We have

$$P_3 = P_1^4 + (P_2+1)^2,$$

We have Jiang function

$$J_3(\omega) = \prod_{3\le P} (P^2 - 3P + 3 - \chi(P)) \ne 0,$$

Where $\chi(P) = P$ if $P \equiv 1$ (mod 4); $\chi(P) = P - 4$ if $P \equiv 1$（mod 8）; $\chi(P) = -P + 2$ otherwise.

Since $J_n(\omega)\neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime.

We have the best asymptotic formula

$$\pi_2(N,3)=\left|\{P_1,P_2: P_1,P_2\le N, P_1^4+(P_2+1)^2 \text{ prime}\}\right|$$

$$\sim \frac{J_3(\omega)\omega}{8\Phi^3(\omega)}\frac{N^2}{\log^3 N}.$$

It is the simplest theorem which is called Friedlander-Iwaniec problem [16].

（2）Let $n=4m$, We have

$$P_3=P_1^{4m}+(P_2+1)^2,$$

where $m=1,2,3,\cdots$.

We have Jiang function

$$J_3(\omega)=\prod_{3\le P\le P_i}(P^2-3P+3-\chi(P))\neq 0,$$

where $\chi(P)=P-4m$ if $8m|(P-1)$; $\chi(P)=P-4$ if $8|(P-1)$; $\chi(P)=P$ if $4|(P-1)$; $\chi(P)=-P+2$ otherwise.

Since $J_3(\omega)\neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime. It is a generalization of Euler proof for the existence of infinitely many primes.

We have the best asymptotic formula

$$\pi_2(N,3)\sim \frac{J_3(\omega)\omega}{8m\Phi^3(\omega)}\frac{N^2}{\log^3 N}.$$

（3）Let $n=2b$. We have

$$P_3=P_1^{2b}+(P_2+1)^2,$$

where b is an odd.

We have Jiang function

$$J_3(\omega)=\prod_{3\le P}(P^2-3P+3-\chi(P))\neq 0,$$

where $\chi(P)=P-2b$ if $4b|(P-1)$; $\chi(P)=P-2$ if $4|(P-1)$; $\chi(P)=-P+2$ otherwise.

We have the best asymptotic formula

$$\pi_2(N,3) \sim \frac{J_3(\omega)\omega}{4b\Phi^3(\omega)} \frac{N^2}{\log^3 N}.$$

（4）Let $n = P_0$, We have

$$P_3 = P_1^{P_0} + (P_2 + 1)^2,$$

where P_0 is an odd prime.

We have Jiang function

$$J_3(\omega) = \prod_{3 \le P} (P^2 - 3P + 3 - \chi(P)) \ne 0,$$

where $\chi(P) = P_0 + 1$ if $P_0 \mid (P-1)$; $\chi(P) = 0$ otherwise.

Since $J_3(\omega) \ne 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is also a prime.

We have the best asymptotic formula

$$\pi_2(N,3) \sim \frac{J_3(\omega)\omega}{2P_0\Phi^3(\omega)} \frac{N^2}{\log^3 N}.$$

The Jiang function $J_n(\omega)$ is closely related to the prime distribution. Using $J_n(\omega)$ we are able to tackle almost all prime problems in the prime distribution.

Acknowledgements

The Author would like to express his deepest appreciation to R. M. Santilli,G. Weiss, L. Schadeck, A. Connes, M. Huxley and Chen I-wan for their helps and supports.

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